$\frac{1}{2} \times \overline{Zx + ZP - Px} : \overline{\text{cos. } \frac{1}{2} xZP}^2$; hence, the azimuth xZP from the north is known.

Ex. Given the lat. 34°. 55′ N. sun's declination 22°. 22′. 57″ N, and true altitude 36°. 59′. 39″, to find the apparent time. Here, $ZP = 55°. 5'$, $Zx = 53°. 0'. 21''$, $Px = 67°. 37'. 3''$; hence,

$Px =$ 67°. 37′. 3″	arith. comp. of sine		0,034019
$ZP =$ 55. 5. 0	arith. comp. of sine		0,086193
$Zx =$ 53. 0. 21			
Sum 175. 42. 24.			
$\frac{1}{2}$ Sum 87. 51. 12	sine	- -	9.999694
$Zx =$ 53. 0. 21			
Dif. 34. 50. 51	sine	- -	9.756932
			2)19.876838
			9.938419 the cosine of

29°. 47′. 44″ half the angle ZPx, $\therefore ZPx = 59°. 35'. 28''$, which reduced into time gives 3*h*. 58′. 22″ the time from apparent noon.

93. If the error in altitude be given, we may thus find the error in time. Let mn be parallel to the horizon, and nx represent the error in altitude; then as the calculation of the time is made upon supposition that there is no error in the declination, we must suppose the body to be at m instead of x, and consequently the angle mPx, or the arc qr, measures the error in time.

Now $nx : xm :: \text{sin. } nmx : \text{rad.}$
$xm : qr :: \text{cos. } rx : \text{rad.}$ (13).

hence, $nx : qr :: \text{sin. } nmx \times \text{cos. } rx : \text{rad.}^2 \therefore qr = nx \times \frac{\text{rad.}^2}{\text{sin. } nmx \times \text{cos. } rx}$; but $ZxP = nmx$, nxm being the complement of both; also sin. ZxP, or nmx : sin. ZP :: sin. xZP : sin. xP, or cos. rx, $\therefore$ sin. $nmx \times$ cos. $rx =$ sin. $ZP \times$ sin. xZP; hence, $qr = nx \times \frac{\text{rad.}^2}{\text{sin.} ZP \times \text{sin.} xZP.} = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azi.}}$ Hence, the error is least on the prime vertical. All altitudes therefore to deduce the time from, ought to be taken on, or as near to, the prime vertical as possible. In lat. 52°. 12′, if the error in alt. at an azi. 44°. 22′ be 1′, then $qr = 1' \times \frac{1^2}{,612 \times ,699} = 2',334$ of a degree $= 9'',336$ in time.

Hence, the perpendicular ascent of a body is quickest when it is on the prime vertical, for nx varies as sin. azi. when qr and the lat. are given.

94. Given the latitude of the place and the sun's declination, to find the

time when twilight begins. Twilight is here supposed to begin when the sun is 18° below the horizon; hence draw the circle hyk parallel to the horizon and 18° below it, and twilight will begin when the sun comes to y, and $Zy=108°$; hence, sin. $yP \times$ sin. ZP : rad.^{2} :: sin. $\frac{1}{2} \times \overline{PZ+Py+108°} \times$ sin. $\frac{1}{2} \times \overline{PZ+Py-108°}$: $\overline{\text{cos. } \frac{1}{2} yPZ}^2$, hence yPZ is known, which converted into time gives the time from *apparent* noon.

95. To find the time when the apparent diurnal motion of a fixed star is perpendicular to the horizon. Let yx be the parallel described by the star; draw the vertical circle Zh touching it at o, and when the star comes to o its motion is perpendicular to the horizon; and as the angle ZoP is a right one, we have, (*Trig. Art.* 212.) rad. : tan. oP :: cot. PZ : cos. ZPo, that is, *rad. : cot. dec. :: tan. lat. : cos. ZPo*, which converted into time (Tab. 1.) gives the time from the star's being on the meridian. Hence, the time of the star's coming to the meridian being found by Art. 105. the time required will be known.

96. To find the time of the shortest twilight. Let ab be the parallel of the sun's declination at the time required, draw cd indefinitely near and parallel to it, and TWa parallel to the horizon 18° below it; then vPw, sPt measure the twilight on each parallel of declination, and when the twilight is shortest, the increment of the hour angle being $=0$, these must be equal; hence, $vPr=wPz$, therefore $vr=wz$; and as $rs=tz$, and the angles r and z are right ones, $rvs=zwt$; but $Pvr=90°=Zvs$, take Zvr from both, and $PvZ=rvs$; for the same reason $PwZ=zwt$; hence, $PvZ=PwZ$. Take $ve=wZ=90°$, then as $Pv=Pw$, and the angle $Pve=PwZ$, therefore $Pe=PZ$; let fall the perpendicular Py and it will bisect the base eZ. Then (Trig. Art. 212.) cos. $Py=\dfrac{\text{cos. } Pv}{\text{cos. } vy}=\dfrac{\text{cos. } Pv}{\text{sin. } ey}$; also, cos. $Py=\dfrac{\text{cos. } Pe}{\text{cos. } ey}=\dfrac{\text{cos. } PZ}{\text{cos. } ey}$; hence, $\dfrac{\text{cos. } Pv}{\text{sin. } ey}=\dfrac{\text{cos. } PZ}{\text{cos. } ey}$, $\therefore$ cos. Pv, or sin. hv, $=$cos. $PZ \times \dfrac{\text{sin. } ey}{\text{cos. } ey}=$cos. $PZ \times$ tan. ey, hence, rad. : cos. PZ, or sin. lat. :: tan. $ey=9°$: sin. hv the sun's declination at the time of shortest twilight. Because PZ is always less than 90°, and $Zy=9°$, therefore Py is always less than 90°, and therefore its cosine is positive; also, vy is always greater than 90°, therefore its cosine is negative; hence, cos. Pv ($=$cos. $Py \times$ cos. vy) is negative, consequently Pv is greater than 90°, therefore the sun's declination is *south*. This is M. Cagnoli's Investigation.

97. To find the length of the shortest twilight. As $wPZ=vPe$, therefore $ZPe=vPw$ measuring the shortest time. Now sin. PZ, or cos. lat. : rad. :: sin. $Zy=9°$: sin ZPy, which doubled gives ZPe, or vPw, which converted into time gives the length of the shortest twilight.

Ex. To find the time of the year at Cambridge, when the twilight is shortest; and the length of that twilight.

Rad. - - - - - - - - - - - - - - -	10,0000000
Sin. 52°. 12'. 35" - - - - - - - - - - -	9,8977695
Tan. 9°. - - - - - - - - - - - - - - -	9,1997125
Sin. 7°. 11'. 25" dec. - - - - - - - - -	9,0974820

This declination of the sun gives the time about March 2, and October 11.

Cos. 52°. 12'. 35" - - - - - - - - - - -	0,2127004 *A.C.*
Sin. 9° - - - - - - - - - - - - - - -	9,1943324
Rad. - - - - - - - - - - - - - - - -	10,0000000
Sin. 14°. 47'. 27" - - - - - - - - - - -	9,4070328

The double of this gives 29°. 34'. 54", which converted into time gives 1*h*. 58'. 20" for the duration of the shortest twilight, it being supposed to end when the sun is 18° below the horizon.

98. To find the sun's declination when it is just twilight all night. Here the sun at a must be 18° below the horizon; hence, $18° + \text{dec. } Qa = RQ = EH =$ comp. of lat. of place, and the sun's dec. = comp. lat. − 18°; look therefore into the *Nautical Almanac*, and see on what days the sun has this declination, and you have the time required. The sun's greatest declination being 23°. 28', it follows, that if the complement of latitude be greater than 41°. 28', or if the latitude be less than 48°. 32', there can never be twilight all night. If the sun be on the other side of the equator, then its dec. = 18°. − comp. lat. FIG. 5.

99. If the spectator be between E and L, and the sun's declination Ee be greater than EZ, then the sun comes to the meridian at e to the *north* of its zenith; and if we draw the secondary Zqm touching the parallel ae of declination described by the sun, then Rm is the greatest azimuth from the north which the sun has that day, the azimuth increasing till the sun comes to q, and then decreasing again, and the sun has the same azimuth twice in the morning. If therefore we draw the straight line Zv perpendicular to the horizon, the shadow of this line, being always opposite to the sun, would first recede from the south point H and then approach it again in the morning, and therefore would go backwards upon the horizon. But if we consider PP' as a straight line, or the earth's axis produced, the shadow of that line would not go backwards upon that plane, because the sun always continues to revolve about that line, and therefore its shadow must always go forwards; whereas the sun does not revolve about the perpendicular Zv. Hence it appears, that FIG. 11.

the shadow of the sun upon a dial can never go backwards, because the gnomon of a dial is parallel to PP', and therefore the sun must always revolve about the gnomon. The time when the azimuth is greatest is found from the right angled triangle PqZ, by saying, rad. : tan. qP :: cot. PZ : cos. ZPq, or *rad.* : *cot. dec.* :: *tan. lat.* : *cos.* PZq the hour angle from apparent noon.

100. It has hitherto been supposed, that it is 12 o'clock when the sun comes to the meridian, and that the clock goes just 24 hours in the interval of the sun's passage from any meridian till it returns to it again. But if a clock be thus adjusted for one day, it will not continue to show 12 o'clock every day when the sun comes to the meridian, because the intervals of time from the sun's leaving any meridian till it returns to it again, are not always equal; this difference between the sun and the clock is called the *Equation of Time*, as will be explained in Chap. IV. Hence, when the clock does not agree with the sun, any arc xe is not the measure of the time from 12 o'clock, but from the time when the sun comes to the meridian, or from *apparent* noon*.

101. In the same manner as we find the hour angle for the sun, we may also find it for any fixed star or planet, its altitude and declination being given; but when the hour angle is thus found, it is necessary to know the time when the body is upon the meridian in order to find the time from thence, the hour angle being the distance from the meridian; also the method of reducing the hour angle into time will be different. For let E be the earth, $rmsn$ the equator, sr a meridian passing through a fixed star S reduced to the equator; then as the meridian returns to the star in $23h.\ 56'.\ 4''$ after leaving it (127), we have $360°$: hour angle :: $23h.\ 56'.\ 4''$: time from the meridian. Now let P be a planet, and the meridian mn to pass through it; then the meridian will return to that position again in $23h.\ 56'.\ 4''$; now let Pv or Pv' be the planet's motion in right ascension in one day, according as its motion is direct or retrograde, and reduce this into time (t) at the rate of $15°$ for an hour, which will be sufficiently exact for so small an arc, then the meridian returns to the planet again after an interval of $23h.\ 56'.\ 4'' \pm t$; hence, the meridian, after leaving the planet, approaches it at the rate of that time for $360°$, because when the meridian leaves the planet it is then approaching a point $360°$ from it; hence, $360°$: hour angle :: $23h.\ 56'.\ 4'' \pm t$: time from the meridian.

* The conversion of the hour angle into time for the sun at the rate of $15°$ for an hour, by a clock adjusted to mean solar time, is not accurate, because the solar days are not all accurately equal to 24 hours, but to $24h. \pm$ the variation (e) of the equation of time for that day, according as the equation is increasing or decreasing; hence, to reduce the hour angle to give accurately the time from apparent noon, say, $360°$: hour angle ($a°$) :: $24h. \pm e$: time $= \frac{a°}{360°} \times \overline{24h. \pm e}$; for, in this case, the meridian, instead of returning to the sun in $24h.$ returns to it in $24h. \pm e$. This quantity e is sometimes $30''$, and therefore if $a° = 60°$, the correction would be $5''$. A clock is adjusted to *mean* solar time,

102. The hour angle which we have hitherto found for the time at which a body rises, has been upon supposition that the body is upon the rational horizon at the instant it appears; but all bodies in the horizon are elevated by refraction 33′ above their true places; this therefore would make them appear when they are 33′ below the rational horizon, or $90° + 33'$ from the zenith; also, all the bodies in our system are depressed below their true places by parallax, as will be afterwards explained, therefore from this cause they would not appear till they were elevated above the rational horizon by a quantity equal to their horizontal parallax, or when distant from the zenith 90° − hor. par. Hence, from both causes together, a body becomes visible when its distance ZV from the zenith $= 90° + 33'$ − hor. par. V being the place of the body when it becomes visible, Z the zenith and P the pole; hence, knowing ZV, also ZP the complement of latitude and PV the complement of declination, we can find the hour angle ZPV. A fixed star has no parallax, therefore $ZV = 90°.\ 33'$. FIG. 13.

103. If the body sensibly alter its declination in a few hours, as the moon does, the time of its rising may be thus found. Let w be the place of the moon on the meridian, v when in the horizon, and d the point when it becomes visible; draw ade parallel to EQ, and ew is the change of declination in the time from rising to the meridian. Now from knowing the time (105) of passing the meridian, and the declination at noon, with the change of declination in the interval of the passages of the moon over the meridian by the Nautical Almanac, compute the change of declination in the interval between noon and the time of the moon's transit, and you will get the moon's declination at the time of its transit. To that declination compute the hour angle upon supposition that the declination continued the same as on the meridian, which will be nearly the angle wPd. From the Nautical Almanac find the change (v) of declination in the interval (t) of time from the moon's passage over the meridian till it returns to it again; then say, 360° : hour angle :: v : the change of declination in describing that angle, which added to or subtracted from the declination at the time of passing the meridian gives very nearly the declination at rising; to which compute the hour angle and convert it into time as before and subtract it from the time of passing the meridian, and it gives very nearly the time of rising; and if greater accuracy should be required, the operation may be repeated by taking this hour angle. FIG. 14.

Ex. To find at what time the moon rose at Greenwich on July 1, 1767. The latitude of Greenwich is $51°.\ 28'.\ 40''$, and (105) the moon passed the meridian at $4h.\ 2'.\ 9''$; now $t = 24h.\ 40'$, and $v = 5°.\ 28'$; hence, $24h.\ 40' : 4h.\ 2'.\ 9'' :: 5°.\ 28' : 53'.\ 38''$ the change of declination in $4h.\ 2'.\ 9''$, which, as the declination is decreasing, subtracted from $5°.\ 22'$, the moon's north declination at noon, leaves $4°.\ 28'.\ 22''$ for the moon's declination when it was on the meridian; hence we take $Pd = 85°.\ 31'.\ 38''$, also $PZ = 38°.\ 31'.\ 20''$; and as the

moon's hor. parallax $= 54'. 21''$, and refraction $33'$, we have $Zd = 89°. 38'. 39''$, hence the angle $ZPd = 95°. 3'. 50''$. Hence, $360° : 95°. 3'. 50'' :: 5°. 28' : 1°. 26'. 37''$ the change of declination in the time of describing $95°. 3'. 50''$, which added to $4°. 28'. 22''$ gives $5°. 54'. 59''$ for the declination at the time of rising, very nearly; hence, $Pd = 84°. 5'. 1''$, therefore the angle $ZPd = 96°. 54'. 2''$; hence, $360° : 96°. 54'. 2'' :: 24h. 40' : 6h. 38'. 22''$ the time of describing the angle ZPd, which subtracted from $4h. 2'. 9''$, the time when the moon was on the meridian, gives the time of rising $21h. 23'. 47''$, answering to $9h. 23'. 47''$ in the morning apparent time.

104. In determining the time when any body rises, or when it is at any known altitude or position, it has been supposed that we know the time at which it comes to the meridian; the determination of this circumstance must therefore be next explained.

105. Let a clock be adjusted to mean solar time, which we may therefore consider as the time from the sun's leaving the meridian till it returns to it again, where great accuracy is not required, the difference being only the variation of the equation of time in 24 hours. Let S and P be the places of the sun and a planet reduced to the equator; then the meridian sr approaches the sun at the rate of $15°$ in an hour; for when it leaves the sun at S it may be considered as approaching a point at that time $360°$ from it, and which it comes up to in 24 hours; hence if any other point were moving forwards with the velocity of the sun, the meridian would approach it at the same rate. Therefore if the planet at P move forwards with a different velocity from that of the sun, the interval of their passages over any meridian will be the same as if we supposed the sun to be at rest and the planet to move with its own proper motion *minus* that of the sun, the planet's motion in right ascension being *greater* than that of the sun. Let x be the difference of their motions in right ascension in 24 hours reduced into time, and $t = SP$ reduced also into time in like manner, the planet being at P at the time the meridian passes through the sun at S; and let v be the place of the planet when the meridian overtakes it, and e be the arc Pv in time; then the motions of the meridian will be 24 and $t + e$, and of the planet in the same times x and e; hence, as we may consider each motion as uniform, $24 : x :: t + e : e, \therefore 24 - x : x :: t : e = \dfrac{tx.}{24 - x}$. This is the case if the planet's motion be greater than the sun's, but if the sun's be greater, then x itself becomes negative, and therefore $-x$ will be positive; hence $e = \dfrac{-tx}{24 + x}$; therefore $t + e = t \pm \dfrac{tx}{24 \mp x} = \dfrac{24t}{24 \mp x}$ the time from apparent noon when the planet passes the meridian, where the upper or lower sign prevails according as the planet's or sun's motion is greatest. If the motion of the planet in right ascension be retrograde, it is manifest that x is the

sum of the motions of the planet and sun in 24 hours, for the bodies moving in opposite directions they approach each other with the *sum* of their motions; let therefore v' be the place of the planet when it comes to the meridian, then the motion of the meridian from its passage through the sun to the planet will be $t-e$; hence $24 : x :: t-e : e$, therefore $24+x : x :: t : e = \frac{tx}{24+x}$; hence, the time required $= t - \frac{tx}{24+x} = \frac{24t}{24+x}$. But as the division by $24 \mp x$ is not so convenient as it would be by 24, therefore resolve $\frac{24t}{24 \mp x}$ into $t \pm \frac{tx}{24} + \frac{tx^2}{24^2} \pm$ &c. where the two first terms will be sufficient for all cases except the moon, where it will be necessary to take the third. For a fixed star, x will represent the increase of the sun's right ascension in 24 hours, and the time required $= \frac{24\,t}{24+x} = t - \frac{t\,x}{24}$. By this method we find, very nearly, the time at which any body comes to the meridian, and hence, by the last articles, we may find the time of its rising, or the time at any given altitude.

Ex. To find the time of the moon's passage over the meridian at Greenwich on July 1, 1767. The sun's *AR.** when on the meridian that day was $6h.\ 40'.\ 25''$, and its daily increase $4'.\ 48''$; also, the moon's *AR.* was $10h.\ 36'.\ 8''$, and its daily increase $42'.\ 28''$. Hence, $t = 10h.\ 36'.\ 8'' - 6h.\ 40'.\ 25'' = 3h.\ 55'.\ 43'' = 3{,}9285$ (Tab. 3.), also, $x = 42'.\ 28'' - 4'\ 48'' = 37'.\ 40'' = 0{,}6277$; hence, $\frac{t\,x}{24} = 6'.\ 10''$; $\frac{tx^2}{24^2} = 10''$; therefore $t + \frac{t\,x}{24} + \frac{t\,x^2}{24^2} = 4h.\ 2'.\ 3''$ the apparent time of passing the meridian.

Where great accuracy of time is required from an observed altitude, the body made use of must be the sun or a fixed star. The method of finding the time by the sun has been already explained (92); and the time by a star may be found by the following method.

106. Find the star's true altitude, and take its declination from the 7th of the Requisite Tables, or from any other tables if it be not there; then in the triangle ZPx (x representing the place of the star) we have ZP the complement of latitude, Px the complement of declination and Zx the complement of the star's altitude, to find the angle ZPx, the star's distance from the meridian, which convert into time. Now the point of the equator which is upon the meridian at any time, is called the *mid-heaven;* therefore the angle ZPx measures the star's distance from the mid-heaven. Hence, if the star be to the *east* of the meridian, *subtract* its distance from the meridian from its *AR.* (adding, if necessary, 24 hours to its *AR.*) and the difference is the *AR.* of the mid-heaven: But if the star be to the *west, add* them together (sub-

FIG. 8.

* *AR.* means right ascension.

tracting 24 hours from the sum, if greater,) and the sum gives the *AR.* of the mid-heaven*. Then find the sun's *AR* at the preceding noon at Greenwich from the Nautical Almanac, and from thence at noon at the given place by the 23d of the Requisite Tables, and subtract it from the *AR.* of the mid-heaven (adding 24 hours to the latter†, if necessary), and the difference would be the apparent time from the preceding noon, or the estimate time, if the sun had had no motion in that time; but as it has moved, find that motion by the 23d of the Requisite Tables, and subtract it, and it gives the *apparent* time required.—Hence, if we apply the equation of time it gives the true time, which compared with the watch, shows how much it is too fast or too slow; and by repeating the observations, the rate of going of the watch may be determined; but this will be further explained in Chap. IV.

Ex. On April 14, 1780, lat. 48°. 56′. N. lon. 66°. W. the true altitude of Aldebaran west of the meridian was 22°. 17′. 50″; to find the apparent time.

Sun's *AR.* for noon at Greenwich by the Nautical Almanac	$1h.\ 31'.\ 1''$
Corrected for the Long. by the 23d of the Requisite Tables ‡	+41
Sun's *AR.* at noon at the given place - - - - - - - - - -	1.31.42

Also by Requisite Table 7. the star's dec. is 16°. 3′ N. Hence $ZP = 41°.\ 4'$, $Zx = 67°.\ 42'.\ 10''$, $xP = 73°.\ 57'$; hence by sph. trig.

$$\begin{aligned} Px &= 73°.\ 57'.\ 0'' \quad \text{arith. comp. of sine } 0.017304 \\ ZP &= 41\ .\ 4\ .\ 0 \quad \text{arith. comp. of sine } 0.182476 \\ Zx &= 67\ .\ 42\ .\ 10 \\ \text{Sum} &= 182\ .\ 43\ .\ 10 \end{aligned}$$

* That this is true for every position of the point aries and place of the star, may be thus shown. Let *E♎* represent the equator, *E* the point on the meridian, ♈, ♈′, ♈″, different positions of the point aries, in respect to the place *A*, *A′* of the star referred to the equator, *A* on the western side of the meridian, and *A′* on the eastern; *B* the point to which the sun is referred; ♈*EB*♎ the direction in which the right ascension is measured. Now suppose the star at *A′*, to the east of the meridian; then, 1. ♈*A′*—*A′E*=♈*E*. 2. ♈″*A′*—*A′E*=—♈″*E*=—24*h*.+♈″ ♎*E*, ∴ ♈″*A*+24*h*.—*A′E*=♈″♎*E*. Now suppose the star at *A*, on the west side; then 1. ♈*A*+*AE*=♈*E*. 2. ♈′♎*A*+*AE*—24*h*.=♈′♎*A*+*A*♈′+♈′*E*—24*h*.=♈′*E*, because ♈′*A*♎*A*+*A*♈′=24*h*.

† For, 1. ♈″*B*♎*E*—♈″*B*=*E*♎*B*. 2. ♈*E*+24*h*.—♈*B*=♈*E*+*EA′B*+*E*♎*B*—♈*B*=*E*♎*B*, because ♈*E*+*EA′B*=♈*B*.

‡ The daily variation of the sun's *AR*, with which you enter the *Requisite Tables*, is taken from the *Nautical Almanac.*

$\frac{1}{2}$ Sum = 91 . 21 . 35 sine - - - - - - 9.999874
Zx = 67 . 42 . 10

Dif. = 23 . 39 . 25 sine - - - - - - 9.603425

2)19.803079

9.901539 the cosine of 37°. 8′. 29″; hence the angle xPZ (or in FIG. 15. the arc AE) = 74°. 16′. 58″, or in time = 4h. 57′. 8″; hence,

Star west of merid. -	4h. 57′. 8″	Estimate Time - -	7h. 48′. 46″
Star's AR. by Req. Tab. 7.	4 . 23. 20	Correc. from Req. Tab. 23.	− 1. 12
AR. of mid-heaven -	9 . 20. 28	Apparent Time required	7. 47. 34
Sun's AR. at noon -	1 . 31. 42		

107. The time of the passage of a star over the meridian may be found (78) from taking the times at which it had equal altitudes on each side of the meridian, and bisecting the interval. If equal altitudes be taken at 8 and 11 o'clock, the star was upon the meridian at half past 9 o'clock. But for the sun this will want a correction, owing to its change of declination, on which account it is not at equal altitudes when equidistant from the meridian. If be be the diurnal arc described by the sun in its ascent to the meridian, and ed in its descent from it, and mn be drawn parallel to HOR, then the sun is at equal altitudes at m and n, and the angle mPn, or the arc qr, measures the difference of the times at m and n from the meridian; when we therefore bisect the interval of the times at which the sun was at m and n, we must correct it by half mPn, or half qr, in order to get the time at which it comes to the meridian. This correction is called the *equation of equal altitudes*. Now (Trig. Art. 264.) if d'' = the variation of the sun's dec. in the interval of the observations, t = tan. lat. v = tan. decl. at noon, s = sine, r = tan. of the hour angle from noon at the time of the observation, taking the half interval of times for the measure of that angle; then $\frac{1}{2}qr = \frac{1}{2}d'' \times \frac{t}{s} \pm \frac{v}{r}$, radius being unity; or as the value of d'' in time is $\frac{d''}{15''}$ seconds, estimated at the rate of 15° for 1 hour, or 15″ for 1 second of time, therefore the correction $= \frac{d}{30} \times \overline{\frac{t}{s} \pm \frac{v}{r}}$ seconds of time, where the sign − is to be used when the lat. and decl. are both north or both south, and + when one is north and the other south. Now in north latitude, when the sun approaches the north pole, or is in the 9th. 10th. 11th. 0th. 1st. 2nd. signs, it is manifest FIG. 16.

from the figure, that the sun, after passing the meridian, will not come to the same altitude as at the observation before, until it be at a greater distance from the meridian; therefore the middle point of time between the observations must be, when the sun has passed the meridian, and the correction must be *subtracted.* When the sun is in the other signs, receding from the north pole, it comes to the same altitude at a less distance from the meridian; therefore the middle point of time must be, before the sun comes to the meridian, and consequently the correction must be *added.* To facilitate this computation, Mr. WALES constructed and computed a set of tables which were published in the Nautical Almanac for 1773; these tables are called *Equation to corresponding altitudes.*

To find the Time the Sun is passing the Meridian, or the horizontal or perpendicular Wire of a Telescope.

108. Let mx be the diameter d'' of the sun, estimated in seconds of a great circle; then, (as the minutes in mx, considered as a small circle, must be greater in proportion as the radius is less, because, when the arc is given, the angle is inversely as the radius), sin. Px, or cos. dec. rx : rad. ∵ seconds d'' in mx of a great circle : the seconds in mx of the small circle ea, which is equal to (13) the seconds in qr = the angle rPq, and therefore the angle $rPq = d''$ divided by cos. dec. (rad. being unity) $= d'' \times$ sec. dec., which measures the time the sun is passing over its diameter, and consequently the time the diameter would be in passing over the meridian; hence (as in Art. 107), the time of passing the meridian $= \dfrac{d'' \times \text{sec. dec.}}{15''}$

Hence qr, the sun's diameter in right ascension, is equal to $d'' \times$ sec. dec. If therefore the sun's diameter $= 32' = 1920''$, and its dec. 20°, its diameter in right ascension $= 1920'' \times 1{,}064 = 34', 2'', 88$. The same will do for the moon, if d'' = its diameter.

109. By Art. 93. $qr = nx \times \dfrac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azi.}} =$ (if $nx = d''$ the sun's diam.) $d'' \times \dfrac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azi.}}$; hence, as before, the time of describing qr, or the time in which the sun ascends perpendicularly through a space equal to its diameter, or the time of passing an horizontal wire, is equal to $\dfrac{d''}{15''} \times \dfrac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azi.}}$ The same expression must also give the time which the sun is in rising. If $d'' = 1980''$ the horizontal refraction, then d'' divided by $15'' = 132''$; hence, refraction accelerates the rising of the sun by $132'' \times \dfrac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azi.}}$

110. The sin. nxm : sin. nmx :: mn : $nx = mn \times \frac{\text{sin. } nmx}{\text{sin. } nxm}$; hence (93), $qr = mn \times \frac{\text{sin. } nmx}{\text{sin. } nxm} \times \frac{\text{rad.}^2}{\text{sin. } nmx \times \text{cos. } rx} = mn \times \frac{\text{rad.}^2}{\text{sin. } nxm \times \text{cos. } rx}$; and if $mn = d''$, we find the time, in which the horizontal motion of the sun is equal to its diameter, to be $\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. } ZxP \times \text{cos. dec}}$, which is therefore the time in which the sun would pass the vertical wire of a telescope.

Dr. MASKELYNE's *Rules to find the Time of the Passage of a Star or Planet from one Wire to another of a transit Instrument.*

111. For a *fixed Star.* Multiply the equatorial interval of time by the secant of the star's declination, and you have the time required. For an arc of the equator, measured on a small circle parallel to it, subtends a greater angle about the earth's axis, in the proportion of rad. : cor. dec. or sec. dec. : radius.

For the *Sun.* Increase the equatorial time of a star by the 365th part (owing to the sun's motion in that time) and you have the equatorial time by the sun; then proceed as for a star.

For a *Planet,* except the moon. Take the difference (d) of $23h.\ 56'$, and the interval of two successive transits of the planet over the meridian, as given in the Nautical Almanac; then say, $24h.$: d :: the time of the passage of a star having the same declination : a fourth number, which added to or subtracted from the time of the passage of a star, according as the interval of the two successive transits is more or less than $23'.\ 56''$, gives the time of the planet's passage.

For the *Moon.* Put $n =$ the equatorial interval by a star, $r =$ daily retardation of the moon's passage over the meridian in minutes; then allowing for the moon's motion, $23h.\ 56'$: $1440' + r'$ :: $n \times \frac{1440' + r'}{23h.\ 56'}$ the time in the equator from wire to wire, seen from the earth's center. Now the time of the image from wire to wire, is *cæteris paribus*, as the angle subtended by the interval of the wires at the object glass, or as its vertical angle, or the angle described by the moon about the supposed place of observation; but the velocity of the moon and the angle described being given, the arc, and therefore the time, is as the distance; hence, the time seen from the center of the earth $(n \times \frac{1440' + r'}{23h.\ 56'})$: time at the spectator :: ☾'s dist. from center : ☾'s dist. from spectator :: sin. ap. Zen. dist. : sin. true zen. dist. therefore the interval of time (t) at the spectator =

$n \times \frac{1440' + r'}{23h.\ 56'} \times \frac{s.\ tr.\ zen.\ dist.}{s.\ ap.\ zen.\ dist.} \times$ sec. ☾'s dec.; hence, Log. $t. = 6{,}84278 + l.\ n +$ L. $(1440 + r) + l.$ Req. Tab. IX. $+ l.$ sec. ☾'s dec. $- 30$.

On the Principles of Dialling.

112. As the apparent motion of the sun about the axis of the earth is at the rate of 15° in an hour, very nearly, let us suppose the axis of the earth to project its shadow into the meridian opposite to that of the sun, and then this meridian will move at the rate of 15° in an hour. Hence, let $zPRpH$ represent a meridian on the earth's surface, POp its axis, z the place of the spectator, $HKRV$ a great circle of which z is the pole; draw the meridians $P1p$, $P2p$, *&c.* making angles with PRp of 15°, 30°, *&c.* respectively; then supposing PR to be the meridian into which the shadow of PO is projected at 12 o'clock, $P1$, $P2$, *&c.* are the meridians into which it is projected at 1, 2, *&c.* o'clock, and the shadow will be projected on the plane $HKRV$ in the lines OR, $O1$, $O2$, *&c.*, and the arcs $R1$, $R2$, *&c.* will measure the angles $RO1$, $RO2$, *&c.* between the 12 o'clock line and the 1, 2, *&c.* o'clock lines. Now in the right angled triangle $PR1$, we have PR (84) the latitude of the place, and the angle $RP1 = 15°$; hence, rad. : tan. 15° :: sin. PR : tan. $R1$; in the same manner we may calculate the arcs $R2$, $R3$, *&c.* In this case we make the earth's axis the gnomon, and the shadow is projected upon the plane $HKRV$. But if we take a plane $abcd$ at z parallel to $HKRV$, and consequently parallel to the horizon at z, and draw zr parallel to POp, then on account of the great distance of the sun we may conceive it to revolve about zr in the same manner as about Pp, and consequently the shadow will be projected upon the plane $abcd$ in the same manner as the shadow of PO is projected upon the plane $HKRV$, and therefore the hour angles are calculated by the same proportion. This is an *horizontal* dial.

113. Now let $NLzK$ be a great circle perpendicular to $PRpHz$, and consequently perpendicular to the horizon at z, and the side next to H is full south. Then, for the same reason as before, if the angles $Np1$, $Np2$, *&c.* be 15°, 30°, *&c.* the shadow of pO will be projected into the lines $O1$, $O2$, *&c.* at 1, 2, *&c.* o'clock, and the angles $NO1$, $NO2$, will be measured by the arcs $N1$, $N2$, *&c.* Hence, in the right angled triangle $pN1$, $pN =$ the complement of the latitude, and the angle $Np1 = 15°$, therefore rad. : tan. 15° :: sin. pN : tan. $N1$; in the same manner we find $N2$, $N3$, *&c.* Hence, for the same reason as for the horizontal dial, if $zabc$ be a plane coinciding with $NLzK$, and st be parallel to Op, st will project its shadow in the same manner on the plane $zabc$ as Op does on the plane $NLzK$, and therefore the hour angles from the 12 o'clock line are computed by the same proportion. This is a *vertical south* dial. In the

same manner the shadow may be projected upon a plane in any position, and the hour angles be calculated.

114. In order to fix an horizontal dial, we must be able to tell the exact time of the sun's coming to the meridian; for which purpose, find the time (92) by the sun's altitude when it is at the solstices, because then the declination does not vary, and set a well regulated watch to that time; then when the watch shews 12 o'clock, the sun is on the meridian; at that instant therefore set the dial to 12 o'clock, and it stands right.

115. Hence we may easily draw a meridian line upon any horizontal plane. Suspend a plumb line so that the shadow of it may fall upon the plane, and when the watch shows 12, the shadow of the plumb line is the true meridian. The common way is to describe several concentric circles upon an horizontal plane, and in the center to erect a gnomon perpendicular to it with a small round well defined head, like the head of a pin; make a point upon any one of the circles where the shadow of the head, by the sun, falls upon it on the morning, and again where it falls upon the same circle in the afternoon; draw two radii from these two points, and bisect the angle which they form, and it will be a meridian line. This should be done when the sun is at the tropic, when it does not sensibly change its declination in the interval of the observation; for if it do, the sun will not (107) be equidistant from the meridian at equal altitudes. This method is otherwise not capable of very great accuracy, as, from the shadow not being very accurately defined, it is not easy to say at what instant of time the shadow of the head of the gnomon is bisected by the circle. If, however, several circles be made use of, and the mean of the whole taken, the meridian may be gotten with sufficient accuracy for all common purposes.

116. To find whether a wall be full south for a vertical south dial, erect a gnomon perpendicular to it and hang a plumb line from it; then when the watch shows 12, if the shadow of the gnomon coincide with the plumb line, the wall is full south.

CHAP. III.

TO DETERMINE THE RIGHT ASCENSION, DECLINATION, LATITUDE AND LONGITUDE OF THE HEAVENLY BODIES.

Art. 117. THE foundation of all Astronomy is to determine the situation of the fixed stars, in order to find, by a reference to such fixed objects, the places of the other bodies at any given time, and from thence to deduce their proper motions. The positions of the fixed stars are found from observation, by finding their right ascensions and declinations by means of the transit telescope and astronomical quadrant, as explained in my *Treatise on Practical Astronomy;* and then by computation their latitudes and longitudes may be found.

118. Now as the earth revolves uniformly about its axis, the apparent motion of all the heavenly bodies, arising from this motion of the earth, must be uniform; and as this motion is parallel to the equator, the interval of the times, in which any two stars pass over any meridian, must be in proportion to the arc of the equator intercepted between the two secondaries passing through them, because (13) this arc of the equator contains the same number of degrees as the arc of any small circle parallel to it and comprehended between the same secondaries; and therefore, if one increase uniformly, the other must. Hence, the right ascension of stars passing the meridian at different times will differ in proportion to the difference of the times of their passing; and as the clock is supposed to go uniformly, we have the following rule: As the interval of the times of the passage of any fixed star over the meridian : the interval of the passage of any two stars :: 360° : their *apparent* difference of right ascensions; which corrected for their aberration in right ascension, gives their *true* difference of right ascensions. By the same method we may find the difference of right ascensions of the sun or moon, when they pass the meridian, and a star, and therefore if that of the star be known, that of the sun or moon will; which will be rendered more exact, if we compare them with several stars and take the mean; remembering to apply the star's aberration in right ascension to the *apparent*, in order to get the *true* difference. When we thus determine the sun's right ascension from that of a star, the sun's aberration, which in longitude is always 20″, is not here considered, because the sun's place in the tables is put down as affected by aberration; and the use of observing the sun's right ascension is to compare it with the tables in order to find their error.

119. Now to determine the right ascension of a fixed star, Mr. FLAMSTEAD proposed a method, by comparing the right ascension of the star with that of the sun when near the equinoxes, and having the same declination; and as this method has not been explained, we shall give a very full explanation thereof,

together with an example. Let $AGCKE$ be the equator, $ABCWE$ the ecliptic, FIG. 19.
S the place of the star, and Sm a secondary to the equator, and let the sun be at P, very near to A, when it is on the meridian, and take $CT = PA$, and draw PL, TQ perpendicular to AGC, and QL parallel to AC; then the sun's declination is the same at T as at P. Observe the meridian altitude of the sun when at P, and also the time of the passage of its center over the meridian; observe also at what time the star passes over the meridian, and then (118) find the apparent difference Lm of their right ascensions. When the sun approaches near to T, observe its meridian altitude for several days, so that on one of them, at t, it may be greater and on the next day, at e, it may be less than the meridian altitude at P, so that in the intermediate time it may have passed through T; and drawing tb, es perpendicular to $AGCE$, observe on these two days, the differences bm, sm of the sun's right ascension and that of the star; draw also sv parallel to Qo. Hence, to find Qb, we may consider the variation both of the right ascension and declination to be uniform for a small time, and consequently to be proportional to each other; hence, vb (the change of meridian altitudes in one day) : ob (the difference of the meridian altitudes at t and T, or the difference of declinations) :: sb (the difference of sm, bm found by observation) : Qb, which added to bm, or subtracted from it, according to the situation of m, gives Qm, to which add Lm, or take their difference, according to circumstances, and we get QL, which subtracted from AGC, or 180°, half the remainder will be AL the sun's right ascension at the first observation, to which add Lm and we get the star's right ascension at the same time. Instead of finding bQ, we might have found sQ, by taking $TQ - es$ for the second term, and from thence we should have gotten Qm. Thus we should get the right ascension of a star, upon supposition that the position of the equator had remained the same, and the apparent place of the star had not varied, in the interval of the observations. But the intersection of the equator with the ecliptic has a retrograde motion, called the Precession of the Equinoxes; also, the inclination of the equator to the ecliptic is subject to a variation, called the Nutation; and from the Aberration of the star, its apparent place is continually changing. The effects of all these circumstances in changing the right ascension of the star will be explained and investigated in their proper places. Now Tables VII. and VIII. (see Vol. II.) contain these corrections for 36 principal stars; that is, if the *mean* right ascension of any star be taken for the beginning of the year, and these corrections be applied to it, according to their signs, for any day, the result gives the *apparent* right ascension of the star for that day.

120. Let therefore $ABCE$ be the ecliptic, $AGCE$ the position of the equator at the first observation when the sun was at P, and $agcd$ the position of the equator at the time of the observation at the other equinox, and take $TC = PA$, FIG. 20.

and draw TQ perpendicular to $AGCE$, as before, and draw Qq parallel to ABC, and tqr perpendicular to $AGCE$; let Ae be also perpendicular to $agcd$. Now as the position of the equator and the apparent place of the star are altered in the time between the two observations, let m be the point where a secondary from the apparent place of the star to the equator at the first observation would cut it, and v the place at the second observation, and draw vw perpendicular to $AGCE$; then Am is the apparent right ascension of the star at the first observation, and av at the second. Also, the sun must be at t when it has the same declination tq at the second observation as it had at the first, and consequently qv is the apparent difference of right ascensions of the sun at t and star, which difference is found by observation in the same manner as the difference at T was before found, when the equator was fixed. Also, as $Qq = Cc = Aa$, and the angle $qQr = cCQ = Aae$, we have $Qr = ae = Aa \times \cos. Aae$. Now if we put M for the *mean* right ascension of the star at the beginning of the year, and S for the sum of all the corrections due at the time of the first observation, and s for the sum due at the second; then, from what we have already explained in the last article, $M + S = Am$, $M + s = av$, hence, if we take the former from the latter, supposing s to be greater than S, we have $s - S = av - Am = ae + ev - Aw - wm$ (m lying beyond w); but $ev = Aw$; hence, $s - S = ae - wm$; consequently $wm = ae - \overline{s - S}$. Now qv, or rw, is known, hence we know rm, and as Qr is known, Qm will be known; and as we also know Lm, we get the value of QL*, with which we proceed, as before, to get the star's right ascension. The great advantage of this method, is, that it does not depend upon any determination of the latitude of the place, declination of the sun or accuracy in the divisions of the instrument. If the latitude be known, we may find the declination from the meridian altitude, it being, from Art. 87, equal to the difference between the meridian altitude and the complement of latitude, and then one observation at the second equinox will be sufficient, because the daily variation of the declination and right ascension may be taken from the Nautical Almanac. Having thus determined the right ascension of one star, the right ascension of all the heavenly bodies may from thence be found (118).

If the right ascension of a star, which is not in these tables, should be required, the corrections must be computed by the Rules which we shall give in their proper places. If the right ascension of the star be first computed without considering these corrections, it will be sufficiently accurate to compute the corrections from, and then they may be applied.

* In all these cases, if you draw the figure and put the star in its proper place, and put m and w in their proper situations, which may be done by observing whether ew or Am be the greater, you will immediately see what quantities are to be added together, and what subtracted. This figure is drawn for the Example.

Ex. Let it be required to find the right ascension of *Pollux* on March 24, in the year 1768, from Dr. MASKELYNE's observations.

On March 24, Pollux passed the meridian at $7h.\ 31'.\ 38''$; and on the 25, at $7h.\ 31'.\ 37'',66$; on the same day the sun passed at $0h.\ 16'.\ 35'',5$; hence, the apparent difference of the *AR's.* of the sun and Pollux on the 24th, allowing for the error of the clock (122), was $7h.\ 15'.\ 2'',46 = 108°.\ 45'.\ 36'',9 = Lm.$ Now on March 24,

Appar. zen. dist. ☉ L. L. - -	49°. 58'. 58'',7
Semidiam. - - - - -	−16. 4, 4
Appar. zen. dist. ☉ cen. - - - -	49. 42. 54, 3
Parallax - - - - - -	− 6, 7
Refr. cor. for Bar. and Ther. - -	+ 1. 10, 4
True zen. dist. ☉ cen. - - -	49. 43. 58
True meridian altitude - - -	40. 16. 2

To find when the sun had the same meridian altitude, or declination, just before it came to the next equinox, let us take Sept. 18, on which we find,

Appar. zen. dist. ☉ L. L. - - -	50°. 8'. 37'',8
Semidiam. - - - - - -	− 15. 59, 4
Appar. zen. dist. ☉ cen. - - -	49. 52. 38, 4
Parallax - - - - - -	− 6, 7
Refr. cor. for Bar. and Ther. - -	+ 1. 5, 8
True zen. dist. ☉ cen. - - -	49. 53. 37, 5
True meridian altitude - - -	40. 6. 22, 5

As this altitude is less than that on March 24, the instant of time when the sun had the *same* declination as on the 24th must be *before* the 18th; therefore as the sun on the 18th had gotten beyond that point where its declination was the same as at P, we must, from the difference of the right ascensions of the sun and star observed on that day, subtract the increase of the sun's right ascension between the 18th and that point of time when the declination was the same as at P, in order to get the difference of the apparent right ascensions at the time when the sun's declination was the same as at P. We may also observe, that the difference of any two true meridian altitudes is the same as the difference of the declinations at the same times. Now as the sun's altitude was not observed on the 17th, we will take the change of declination for that day from the Nautical Almanac, which is $23'.\ 20''$; also, the increase of the sun's *AR.* for that day was $3'.\ 36''$ in time, or $54'$ in space. The difference of the

true meridian altitudes, or the difference of declinations on March 24, and Sept. 18, was 9′. 39″,5; hence, 23′. 20″ : 9′. 39″,5 :: 54′ : 22′. 21,″4, the increase of the sun's right ascension from the time before the 18th at which the declination was the same as on March 24, to the 18th. On Sept. 18, Pollux passed the meridian at 7*h*. 30′. 39″,9, and on the 19th at 7*h*. 30′. 40″. On the 18th the sun passed at 11*h*. 44′. 53″,33; therefore the apparent difference of the *AR*'s of the sun and Pollux on that day, allowing for the error of the clock (122), was 4*h*. 14′. 13″,5 = 63°. 33′. 22″,5, from which subtract 22′. 21″,4, and we have 63°. 11′. 1″,1 = *qv*. Now to get the correction in Table VIII. we must have the place of the moon's ascending node, which, from the Lunar Tables, is found to be 9ˢ. 17°. 45′. 28″ on March 24, and 9ˢ. 8°. 19′. 54″ on Sept. 18. Hence,

March 24,	Correction from Table VII. } red. to space	+	19″,2
	——— VIII. }	+	19, 5
		+	38, 7 = *S*
Sept. 18,	Correction from Table VII. } red. to space	+	31″,3
	——— VIII. }	+	20
		+	51, 3 = *s*

Hence, *s* − *S* = 12″,6.

Prec. of Equin. from March 24, to Sept. 18, Table XV. - - - - - - -	24″,9
Variation of the equat. of equinoxes, Table XVI.	+ 0, 7
True Precession in the interval - - - - - - - - - -	25, 6 = *Aa*
Cos. 23°. 28′ - - - - - - - - - - - - - - - - - -	,917
	23, 4 = *ae*

Hence, *mw* = 23″,4 − 12″,6 = 10″,8; therefore *rm* = *rw* − *mw* = *qv* − *mw* = 63°. 10′. 49″,3; to this add *Qr* = 23″,4, and we have *Qm* = 63°. 11′. 12″,7, which being added to *Lm* = 108°. 45′. 36″,9 we have *LQ* = 171°. 56′. 49″,6, which subtracted from 180°, half the difference is 4°. 1′. 35″,2 = *AL* the sun's right ascension on March 24, to which add *Lm* = 108°. 45′. 36″,9 and we get 112°. 47′. 12″,1 the apparent right ascension of Pollux at the same time; and if from this we subtract 38″,7 the equation at that time, we get 112°. 46′. 33″,4 for its *mean* right ascension. This conclusion differs a little from that determined by Dr. Maskelyne in Table VI, from the mean of seven observations.

121. But the method made use of by DR. MASKELYNE in settling the right ascensions of the stars, though founded upon the same principle as this of Mr. FLAMSTEAD, is different in its process, and procured him the advantage of a greater number of observations, both of the sun and stars, in the same time, and consequently enabled him to fix the right ascension of the stars with greater accuracy in a shorter time. He took α *Aquilæ* for his fundamental star, and assumed its right ascension as settled by Dr. BRADLEY, reducing it to the time of his observations by the mean precession, and afterwards making the following correction. By comparing a great many observed transits of such stars as he thought proper to select, with that of Aquilæ, in various parts of the year, and applying the proper equations, he obtained their mean right ascensions relative to that of α Aquilæ assumed, or affected with the same error; and comparing the transits of the sun near the equinoxes with those of the above mentioned stars observed on the same day, he obtained the sun's right ascension relative to that of α Aquilæ assumed. From the observed zenith distances of the sun on the same days, corrected for refraction, parallax and the error of the line of collimation, with the apparent obliquity of the ecliptic at the time, he deduced the sun's right ascensions: and then by comparing the sun's right ascensions deduced from the observed transits with those deduced from his observed zenith distances at equal or nearly equal declinations of the same kind near both equinoxes, he deduced the error of the assumed right ascension of α Aquilæ, which came out 3″,8 additive. He observed further, that in the interval of 12 years, which passed between the settling of Dr. BRADLEY's Catalogue about 1755 and his own about 1767, the precession in right ascension was diminished by 2″,16 by the action of the planets. Therefore if this had been allowed in assuming the right ascension of α Aquilæ from Dr. BRADLEY's determination, the correction of the right ascension of α Aquilæ would have come out 5″,96 additive, or at the rate of $\frac{1}{2}''$ a year, which agrees very well with the annual proper motion of α Aquilæ deduced from other observations. Dr. MASKELYNE has also given the following method.

Assume the *mean AR* of the star at the beginning of the year, and thence, by applying the equations, compute its *apparent AR* on two days of the year when the sun has nearly equal declinations on the same side of the equator, from two declinations observed; and then by the observed difference of the transits of the sun and star, compute the two apparent *AR's* of the sun and star; call this by *the star*. Correct the observed zenith distances of the sun by the correction of the line of collimation (if necessary), refraction and parallax, and you will obtain its apparent zenith distances, affected only by an error in the latitude of the place, making an error in the declination. To the mean obliquity of the ecliptic at the beginning of the year, apply the proportional part of the annual diminution, the correction for the day of the year,

and the equation depending on the place of the moon's node, and you will have the apparent obliquity, with which and the two declinations of the sun before found, compute the two AR's by the sun; call this by *the declination.* Subtract the sun's AR by *the star* from his AR by *the declination* near the vernal equinox, and call the difference a put down with its proper sign. Do the same for the autumnal equinox, and call the difference d. Then $\frac{1}{2}(a+b)$ is the correction of the mean AR of the star at the beginning of the year. This correction being applied to the two AR's of the sun by the star, will give the apparent AR's of the sun at those times. For let $A =$ *app.* AR of ☉ at P by *the star*, A' that at T, $B =$ ☉s AR at P by *the declination*, $B' =$ that at T'; $y =$ correction to be applied to correct the computed declination of the sun, and let $1 : n ::$ ☉s error (y) in decl. : corresponding error in $AR = ny$. Now an increase of declination, *increases* the AR in the *first* quadrant, and *decreases* it in the *second*; hence, an increase (ny) of AR in the first quadrant, makes it $B+ny$, and in the second, $B'-ny$; these we may consider as the true AR's of the ☉ from *the declination*; also, the true AR's from *the star* (putting $x =$ the correction of the mean AR of the star at the beginning of the year) are $A+x$ and $A'+x$; hence, $A+x=B+ny$, $A'+x=B'-ny$, and $x=\frac{1}{2}(B-A+B'-A')$; but $a = B-A$, $x=B'-A'$; therefore $x=\frac{1}{2}(a+b)$. Further, $y=\frac{1}{2}n(B-B'+A'-A)$ the error in declination. But $1 : n :: \overline{\dot{PL}} :: \overline{\dot{AP}}$; now sin. $AP =$ tan. $PL \times$ cot. A, therefore $\overline{\text{sin.}\ \dot{AP}} = \overline{\dot{AP}} \times$ cos. $AP = \overline{\dot{AP}} \times \overline{\text{cos.}\ PL}^2 \times$ cot. A, and $1 : n ::$ cos. $AP : \overline{\text{sec.}\ PL}^2 \times$ cot. A; hence, $y=\frac{1}{2}(B-B'+A'-A) \times$ cos. $AR \times \overline{\text{cos. dec.}}^2 \times$ tan. obl. ecl.

By making a great number of observations of this kind, and taking the mean, the AR of a star may be very accurately determined. Dr. MASKELYNE observed, that this method is more simple than that of Dr. BRADLEY, or DE LA CAILLE, though on the same principle, first introduced by FLAMSTEAD.

122. The practical method of finding the right ascension of a body from that of a fixed star, by a clock adjusted to sidereal time, is thus. Let the clock begin its motion from $0h.\ 0'.\ 0''$ at the instant the first point of Aries is on the meridian; then, when any star comes to the meridian, the clock would show the apparent right ascension of the star, the right ascension being estimated in time at the rate of 15° an hour, provided the clock was subject to no error, because it would then show at any time how far the first point of Aries was from the meridian. But as the clock is necessarily liable to err, we must be able at any time to ascertain what its error is, that is, what is the difference between the right ascension shown by the clock and the right ascension of that point of the equator which is at that time on the meridian. To do this, we must, when a star, whose apparent right ascension is known, passes the meridian, compare its apparent right ascension with the right ascension shown by the clock, and

the difference will show the error of the clock. For instance, let the apparent right ascension of *Aldebaran* be 4*h*. 23'. 50" at the time when its transit over the meridian is observed by the clock, and suppose the time shown by the clock to be 4*h*. 23'. 52", then there is an error of 2" in the clock, it giving the right ascension of the star 2" more than it ought. If the clock be compared with several stars* and the mean error taken, we shall have, more accurately, the error at the mean time of all the observations. These observations being repeated every day, we shall get the rate of the clock's going, that is, how fast it gains or loses. The error of the clock, and the rate of its going, being thus ascertained, if the time of the transit of any body be observed, and the error of the clock at the time be applied, we shall have the right ascension of the body. This is the method by which the right ascension of the sun, moon and planets are regularly found in Observatories.

Ex. On April 27, 1774, the following observations were made at Greenwich: *α Serpentis* passed the meridian at 15*h*. 31'. 28",76, the moon's second limb passed at 15*h*. 59'. 7",76, and *Antares* at 16*h*. 13'. 55",02 sidereal time; to find the moon's right ascension.

First, to find the error of the clock by the transit of the stars.

Mean *AR*. of *α serpentis* at begin. of 1790 by Tab. VI.	15ʰ. 33'. 55", 84
Precession in 16 years by Tab. VI. - - -	− 46, 94
Mean *AR*. at begin. of 1774 - - - - -	15. 33. 8, 90
Cor. for aber. and prec. to April 27, by Tab. VII.	+ 2, 12
Cor. for nutation by Tab. VIII. - - - - -	− 0, 23
App. *AR*. by the tables - - - - - -	15. 33. 10, 79
App. *AR*. by the clock - - - - - - -	15. 31. 28, 76
Error of the clock by *α serpentis* too slow - -	1. 42, 03
Mean *AR*. of *Antares* at begin. of 1790 by Tab. VI.	16. 16. 33, 24
Precession in 16 years by Tab. VI. - - -	− 58, 45
Mean *AR*. at begin. of 1774 - - - - -	16. 15. 34, 79
Cor. for aber. and prec. to April 27, by Tab. VII. -	+ 2, 38
Cor. for nutation by Tab. VIII. - - - - - -	− 0, 09

* The stars used for this purpose at the Observatory at Greenwich are those in Tab. VI. whose *AR*'s Dr. MASKELYNE settled to a very great degree of accuracy. As many of these as conveniently can, are observed every day, in order to ascertain the going of the clock, and for no other purpose.

App. *AR.* by the tables - - - - -	16. 15. 37, 08
App. *AR.* by the clock - - - - -	16. 13. 55, 02
Error of the clock by *antares* too slow - - - -	1. 42, 06

The mean of these two errors gives 1'.42",045 for the error at the middle between the times of the transits of the two stars, or at 15*h*. 52'. 41", 89. Now from knowing the error of the clock at this time, and the rate of its going, we must find the error at the time the moon passed, which may, in this case, be considered the same, the times being nearly equal. Hence,

Moon passed the meridian by the clock - - - -	15h. 59' 7",75
Error of the clock, too slow - - - - - -	+ 1. 42,045
True *AR.* of the moon's 2d limb - - - - -	16 . 0. 49, 795
Do. in degrees - - - - - - - - -	8s. 0°. 12'. 26",9
Moon's semid. in *AR.* (109) - - - - - -	− 17. 13, 5
True *AR.* of the moon's center - - - - -	7 . 29. 55. 13, 4

The error of the clock is generally determined by a greater number of stars, when they can be observed; and the mean error from day to day gives the rate of its going, from which we may find the error at any other time. For instance, on August 8, 1769, I found, from taking the mean of the errors of four stars, that the mean error of the clock was 2",32, too fast, at 16*h*. 21'. 18", being the mean of all the times when the stars were observed; and on the 9th the error was 2",09, too fast, at 13*h*. 52'. 58", the mean of all the times. Also *Jupiter* passed the meridian on the 9th at 14*h*. 49'. 10",4. Now the interval between the 8*d*. 16*h*. 21'. 18" and 9*d*. 13*h*. 52'. 58" is 21*h*. 31'. 40", in which time the clock lost 0",23; also, the interval between 13*h*. 52'. 58" and 14*h*. 49'. 10",4 is 56'. 12",4; hence, 21*h*. 31'. 40" : 56'. 12",4 :: 0",23 : 0",009, which is what the clock lost in the second interval; therefore when Jupiter passed the meridian, the clock was 2",09 − 0",009 = 2",08 too fast, which subtracted from 14*h*. 49'. 10",4 gives 14*h*. 49'. 8",32, the apparent right ascension of Jupiter. To the apparent *AR.* apply the aberration in *A.R.* and you get the *true AR.*

123. The right ascension of the heavenly bodies being thus ascertained, the next thing to be explained is, the method of finding their declinations. Take the apparent altitude of the body, when it passes the meridian, by an astronomical quadrant, as explained in my *Treatise on Practical Astronomy;* correct it for parallax and refraction, and for the error of the line of colllima-

tion of the instrument, if necessary, and you get the true meridian altitude, the difference between which and the altitude of the equator (87) (which is equal to the complement of the latitude, previously determined) is the declination required.

Ex. On April 27, 1774, the zenith distance of the moon's lower limb when it passed the meridian at Greenwich was 68°. 19′. 37″,3; its parallax in altitude was 56′. 19″,2, allowing for the spheroidical figure of the earth; the barometer stood at 29, 58, and the thermometer at 49; to find the declination.

Observed zenith distance of L. L. - -	68°. 19′. 37″,3
Refr. cor. for bar. and ther. Tab. XI. XII.	+ 2. 23
	68 . 22 . 00, 3
Parallax - - - - - - - -	− 56 . 19, 2
True zenith distance of L. L. - -	67 . 25 . 41, 1
Semidiameter - - - - - - -	− 16 . 35
True zenith distance of the center - -	67 . 9 . 6, 1
Latitude - - - - - - -	51 . 28 . 40
Declination *south* - - - -	15 . 40 . 26, 1

The *horizontal* parallax and semidiameter may be taken from the Nautical Almanac; and the parallax in altitude may be found, as will be explained when we come to treat of the Parallax, and then the correction is to be applied to the semidiameter, from Table XIII.

124. To find the latitude and longitude from the right ascension and declination, or the converse, we have the following admirable Rules, given by Dr. MASKELYNE.

Given the Right Ascension and Declination of an Heavenly Body, and the Obliquity of the Ecliptic, to find its Latitude and Longitude.

1. The *sine of *AR.* + cotang. decl. − 10, = cotang. of arc A, which call *north* or *south*, according as the declination is *north* or *south*. 2. Call the obliquity of the ecliptic *south* in the 6 first signs of *AR*, and *north* in the 6 last. Let the sum of arc A and obl. eclip. according to their titles, = arc B with its proper title†. 3. The arith. comp. of cos. arc A + cos. arc B + tan. *AR.*—

* By sine, tang. &c. is meant log. sine, log. tang. &c.

† If one be *north* and the other *south*, the proper title is that belonging to the greater of the two, and in this case, arc B is their difference, one being considered as negative to the other.

10, = tan. of the *longitude*, of the same kind as *AR*, unless arc B be more than 90°, in which case, the quantity found of the same kind as *AR*. must be subtracted from 12 signs or 360°. 4. The sine of longitude + tan. arc B − 10, = tan. of the required *latitude*, of the same title as arc B. N. B. If the longitude come out near 0°, or near 180°, for the sine of long. in the last operation, substitute tan. long. + cos. long. − 10,*; or the last operation will be, tan. long. + cos. long. + tan. arc B − 20, = tan. lat. The tan. long. is already given.

Given the Latitude and Longitude of an Heavenly Body, and the obliquity of the Ecliptic, to find its Right Ascension and Declination.

1. Sine long. + cot. lat. − 10, = cot. arc A, which call *north* or *south*, according as the lat. is *north* or *south*. 2. Call the obliquity of the ecliptic *north* in the first semicircle of longitude, and *south* in the second. Let the sum of arc A and obl. eclip. according to their titles, = arc B with its proper title. 3. The arith. comp. of cos. arc A + cos. arc B + tan. long. − 10, = tan. of *right ascension*, of the same kind as the longitude, unless arc B be more than 90°, in which case, the last quantity found of the same kind as the longitude, must be subtracted from 12 signs or 360°. 4. The sine of *AR*. + tan. arc B − 10. = tan. of the required *declination*, of the same title as arc B. N. B. If *AR*. come out near 0°, or near 180°, for the sine *AR*. in the last operation, substitute tan. *AR*. + cos. *AR*. − 10; or the last operation will be tan. *AR*. + cos. *AR*. + tan. arc B − 20, = tan. declination. The tan. *AR*. is already given.

DEMONSTRATION. Let s be the body, ΥC the ecliptic, ΥQ the equator, sr, sn perpendicular to ΥC, ΥQ. Then rad. : sin. Υn :: cot. sn : cot. $s\Upsilon n$, hence, log. sin. Υn + log. cot. $sn - 10$, = log. cot. $s\Upsilon n$ arc A. Hence, $s\Upsilon n \mp Q\Upsilon C = s\Upsilon r$ arc B. Also

$$\left.\begin{array}{l}\text{cos. } s\Upsilon n : \text{rad.} :: \text{tan. } n\Upsilon : \text{tan. } s\Upsilon \\ \text{rad.} : \text{cos. } s\Upsilon r :: \text{tan. } s\Upsilon : \text{tan. } r\Upsilon\end{array}\right\} \text{Trig. Art. 219.}$$

$$\therefore \text{cos. } s\Upsilon n : \text{cos. } s\Upsilon r :: \text{tan. } n\Upsilon : \text{tan. } r\Upsilon = \frac{\text{cos. } s\Upsilon r \times \text{tan. } n\Upsilon}{\text{cos. } s\Upsilon n}; \text{ hence, ar. co.}$$

log. cos. $s\Upsilon n$ + log. cos. $s\Upsilon r$ + log. tan. $n\Upsilon - 10$, = log. tan. $r\Upsilon$ the *longitude*. And (Trig. Art. 210), rad. : sin. $r\Upsilon$ :: tan. $r\Upsilon s$: tan. sr; hence, log. sin. $r\Upsilon$ + log. tan. $r\Upsilon s - 10$, = log. tan. sr the *latitude*. And in whatever position we take s, these conclusions will give the rule as stated above. If we consider ΥC as the equator and ΥQ the ecliptic, the demonstration will do for the second rule.

* For the reason of this correction in extreme cases, see Dr. MASKELYNE's excellent Introduction to TAYLOR's Logarithms.

Ex. Given the true *A.R.* of the moon's center 7*s*. 29°. 55′.13″,4. and its declination 15°. 40′. 26″,1 *south*, as determined in the two last Examples; to find its latitude and longitude*.

By Dr. MASKELYNE's observations, the mean obliquity of the ecliptic at the beginning of the year 1784, was 23°. 28′. 0″,2, and as its gradual diminution is at the rate of $\frac{1}{2}$ a second in a year, the mean obliquity at the beginning of 1774 was 23°. 28′. 5″,2, which corrected by Tab. IX. X. gives 23°. 27′. 55″,8 for the obliquity at the time of observation.

Sine of right asc. -	7ˢ. 29°. 55′. 13″,4 -	9.9371817
Cotan. of decl. - -	15 . 40. 26, 1 -	10.5519183
Cotan. arc A *south* - -	17 . 57. 57, 8 -	10.4891000
Obliq. ecl. *north* - - -	23 . 27. 55, 8	
Arc B *north* - - -	5 . 29. 58, 0 cos.	9.9979964
Arith. comp. of log. cos. arc A	- - -	0.0217102
Tang. of right asc. - -	- - -	10.2371744
Tang. of *longitude* - -	8ˢ. 1°. 2′. 7″,4 -	10.2568810
Sine of longitude - -	- - - -	9.9419678
Tang. of arc B - - -	- - - -	8.9835328
Tang. of *latitude* north -	- 4°. 48′. 54″,1 -	8.9255006

In like manner, the right ascensions and declinations of the fixed stars being found from observation, their latitudes and longitudes may be computed, and thus a catalogue of all the fixed stars may be made for any time. But as both the equator and ecliptic are subject to a change in their positions, the right ascension, declination, latitude and longitude of all the fixed stars will vary. Hence, if their annual variations be computed, as will be afterwards explained, their right ascensions, &c. may be found at any other time.

125. If the body be the sun at s', whose right ascension and declination are given, to find its longitude; then sin. $s'♈n$: rad. :: sin. $s'n$: sin. $♈s'$, that is, *sin. obl. ecl.* : *rad.* :: *sin. decl.* : *sin. longitude.* Or, cos. $s'♈n$: rad. :: tan. $♈n$: tan. $♈s'$, that is, *cos. obl. ecl.* : *rad* :: *tan. right asc.* : *tan. longitude.* The sun, being always in the ecliptic, has no latitude.

To find the angle of Position.

126. Let p be the pole of the ecliptic $♈L$, P the pole of the equator $♈C$, FIG. 22.

* In making trigonometrical calculations, it will save time, when the same arcs occur, to take out all their logarithms at once, to avoid the trouble of turning to them again. The Computer therefore, before he begins his operation, should put it down in its proper order, leaving it to be filled up by the logarithms; he will then see what arcs are repeated, and he may, at one opening of the tables, take out all their logarithms and put them down in their proper places.

S a star, draw the great circles *pPLC*, *pSD*, *PSBA*, and (53) *PSp* is the angle of position. Now the angle *PpS*, or (12) *DL*, is the complement of longitude ♈*D*; the angle *pPS* is the supplement of *APC*, or of *AC* (12), which is the complement of the right ascension ♈*A* of the star; *pP* is the obliquity of the ecliptic; *PS* is the complement of declination, and *pS* the complement of the latitude of the star. Hence, if the longitude and declination of a star be given, we have, sin. *PS* : sin. *PpS* :: sin. *Pp* : sin. *PSp*, that is, *cos. star's dec.* : *cos. its long.* :: *sin. obl. ecl.* : *sin. angle of Position.* If the latitude and declination of the star be given, we know *pS* and *PS* their complements, and *Pp*; hence, $\sin. pS \times \sin. PS : \text{rad.}^2 :: \sin. \tfrac{1}{2} \times \overline{SP + Sp + Pp} \times \sin. \tfrac{1}{2} \times \overline{SP + Sp - Pp} : \cos. \tfrac{1}{2} \angle PSp^2$. Or of the right ascension, declination, latitude and longitude of the star, any two being known, we shall know three parts of the triangle *PpS*, and consequently the angle *PSp* may be found. If *S* be the sun, $pS = 90°$, and the triangle may be solved by the circular parts.

CHAP. IV.

ON THE EQUATION OF TIME.

Art. 127. HAVING explained, in the last Chapter, the practical methods of determining the place of any body in the heavens, we come next to the consideration of another circumstance not less important, that is, the irregularity of time as measured by the sun. The best measure of time which we have, is a clock regulated by the vibration of a pendulum. But however accurately a clock may be made, it must be subject to go irregularly, partly from the imperfection of the workmanship, and partly from the expansion and contraction of the materials by heat and cold, by which the length of the pendulum, and consequently the time of vibration, will vary. As no clock therefore can be depended upon for keeping time accurately, it is necessary that we should be able to ascertain at any time, how much it is too fast or too slow, and at what rate it gains or loses. For this purpose it must be compared with some motion which is uniform, or of which, if it be not uniform, you can ascertain the variation. The motions of the heavenly bodies have therefore been considered as most proper for this purpose. Now the earth revolving uniformly about its axis, the apparent diurnal motion of the fixed stars about the axis must be uniform. If a clock therefore be adjusted to go 24 hours from the passage of any fixed star over the meridian till it returns to it again, its rate of going may be at any time determined by comparing it with any fixed star, and observing whether the interval continues to be 24 hours; if not, the difference shows how much it gains or loses in that time. A clock adjusted to go 24 hours in this interval is said to be adjusted to *sidereal* time. But if we compare a clock with the sun, and adjust it to go 24 hours from the time the sun leaves the meridian on any day, till it returns to it the next day, which is a *true* solar day, the clock will not, even if it go uniformly, continue to agree with the sun, that is, it will not show 12 when the sun comes to the meridian.

128. For let P be the pole of the earth, $vwyz$ its equator, and let the earth revolve about its axis in the order of the letters $vwyz$; ♈DLE the celestial equator, and ♈CL the ecliptic, in which the sun moves according to that direction. Let a, m, be the sun when on the meridian of any place on two successive days, and draw $Pvae$, $Prmh$, secondaries to the equator, and let the spectator be at s on the meridian Pv, with the sun at a on his meridian. Then when the earth has made one revolution about its axis, Psv is come again into the same position; but the sun having moved forward to m, the earth has still FIG. 23.

to describe the angle vPr in order to bring the meridian Psv into the position Pr, so that the sun may be again in the spectator's meridian. Now the angle vPr is measured by the arc eh, which is the increase of the sun's right ascension in a *true* solar day; hence, the length of a *true* solar day is *equal to the time of the earth's rotation about its axis + the time of its describing an angle equal to the increase of the sun's right ascension in a true solar day.* Now if the sun moved uniformly in the equator ΥDLE, this increase eh would be always the same in the same time, and therefore the solar days would be always equal; but the sun moves in the ecliptic ΥCL, and therefore *if* its motion were *uniform*, equal arcs am upon the ecliptic would not give equal arcs eh upon the equator*. But the motion of the sun is not uniform, and therefore am, described in any given time, is subject to a variation, and which also must necessarily make eh variable. Hence, the increase eh of the sun's right ascension in a day varies from two causes, that is, from the obliquity of the ecliptic to the equator, and from the unequal motion of the sun in the ecliptic. The length therefore of a true solar day, is subject to a continual variation; consequently a clock adjusted to go 24 hours for any one true solar day, would not continue to show 12 when the sun comes to the meridian, because the intervals by the clock would continue equal (the clock being supposed neither to gain nor lose), whilst the intervals of the sun's passage over the meridian would vary.

129. As the sun moves through 360° of right ascension in $365\frac{1}{4}$ days very nearly, therefore $365\frac{1}{4}$ days : 1 day :: 360°. : 59′. 8″,2 the increase of right ascension in one day, if the increase were uniform, or it would be the increase in a *mean* solar day, that is, if the solar days were all equal. If therefore a clock be adjusted to go 24 hours in a *mean* solar day, it cannot continue to coincide with the sun, that is, to show 12 when the sun is on the meridian; but the sun will pass the meridian, sometimes *before* 12 and sometimes *after*. This difference is called the *Equation of Time*. A clock thus adjusted is said to be adjusted to *mean solar time*†. The time shown by the clock is called *true* or *mean* time, and that shown by the sun is called *apparent* time. What we call *apparent* time the French call *true*.

* For draw mt parallel to eh, and suppose ma to be indefinitely small; then by plain trigon.

$ma : mt$ :: rad. : sin. mat, or Υae,

$mt : eh$ :: cos. ae : rad. (13)

∴ $ma : eh$ :: cos. ae : sin. Υae :: (because Trig. Art. 212. sin. $\Upsilon ae = \frac{\text{cos. } a\Upsilon e \times \text{rad.}}{\text{cos. } ae}$) $\overline{\text{cos. } ae^2}$: cos. $a\Upsilon e \times$ radius; hence, the ratio of ma to eh is variable.

† As the earth describes an angle of 360°. 59′. 8″,2 about its axis in a *mean* solar day of 24 hours, and an angle of 360° in a *sidereal* day, therefore 360°. 59′. 8″,2 : 360° :: 24*h*. : 23*h*. 56′. 4″,098 the length of the sidereal day in mean solar time, or the time from the passage of a fixed star over the meridian till it returns to it again.

130. A clock adjusted to go 24 hours in a *mean* solar day, would coincide with an imaginary star moving uniformly in the equator with the sun's mean motion 59′. 8″,2 in right ascension, if the star were to set off from any given meridian when the clock is 12; that is, the clock would always show 12 when the star came to the meridian, because the interval of the passages of this star over the meridian would be a *mean* solar day. This star therefore, if we reckon its motion from the meridian in time at the rate of 1 hour for 15°, would always coincide with the clock; that is, when the clock shows 1 hour, the star's motion would be 1 hour; when the clock shows 2 hours, the star's motion would be 2 hours; and so on. Hence, this star may be substituted instead of the clock; therefore when the sun passes the given meridian, the difference between its right ascension and that of the star, converted into time, is the difference between the time when the sun is on the meridian and 12 o'clock, or the equation of time; because the given meridian passes through the star at 12 o'clock, and its motion in respect to that star is at the rate of 15° in an hour (132).

131. Now to compute this equation of time, let $APLS$ be the ecliptic, ALv the equator, A the first point of aries, P the sun's apogee, S any place of the sun, draw Sv perpendicular to the equator, and take $An=AP$. When the sun sets out at P, let the imaginary star set out at n with the sun's mean motion in right ascension, or longitude, or at the rate of 59′. 8″,2 in a day, and when n passes the meridian let the clock be adjusted to 12, as described in the last Article: These are the corresponding positions of the clock and sun, as assumed by Astronomers. Take $nm=Ps$, and when the star comes to m, the place of the sun, if it moved uniformly with its mean motion, would be at s, but at that time let S be the place of the sun. Now let the sun S, and consequently v, be on the meridian; then as m is the place of the imaginary star at that instant, mv is the equation of time. The sun's mean place is at s, and as $An=AP$, and $nm=Ps. \therefore Am=APs$, consequently $mv=Av-Am=Av-APs$. Let a be the *mean* equinox, and draw az perpendicular to AL; then $Am=Az+zm=Aa\times\cos. aAz+zm=\frac{11}{12}Aa+zm$; hence, $mv=Av-zm-\frac{11}{12}Aa$; but Av is the sun's true right ascension, zm is the mean right ascension, or mean longitude, and $\frac{11}{12}Aa$ (Az) is the equation of the equinoxes in right ascension; hence, the equation of time is equal to *the difference of the sun's true right ascension, and its mean longitude corrected by the equation of the equinoxes in right ascension.* When Am is *less* than Av, *mean* time precedes *apparent*, and when *greater*, *apparent* time precedes *mean;* for as the earth turns about its axis in the direction Av, or in the order of right ascension, that body whose right ascension is least must come to the meridian first. That is, when the sun's true right ascension is *greater* than its mean longitude corrected as above, we must *add* the equation of time to apparent, to get the mean time; and when it is *less*, we must *subtract.* To convert mean FIG. 24.

time into apparent, we must *subtract* in the *former* case and *add* in the *latter*. This Rule for computing the equation of time was first given by Dr. MASKELYNE in the *Phil. Trans.* 1764.

132. As a meridian of the earth, when it leaves m, returns to it again in 24 hours, it may be considered, when it leaves that point, as approaching a point at that time 360° from it, and at which it arrives in 24 hours. Hence, the relative velocity with which a meridian accedes to or recedes from m is at the rate of 15° in an hour. Therefore when the meridian passes through v, the arc vm reduced into time at the rate of 15° in an hour, gives the equation of time at that instant. Hence, the equation of time is computed for the instant of *apparent* noon. Now the time of apparent noon in mean solar time, for which we compute, can only be known by knowing the equation of time. To compute therefore the equation on any day, you must assume the equation the same as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, sufficiently accurate for the purpose of computing the equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1, 1792, for the meridian of Greenwich, by MAYER's Tables.

The equation on July 1, 1788, was, by the *Nautical Almanac*, 3'. 28'', to be added to apparent noon, to give the corresponding mean time; hence, for July 1, 1792, at 0*h*. 3'. 28'' compute the true longitude*.

	Mean. Long. ☉	Long. ☉'s Apog.	N°.1.	N°.2.	N°.3.	N°.4.
Epoch for 1792.	9s. 10°. 50'. 0'',7	3s. 9°. 23'. 46''	241	227	123	478
Mean Mot. July 1,	5. 29. 23. 16, 2	33	163	456	312	27
3'	7, 4					
28''	1, 1					
Mean Longitude	3. 10. 13. 25, 4	3. 9. 24. 19	404	683	435	505
Equat. of Center	— 1. 37, 1	3. 10. 13. 25,4				
Equat. ☽ I.	+ 4, 5					
♃ II.	— 4, 7	49. 6,4	Mean Anomaly.			
♀ III.	+ 3,65					
☊ IV.	— 0, 6					
True Longitude	3. 10. 11. 51,15					

* The reason of this operation will appear, when we come to the construction and use of the Solar Tables.

With this true longitude and obliquity 23°. 27′. 48″,4 of the ecliptic, the true right ascension of the sun is found to be 3^s. 11°. 5′. 41″,25; also, the equation of the equinoxes in longitude = – 0″,6; hence,

The mean longitude - - -	3^s. 10°. 13′. 25″,4
$\frac{11}{12}$ of – 0″,6 - - - - -	– 0,55
Mean longitude corrected - -	3. 10. 13. 24,85
True right ascension - - -	3. 11. 5. 41,25
Equation - - - - - -	52. 16, 4

which converted into time gives 3′. 29″,1 the true equation of time; which must be added to apparent to give the true time, because the true right ascension is greater than the mean longitude.

133. The sun's apogee P has a progressive motion, and the equinoctial points A, L, have a regressive motion; the inclination also of the equator to the ecliptic is subject to a constant variation. Hence, the same Table of the equation of time cannot continue to serve for the same degree of the sun's longitude. Also, the sun's longitude at noon at the same place is different for the same days on different years, and it is for apparent noon that the equation is computed. For these reasons, the equation of time must be computed anew for every year.

134. Whenever it is required to make any calculations from Astronomical Tables, and the time given is apparent time, the equation of time must be applied to convert it into mean time, and for that time the computations must be made, the Tables being disposed according to mean motions. Thus, if it were required to find the sun's place on any day at apparent noon, the equation of time that day at apparent noon must be applied to 12 o'clock, and then the sun's place computed from the tables for that time. All the articles in the Nautical Almanac answering to noon, are computed in the same manner.

135. A clock adjusted to sidereal time begins at $0h$. 0′. 0″ when the true equinox A is upon the meridian; therefore the distance of the meridian from A measures sidereal time. A clock adjusted to mean solar time begins at $0h$. 0′. 0″ when m is upon the meridian. Let x be a point of the equator through which the meridian passes at any time, then Ax is the sidereal time; and let t be the place of the imaginary star at the same instant, and y its place when the meridian coincided with it; then (132) the arc xt is the measure of the time from the mean noon. Hence, to get xt, subtract the sun's mean right ascension Ay in time at noon on the given day from the time Ax shown by the sidereal clock, and you get xy, which is nearly the time xt from mean noon;

from this subtract ty, the sun's mean motion in right ascension in the interval xy of sidereal time, and you have xt the time from mean noon by a clock adjusted to mean solar time. To facilitate this computation, Dr. MASKELYNE has given two Tables; Table XVII. (Vol. II.) shows the mean motion of the sun in right ascension for every day of the year; Table XVIII. is the mean motion of the sun in right ascension in time to hours and minutes of sidereal time. Hence, from the Solar Tables, take the epoch of the sun's mean longitude for the year, and convert it into time, and add it to the time in Table XVII, corresponding to the given day, and correct it by Table XIX, and it gives the sun's mean longitude, or mean right ascension, expressed in sidereal time, reckoned from the true equinox, at the mean noon of the proposed day: This subtracted from the proposed sidereal time, gives the mean time nearly, with which Table XVIII. is to be entered, and the number taken out of it, being the sun's mean motion since the mean noon, subtracted from the mean time found nearly, will give the mean time correct. It is to be observed, that the mean time found nearly, or before it is corrected by Table XVIII, is a portion of sidereal time, being the interval by the clock between the transit of the imaginary star, and the proposed instant; and therefore to shorten the operation, Table XVIII. is made to be entered with sidereal time, instead of mean time, commonly used in Astronomical Tables. Dr. MASKELYNE also gave another Table of the epoch of the sun's mean right ascension in time for the beginning of the year; but as that can be taken from our Tables of the sun's motion, the mean right ascension and mean longitude being the same, it is not here given.

Ex. On July 1, 1790, the time by the sidereal clock was 11*h*. 20′. 14″, to find the mean solar time.

Epoch of sun's *AR*. 1790 - -	18*h*. 41′. 15″,9
Mean mot. in *AR*. to July 5, Tab. XVII.	12. 13. 19, 3
Equat. equin. Tab. XIX. - - -	+ 0,66
☉'s mean. long. at mean noon - -	6. 54. 35, 86
Sidereal time given - - - -	11. 20. 14.
Mean time nearly - - - -	4. 25. 38, 14
Cor. by Tab. XVIII. - - -	43, 5
Mean solar time - - - -	4. 24. 54, 64

Hence, if the mean solar time be given, for instance, 4*h*. 24′. 54″,64, we may thus find the sidereal time. To get the correction from Table XVIII, corresponding to mean time nearly, first get it for mean solar time, which is 42″,39, and add it to the mean solar time, and we have 4*h*. 25′. 37″,03, which is very near

what we call, mean time nearly; corresponding therefore to this time, take out the correction from Table XVIII, which is $43',5$, and add it to the given mean solar time, and we get $4h.\ 25'.\ 38'',14$ correctly for what we call mean time nearly; add this to $6h.\ 54'.\ 35'',86$, the sun's mean longitude at noon, and it gives $11h.\ 20'.\ 14''$ the sidereal time required.

136. Whenever the time is computed from the sun's altitude, that time must be apparent time, because we compute it from the time when the sun comes to the meridian, which is noon, or 12 o'clock, apparent time. Hence also, the time shown by a dial is apparent time, and will differ from the time shown by a well regulated watch or clock, by the equation of time. A clock or watch may therefore be regulated by a good dial, by applying the equation, as before directed, to the apparent time shown by the dial, and it will give the *mean* time, or that which the clock or watch ought to show.

137. Mr. WOLLASTON has proposed to regulate a watch or clock by a dial constructed to show mean noon, or 12 o'clock by a watch or clock. A ray of light through a small hole being let into a dark chamber upon the floor, draw a meridian upon the floor corresponding to the hole, on which therefore the sun's rays will always fall when the sun comes to the meridian. On each side of this line, for every day of the year, make a point where the image of the sun is at 12 o'clock mean time, by a clock or watch regulated for that purpose; through all these points draw a curve, and then you may regulate your clock or watch by setting it to 12 when the image of the sun falls on that curve. To prevent any mistake, put the months against the different parts of the curve on which the ray falls in them. Or the same may be done on any horizontal plane, by erecting a piece of brass, and making a small hole for the sun to shine through. The curve may also be laid down by calculation, as Mr. WOLLASTON has shown; and if it be drawn with great care, it will be sufficiently accurate for regulating all common clocks; and it has this advantage over that of correcting them by a common sun dial, that as the months are put to the curve, you cannot easily make a mistake; whereas, in applying the equation of time to a dial, a person, ignorant of these matters, is very apt to apply it wrong.

138. The Equation of Time was known to, and made use of by PTOLEMY. TYCHO employed only one part, that which arises from the unequal motion of the sun in the ecliptic; but KEPLER made use of both parts. He further suspected, that there was a third cause of the inequality of solar days, arising from the unequal motion of the earth about its axis. But the Equation of Time, as now computed, was not generally adopted till 1672, when FLAMSTEAD published a Dissertation upon it, at the end of the works of HORROX.

CHAP. V.

ON THE LENGTH OF THE YEAR, THE PRECESSION OF THE EQUINOXES FROM OBSERVATION, AND THE OBLIQUITY OF THE ECLIPTIC.

Art. 139. FROM comparing the sun's right ascension every day with the fixed stars lying to the east and west, the sun is found constantly to recede from those on the west, and approach to those on the east; and the interval of time from its leaving any fixed star till it returns to it again is called a *sidereal* year, being the time in which the sun completes its revolution amongst the fixed stars, or in the ecliptic. But the sun, after it leaves either of the equinoctial points, returns to it again in a less time than it returns to the same fixed star, and this interval is called a *solar* or *tropical* year, because the time from its leaving one equinox till it returns to it, is the same as from one tropic till it comes to the same again. This is the year on which the return of the seasons depends.

On the Sidereal Year.

140. To find the length of a *sidereal* year. On any day take the difference between the sun's right ascension when it passes the meridian and that of a fixed star; and when the sun returns to the same part of the heavens the next year, compare its right ascension with the same star for two days, one when their difference of right ascensions is less and the other when greater than the difference before observed; and let D be the increase of the sun's right ascension in this interval of one day; then take the difference (d) between the differences of the sun's and star's right ascensions on the first of these two days and on the day when the observation was made the year before; and let t be equal to the exact time between the transits of the sun over the meridian on the two days; then $D : d :: t :$ the time from the passage of the sun over the meridian on the first day to the instant when it had the same difference of right ascension compared with the star which it had the year before; the interval between these two times gives the length of a sidereal year. The best time for these observations is about March 25, June 20, September 17, December 20, the sun's motion in right ascension being then uniform. Instead of observing the difference of the right ascensions, you may observe that of their longitudes. If instead of repeating the second observations the year after, there be an interval of several years, and you divide the observed interval of time when the difference of their right ascensions was found to be equal, by the number of years, you will have the length of a sidereal year more exact. Or the length may be found thus.

141. Take the time (t) of a star's transit over the meridian by a clock adjusted to mean solar time; then the year after, take the time again on two days, one (m) when it passes the meridian *before*, and the other (n) *after* the time t; then $m-n : m-t :: 23h.\ 56'.\ 4''$: the time from m till the difference between the star's and sun's right ascension was the same as at the first observation; and the interval of these two times is the length of a sidereal year. *Cassini's Elem. d'Astron.* pag. 202.

Ex. On April 1, 1669, at $0h.\ 3'.\ 47''$ mean solar time, M. PICARD observed the difference between the sun's longitude and that of *Procyon* to be $3^s.\ 8°.\ 59'.\ 36''$, which is the most ancient observation of this kind whose accuracy can be depended upon; see *Hist. Celeste, par M. le Monnier*, pag. 37. And on April 2, 1745, M. de la CAILLE found, by taking their difference of longitudes on the 2d and 3d, that at $11h.\ 10'.\ 45''$ mean solar time, the difference of their longitudes was the same as at the first observation. Now as the sun's revolution was known to be nearly 365 days, it is manifest that it had made in this interval 76 complete revolutions in respect to the same fixed star in the space of 76 years $1d.\ 11h.\ 6'.\ 58''$. But in these 76 years, there were 58 of 365 days, and 18 bissextiles of 366 days; that interval therefore contains $27759d.\ 11h.\ 6'.\ 58''$, which being divided by 76, the quotient is $365d.\ 6h.\ 8'.\ 47''$ the length of a sidereal year.

Ex. M. CASSINI observed the transit of *Sirius* over the meridian on May 21, 1717, to be at $2h.\ 38'.\ 58''$; on May 21, 1718, it passed at $2h.\ 40'$, and on the 22d at $2h.\ 36'$; to find the length of the sidereal year.

In this case $t=2h.\ 38'.\ 58''$, $m=2h.\ 40'$, $n=2h.\ 36'$; hence, $4' : 1'.\ 2'' :: 23h.\ 56'.\ 4'' : 6h.\ 10'.\ 59''$, which added to $2h.\ 40'$ the time it passed on May 21, 1718, gives $8h.\ 50'.\ 59''$ for the time on that day when the difference between the sun's and star's right ascensions was the same as on May 21, 1717. Hence this interval is $365d.\ 6h.\ 10'.\ 59''$ for the length of a sidereal year. The mean of these two, gives the length $365d.\ 6h.\ 9'.\ 53''$. But the length of a sidereal year has generally been determined from the length of a tropical year, found as we shall now proceed to explain.

On the Tropical Year.

142. Observe the meridian altitude (a) of the sun on the day nearest to the equinox; then the next year take its meridian altitude on two following days, one, when its altitude (m) is less than a, and the next when its altitude (n) is greater than a, and $n-m$ is the increase of the sun's declination in 24 hours; hence, $n-m : a-m ::$ 24 hours : the interval from the first of the two days till the sun has the same declination as at the observation the year before; because

at that time the sun's declination increases uniformly. Hence we find the time when the sun's place in the ecliptic had the same situation in respect to the equinoctial points, which it had at the time of the observation the year before. Therefore this 4th term being added to the number of days between the two first observations, gives the length of a tropical year. If instead of repeating the second observation the next year, there be an interval of several years, and you divide the interval between the times when the declination was found to be the same, by the number of years, you will get the time more exactly. *Cassini's Elem. d'Astron.* pag. 204.

Ex. M. Cassini informs us, that on March 20, 1672, his Father observed the meridian altitude of the sun's upper limb at the Royal Observatory at Paris, to be 41°. 43′; and on March 20, 1716, he himself observed the meridian altitude of the upper limb, to be 41°. 27′. 10″; and on the 21st to be 41°. 51′. Hence, the difference of the two latter altitudes was 23′. 50″, and of the two former 15′. 50″; hence, 23′. 50″ : 15′. 50″ :: 24 hours : 15*h*. 56′. 39″; therefore on March 20, 1716, at 15*h*. 56′. 39″ the sun's declination was the same as on March 20, 1672. Now the interval between these two observations was 44 years, of which 34 consisted of 365 days each, and 10 of 366; therefore the interval in days was 16070; hence, the whole interval between the equal declinations was 16070 days 15*h*. 56′. 39″, which divided by 44, gives 365*d*. 5*h*. 49′. 0″. 53‴ the length of a tropical year from these observations.

But when we determine the length of a tropical or solar year from the times of the equinoxes, it will want a correction to give the length of a mean tropical or mean solar year; because, from the motion of the sun's apogee, the equation of the orbit at the equinox is not the same in different years, which will affect the time of the return of the sun to the same mean longitude; and therefore will make the apparent solar year different from the mean solar year. This correction therefore gives the time that would have elapsed between the equinoxes, if the apogee had been fixed; this is called the *mean* solar year. To apply this correction to the last Example, we proceed thus.

On March 20, 1672, the place of the sun's apogee was 3*s*. 7°. 7′. 6″ by Cassini, therefore the sun's true anomaly was 8*s*. 22°. 52′. 54″; from which we find that the equation of the center, or the difference between the true and mean anomaly, was 1°. 54′. 42″, showing how much the true anomaly exceeds the mean; subtract this from 0*s*. 0°. 0′. 0″ and we get 11*s*. 28°. 5′. 18″ for the mean longitude of the sun at the time of the equinox. The place of sun's apogee on March 20, 1716, was 3*s*. 7°. 52′. 23″, and therefore its true anomaly was 8*s*. 22°. 7′. 37″, from which the equation of the center was 1°. 54′. 29″, which subtracted from 0*s*. 0°. 0′. 0″ gives 11*s*. 28°. 5′. 31″ for the mean longitude of the sun at the equinox in 1716. Hence, the sun's mean place at the equinox in

the spring 1716 is greater by 13″ than in 1672, and this answers to 5′. 16″ in time; in this interval of time therefore (44 years), there have been 44 mean revolutions + 5′. 16″; and consequently 44 apparent solar years are greater by 5′. 16″ than 44 mean; divide this by 44, the number of years in the interval, and it gives 7″. 11‴ for the length of the *apparent* above the *mean* solar year. Now the length of the apparent solar year was determined to be 365*d.* 5*h.* 49′. 0″. 53‴; hence, from these observations, the length of the *mean* solar year is 365*d.* 5*h.* 48′. 53″. 42‴.

143. The length of a tropical year may also be found by observing the exact time of the equinoxes. To do this we must previously know the latitude of the place, from which we shall know the altitude of the point of the equator on the meridian, it being equal (87) to the complement of latitude. Take the meridian altitude of the sun's center on two days, one when it is less than the complement of latitude and the other when greater; then the sun must have passed the equator in the intermediate time. Take the difference (D) between these altitudes and it gives the increase of the sun's declination in 24 hours; take also the difference (d) between the altitude on the first day and the complement of latitude, and then say, $D : d ::$ 24 hours : to the time from noon on the first day till the sun came to the equator. Repeat this when the sun returns to the same equinox, and the interval of the times gives the length of a tropical year. If an interval of several years be taken, and you divide by the number, it will give the time more accurately. If we take a difference of two days, the third term must be 48*h*. The same may be done by one observation, if we know the rate at which the sun changes its declination in 24 hours, which at the equinox in spring time is found, by the mean of a great number of observations, to be 23′. 40″, and in the autumn to be 23′. 28″. *Cassini's Elem. d'Astr.* pag. 207.

Ex. On March 20, 1672, the sun's meridian altitude at the Royal Observatory at Paris was observed to be 41°. 25′. 56″, from which subtract 41°. 9′. 50″ the meridian altitude of the equator, and there remains 16′. 6″ for the sun's declination; hence, 23′. 40″ : 16′. 6″ :: 24 hours : 16*h.* 19′, the sun's distance in time from the equinox, which, as the sun was past the equinox, subtracted from the 20th gives the 19th day 7*h.* 41′ for the time of the equinox. And in 1731 the time of the equinox was found, in the same manner, to be on Mar. 20, at 14*h.* 45′. In this interval of 59 years there were 13 bissextiles, and consequently the whole number of days in the 59 years was 21548, and therefore the whole interval between the two equinoxes was 21549*d.* 7*h.* 4′, which divided by 59 gives the length of the *apparent* solar year 365*d.* 5*h.* 48′. 53″; from this subtract 7″, the variation of the equation of the orbit in the interval of the observations, and we have the mean length of the solar year 365*d.* 5*h.* 48′. 46″. The interval has here been taken between the *true* equinoxes, whereas we want

to get the length of a tropical year between the *mean* equinoxes in order to get the length of a *mean* tropical year. But in taking a long interval of time, the difference, whether we take the true or mean equinox, will be insensible. Another correction might also be added, when we compare the modern observations with the ancient ones, on account of the precession of the equinoxes being greater now than it was then. From the modern observations the length of a mean solar year appears to be $2'',6$ less than that which is deduced from comparing the same observations with those of HIPPARCHUS.

144. As the sun's declination at the equinoxes changes about $24'$ in 24 hours, an error of $10''$ in the altitude of the sun will cause an error of 10 minutes in the determination of the time of the equinox, and consequently the same error in the length of the year, if it were determined by 2 observations at the interval only of 1 year; but if the interval were 60 years, the error would be only 10 seconds. As the accuracy therefore is very much increased by taking a long interval, let us compare the most ancient observations with the modern ones.

HIPPARCHUS, in the year 145 before J. C. found the time of the equinox to be on March 24, at $11h.\ 55'$ in the morning at Alexandria. In the year 1735, at the Royal Observatory at Paris the time of the equinox was found to be on March 20, at $14h.\ 20'.\ 40''$. Now the difference of the meridians between Paris and Alexandria is, in time, $1h.\ 51'.\ 46''$, which, as Alexandria lies to the east of Paris, being added to $14h.\ 20'.\ 40''$ gives $16h.\ 12'.\ 26''$ the time at Alexandria. Reduce this time to the Julian year, by subtracting 11 days by which the Gregorian is before the Julian, and we have the time of the equinox by this style, on March 10, at $4h.\ 12'.\ 26''$ in the morning. Between these two observations there was an interval of 1880 Julian years, except $14d.\ 7h.\ 42'.\ 34''$. In these years there were 470 bissextiles and the rest common Julian years of 365 days. Therefore if we divide $14d.\ 7h.\ 42'.\ 34''$ by 1880 it gives $10'.\ 58''.\ 10'''$, showing how much the apparent solar year is less than 365 days 6 hours; hence, the length of the *apparent* solar year is $365d.\ 5h.\ 49'.\ 1''.\ 50'''$, to which add $6''.\ 30'''$, being what the *apparent* is less than the *mean* solar year, found as before, and we get $365d.\ 5h.\ 49'.\ 8''.\ 20'''$ the length of the *mean* solar year from these observations. The mean of 10 results from different observations made by HIPPARCHUS, compared with the modern ones, gives the length of the *mean* solar year $365d.\ 5h.\ 48'.\ 49''$.

145. The length of the year may also be found by finding the time when the sun comes to the tropic. For let ADL be the equator, ASL the ecliptic, A aries; find the time (119) when the sun has the same declination mv, nw on each side of the tropic S, and at the same times find also the differences of its right ascension and that of a fixed star s, the sum or difference of which wz, vz, according to the position of z, measures the motion vw of the sun in right ascension, the half of which is wD (SD, sz being perpendicular to AL);

hence we shall get Dz which is equal to $wD \pm wz$. Now to find when the sun comes to D, observe its right ascension at x, either the day before or day after the solstice, compared with the same star, and you have xz, the difference between which and Dz, is Dx. Observe also the increase (d) of the sun's right ascension at that time in 24 hours; then $d : xD :: 24h$: the time of the passage of the sun from x to D, which added to or subtracted from the time at x, according as vx is less or greater than vD, gives the time when the sun in right ascension is at D, or when it is in the solstice S. *Cassini's Elem. d'Astron.* pag. 238.

Ex. According to Cassini on May 29, 1737, the altitude of the sun's upper limb, when it passed the meridian, was 63°. 6'. On July 14, its altitude on the meridian was 63. 7', and on the 15th it was 62°. 57'. 35''; it was therefore diminished 9'. 25'' in one day, and on the 14th its altitude was 1' greater than on May 29; hence 9'. 25'' : 1' :: 24*h.* : 2*h.* 32'. 55''; which added to the 14th gives 2*h.* 32'. 55'' for the time when the altitude, and consequently the declination, was the same as on May 29. On the same May 29, the difference vz of the right ascension of the sun and *Sirius* was 32°. 9'. 8''. On July 14, the difference was 15°. 16'. 4'' when the sun was on the meridian; and as the increase of the sun's right ascension was then 1°. 0'. 45'' in 24 hours, we have 24*h.* : 2*h.* 32'. 55'' :: 1°. 0'. 45'' : 6' : 40'', which added to 15°. 16'. 4'' gives $zw = 15°. 22'. 44''$ the difference of the right ascensions of the sun and Sirius on July 14, at 2*h.* 32. 55''. But as Sirius passed the meridian before the sun, z in this case will fall between D and w, and therefore $vw = vz + zw = 47°. 31'. 52''$, hence, $Dw = 23°. 45'. 56''$, from which take zw and we get $Dz = 8°. 23'. 12''$ the distance of Sirius in right ascension from the solstice. Now on June 21, Sirius passed the meridian at 0*h.* 33'. 34'', at which time the difference zx of its right ascension and that of the sun was 8°. 23'. 30'', and consequently $xD = 18''$, showing what the sun wants in right ascension of the solstice. Now taking the increase of the sun's right ascension at that time to be 62',25 in 24 hours, we have 62',25 : 18'' :: 24*h.* 6'. 56'', which added to 0*h.* 33'. 34'' gives 0*h.* 40'. 30'' on the 21st for the time of the solstice. Hence, by finding the interval of two solstices, we get the length of a tropical year. After getting the apparent solar year, we get the mean solar year, by applying to it the variation of the equation of the center, for the same reason that we made a similar correction at the equinoxes.

M. Cassini, by comparing a solstice observed at Athens on June 27, 431 years before J. C. with one observed at Paris on June 21, 1717, found the length of a mean solar year to be 365*d.* 5*h.* 49'. 48''. 39'''. By comparing one observed at Alexandria on June 24, 140 years after J. C. with one observed at Paris on June 20, 1732, the length was found 365*d.* 5*h.* 47'. 36''. By solstices

observed at Nuremberg in 1487, 1493, 1498, 1503, and one at Paris on 1731, the length is found to be 365*d*. 5*h*. 48′. 31″. By comparing 14 solstices observed at Uranibourg with as many observed at Paris, he found the length of the mean solar year to be 365*d*. 5*h*. 48′. 52″. The accuracy of these observations appears from hence, that of the 14 determinations, only 1 differed 20″, 1 differed 15″, 1 differed 11″, and all the others less.

If we take a mean of all the mean solar years as determined by CASSINI from the equinoxes, leaving out 2 which differ very much from all the rest, we have the length of a mean solar year 365*d*. 5*h*. 48′. 51″$\frac{1}{2}$. If we do the same by those determined from the solstices, the length comes out 365*d*. 5*h*. 48′. 42″$\frac{1}{2}$; the mean of which gives 365*d*. 5*h*. 48′. 47″ the length of a mean solar year.

146. M. de la LANDE, in a Piece entitled *Memoire sur la veritable Longueur de l' Année Astronomique*, which gained the prize proposed by the Royal Society at Copenhagen for the year 1780, has, by comparing a great number of the most distant observations, and those which could be most depended upon for their accuracy, determined the length of the mean solar year to be 365*d*. 5*h*. 48′. 48″, differing only 1″ from our determination from CASSINI.

To find the Precession of the Equinoxes from Observation.

147. The sun returning to the equinox every year before it returns to the same point of the Heavens, shows that the equinoctial points have a retrograde motion, which, as we shall prove, arises from the motion of the equator, caused by the attraction of the sun and moon upon the earth in consequence of its spheroidical figure. The effect of this is, that the longitude of the stars must constantly increase; and hence by comparing the longitude of the same stars at different times, the motion of the equinoctial points, or the precession of the equinoxes, may be found.

148. HIPPARCHUS was the first person who observed this motion, by comparing his own observations with those which TIMOCHARIS made 155 years before. From this he judged the motion to be one degree in about 100 years; but he doubted whether the observations of TIMOCHARIS were accurate enough to deduce any conclusion to be depended upon. In the year 128 before J. C. he found the longitude of *Virgin's Spike* to be 5*s*. 24°; and in the year 1750 its longitude was found to be 6*s*. 20°. 21′, the difference of which is 26°. 21′. In the same year he found the longitude of the *Lion's Heart* to be 3*s*. 29°. 50′; and in 1750 it was 4*s*. 26°. 21′, the difference of which is 26°. 31′. The mean of these two gives 26°. 26′ for the increase of longitude in 1878 years, or 50″. 40‴ in a year, for the precession. By comparing the observations of d'ALBATEGNIUS in the year 878 with those made in 1738, the precession appears

to be 51″. 9‴. From a comparison of 15 observations of TYCHO with as many made by M. de la CAILLE, the precession is found to be 50″. 20‴. But M. de la LANDE, from the observations of M. de la CAILLE compared with those in FLAMSTEAD's Catalogue, determines the secular precession to be 1°. 23′. 45″, or 50″,25 in a year.

149. The precession being given, and also the length of a tropical year, the length of a sidereal year may be found by this proportion; 360° − 50″,25 : 360°. :: 365*d.* 5*h.* 48′. 48″ : 365*d.* 6*h.* 9′. 11″½ the length of the *sidereal* year.

On the Anomalistic Year.

150. The year, called the *anomalistic* year, is sometimes used by Astronomers, and is the time from the sun's leaving its apogee till it returns to it. Now the motion of the sun's apogee is 1′. 2″ every year, in longitude, or in respect to the equinox, according to M. de la LANDE: therefore 1′. 2″ − 50″,25 = 11″,75 the progressive motion of the apogee in a year, and hence the anomalistic must be longer than the sidereal year by the time the sun takes in moving over 11″,75 of longitude at his apogee; but when the sun is in its apogee, its motion in longitude is 58′. 13″ in 24 hours; hence, 58′. 13″ : 11″,75 :: 24 hours : 4′. 50″⅗, which added to 365*d.* 6*h.* 9′. 11½″ gives 365*d.* 6*h.* 14′. 2″⅙ for the length of the anomalistic year. M. de la LANDE determined this motion of the apogee, from the observations of M. de la HIRE and those of Dr. MASKELYNE. CASSINI made it the same. MAYER made it 1′. 6″ in his Tables.

On the Obliquity of the Ecliptic.

151. The method used by Astronomers to determine the obliquity of the ecliptic is that explained in Art. 86. by taking half the difference of the greatest and least meridian altitudes of the sun. The following is the obliquity as determined by different Astronomers.

ERATOSTHENES 230 years *before* J. C.	23°. 51′. 20″
HIPPARCHUS 140 years *before* J. C. -	23 . 51 . 20
PTOLEMY 140 years *after* J. C. - -	23 . 51 . 10
PAPPUS in the year 390 - - -	23 . 30 . 0
ALBATEGNIUS in 880 - - - -	23 . 35 . 40
ARZACHEL in 1070 - - - -	23 . 34 . 0
PROPHATIUS in 1300 - - - -	23 . 32 . 0
REGIOMONTANUS in 1460 - - -	23 . 30 . 0
COPERNICUS in 1500 - - - -	23 . 28 . 24
WALTHERUS in 1490 - - -	23 . 29 . 47
TYCHO in 1587 - - - - -	23 . 29 . 30

CASSINI (the Father) in 1656 - - -	23°. 29'. 2"
CASSINI (the Son) in 1672 - - -	23 . 28 . 54
FLAMSTEAD in 1690 - - -	23 . 28 . 48
De la CAILLE in 1750 - - -	23 . 28 . 19
Dr. BRADLEY in 1750 - - -	23 . 28 . 18
MAYER in 1750 - - - - - -	23 . 28 . 18
Dr. MASKELYNE in 1769 - - -	23 . 28 . 8,5
M. de la LANDE in 1786 - - -	23 . 28 . 0

The observations of ALBATEGNIUS, an Arabian, are here corrected for refraction. Those of WALTHERUS, M. de la CAILLE computed. The obliquity by TYCHO is here put down as correctly computed from his observations. Also the obliquity, as determined by FLAMSTEAD, is corrected for the nutation of the earth's axis. These corrections M. de la LANDE applied.

152. It is manifest from the above observations, that the obliquity of the ecliptic keeps diminishing; and the irregularity which here appears in the diminution we may ascribe to the inaccuracy of the ancient observations, as we know that they are subject to greater errors than the irregularity of this variation. If we compare the first and last observations, they give a diminution of 70" in 100 years. If we compare the last with that of TYCHO, it gives 45". The last compared with that of FLAMSTEAD gives 50". If we compare that of Dr. MASKELYNE with Dr. BRADLEY'S and MAYER'S it gives 50". The comparison of Dr. MASKELYNE'S determination, with that of M. de la LANDE, which he took as the mean of several results, gives 50". We may therefore state the secular diminution of the obliquity of the ecliptic, at this time, to be 50", as determined from the most accurate observations. This result agrees very well with that deduced from theory, as will be shown when we come to treat of the physical cause of this diminution. It must however be observed, that some eminent Astronomers use 50",25.

CHAP. VI.

ON PARALLAX.

Art. 153. THE center of the earth describes that circle in the Heavens which is called the ecliptic; but as the same object would appear in different positions in respect to this circle, when seen from the center and surface, Astronomers always reduce their observations to what they would have been, if they had been made at the center of the earth, in consequence of which, the places of the heavenly bodies are computed as seen from the ecliptic, and it becomes a fixed point for that purpose, on whatever part of the earth's surface the observations are made.

154. Let C be the center of the earth, A the place of the spectator on its surface, S any object, ZH the sphere of the fixed stars, to which the places of all the bodies in our system are referred; Z the zenith, H the horizon; draw CSm, ASn, and m is the place seen from the center, and n from the surface. Now the plane SAC passing through the center of the earth must be perpendicular to its surface, and consequently it will pass through the zenith Z; and the points m, n lying in the same plane, the arc of parallax mn must lie in a circle perpendicular to the horizon, and hence the azimuth is not affected, if the earth be a sphere. Now the parallax mn is measured by the angle mSn or ASC, and by trig. $CS : CA ::$ sin. SAC or SAZ : sin. ASC the parallax $= \frac{CA \times \text{sin. } SAZ}{CS}$. As CA is constant, supposing the earth to be a sphere, the sine of the parallax varies as the sine of the apparent zenith distance directly, and the distance of the body from the center of the earth inversely. Hence, a body in the zenith has no parallax, and at s in the horizon it is the greatest. If the object be at an indefinitely great distance, it has no parallax; hence the apparent places of the fixed stars are not altered by it. As n is the apparent place, and m is called the true place, the parallax depresses an object in a vertical circle. For the same body at different altitudes, the parallax varies as the sine (s) of the apparent zenith distance; therefore if $p=$ the horizontal parallax, and radius be unity, the sine of the parallax $=ps$. To ascertain therefore the parallax at all altitudes, we must first find it at some given altitude. FIG. 26.

155. *First* method, for the *sun*. ARISTARCHUS proposed to find the sun's parallax, by observing its elongation from the moon at the instant it is dichotomized, at which time the angle at the moon is a right angle; therefore we should know the angle which the distance of the moon subtends at the sun;

which diminished in the ratio of the moon's distance from the earth's center to the radius of the earth, would give the sun's horizontal parallax. But a very small error in the time when the moon is dichotomized, (and it is impossible to be very accurate in this) will make so very great an error in the sun's parallax, that nothing can be depended upon from it. VENDELINUS determined the angle of elongation when the moon was dichotomized to be 89°. 45′, from which the sun's parallax was found to be 15″. But P. RICCIOLI found it to be 28″ or 30″ from like observations.

156. *Second* method. HIPPARCHUS proposed to find the sun's parallax from a lunar eclipse, by the following method. Let S be the sun, E the earth, Ev the length of its shadow, mr the path of the moon in a central eclipse. Observe the length of this eclipse, and then, from knowing the periodic time of the moon, the angle mEr, and consequently nEr, will be known. Now the horizontal parallax ErB of the moon being known, we have the angle $Evr = ErB - nEr$; hence we know $EAB = AES - Evr = AES - ErB + nEr$; that is, the sun's horizontal parallax = the apparent semidiameter of the sun − the horizontal parallax of the moon + the semidiameter of the earth's shadow where the moon passes through. The objection to this method is, the great difficulty of determining the angle nEr with sufficient accuracy; for any error in that angle will make the same error in the sun's parallax, the other quantities remaining the same. By this method PTOLEMY made the sun's horizontal parallax 2′. 50″. TYCHO made it 3′.

157. *Third* method, for the *moon*. Take the meridian altitudes of the moon, when it is at its greatest north and south latitudes, and correct them for refraction; then the difference of the altitudes, thus corrected, would be equal to the sum of the two latitudes of the moon, if there were no parallax; consequently the difference between the sum of the two latitudes and the difference of the altitudes will be the difference between the parallaxes at the two altitudes. Now to find from thence the parallax itself, let S, s be the sines of the greatest and least apparent zenith distances, P, p the sines of the corresponding parallaxes; then as, when the distance is given, the parallax varies (154) as the sine of the zenith distance, $S : s :: P : p$, hence, $S - s : s :: P - p : p = \frac{s \times \overline{P-p}}{S-s}$ the parallax at the greatest altitude. This supposes that the moon is at the same distance in both cases; but as this will not necessarily happen, we must correct one of the observations in order to reduce it to what it would have been, had the distance been the same. If the observations be made in those places where the moon passes through the zenith in one of the observations, the difference between the sum of the two latitudes and the zenith distance at the other observation, will be the parallax at that altitude.

158. *Fourth* method. Let a body P be observed from two places A, B in the same meridian, then the whole angle APB is the effect of parallax between the two places. The parallax (154) $APC = hor.\ par. \times \sin.\ PAL$, taking APC for sin. APC, and the parallax $BPC = hor.\ par. \times \sin.\ PBM$; hence $hor.\ par. \times \overline{\sin.\ PAL + \sin.\ PBM} = APB$, $\therefore hor.\ par. = APB$ divided by the sum of these two sines. If the two places be not in the same meridian it does not signify, provided we know how much the altitude varies from the change of declination of the body in the interval of the passages over the meridians. FIG. 28.

Ex. On Oct. 5, 1751, M. de la CAILLE, at the Cape of Good Hope, observed *Mars* to be 1′. 25″,8 below the parallel of λ in aquarius, and at 25° distance from the zenith. On the same day at Stockholm, Mars was observed to be 1′. 57″,7 below the parallel of λ and at 68°. 14′ zenith distance. Hence the angle APB is 31″,9, and the sines of the zenith distances being 0,4226 and 0,9287, the horizontal parallax was 23″,6. Hence, if the ratio of the distance of the earth from Mars to its distance from the sun be found, we shall have the sun's horizontal parallax. Now from comparing the altitudes of the northern limb of Mars with stars nearly in the same parallel observed on the same days at the Cape and at Greenwich, Bologna, Paris, Stockholm, Upsal, Hernosand, the mean of the whole gave 10″,2 for the horizontal parallax of the sun; and rejecting those results which differed the most from the rest, the mean was 9″, 842. From the mean of another set of observations, the result was 9″,575. From the mean of several observations on *Venus* made in like manner, the parallax came out 10″,38. The mean of the three last gives 9″,93 for the horizontal parallax of the sun. FLAMSTEAD, from an observation on Mars, concluded the sun's parallax could not be more than 10″. MARALDI found the same. From the observations of POUND, and Dr. BRADLEY, Dr. HALLEY found it never greater than 12″ nor less than 9″. CASSINI, from his observations on Mars, found it to be between 11″ and 15″. But the most accurate method of determining the sun's parallax is from the transit of Venus over its disc, as will be explained when we treat on that subject.

159. If the earth be a spheroid, let E be the equator; draw GAv, HBr perpendicular to the surface, and compute the angles CAv or LAG, and CBr or MBH by the Rule which we shall give, when we treat of the figure of the earth; subtract these from the observed zenith distances PAG, PBH, and we have the angles PAL, PBM. Now $CP : CA ::$ sin. CAP or PAL : sin. $APC = \frac{CA \times \sin.\ PAL}{CP}$; also, $CP : CB ::$ sin. CBP or PBM : sin. $BPC = \frac{CB \times \sin.\ PBM}{CP}$; and as the parallax is very small, the sum of the two sines will be very nearly the sine of the sum, therefore the sine of $APB =$

$\frac{CA \times \sin.PAL + CB \times \sin.PBM}{CP}$; hence, $CP = \frac{CA \times \sin.PAL + CB \times \sin.PBM}{\sin.\ APB}$.

160. *Fifth* method. Let EQ be the equator, P its pole, Z the zenith, v the true place of the body and r the apparent place as depressed by parallax in the vertical circle ZK, and draw the secondaries Pva, Prb; then ab is the parallax in right ascension, and rs in declination. Now $vr : vs :: 1$ (rad.) : sin. vrs or ZvP, and $vs : ab ::$ cos. va : 1 (13); hence, $vr : ab ::$ cos. va : sin. ZvP, $\therefore$ $ab = \frac{vr \times \sin.\ ZvP}{\cos.\ va}$; but $vr = hor.\ par. \times \sin.\ vZ$ (154), and (Trig. Art. 221.) sin. $vZ : \sin.ZP :: \sin.\ ZPv : \sin.\ ZvP = \frac{\sin.\ ZP \times \sin.\ ZPv}{\sin.\ vZ}$, therefore by substitution, $ab = \frac{hor.\ par. \times \sin.\ ZP \times \sin.\ ZPv}{\cos.\ va}$. Hence for the same star, where the *hor. par.* is given, the parallax in right ascension varies as the sine of the hour angle. Also the $hor.\ par. = \frac{ab \times \cos.\ va}{\sin.\ ZP \times \sin.\ ZPv}$. For the eastern hemisphere, the apparent place b lies on the equator to the east of a the true place, and therefore the right ascension is diminished by parallax; but in the western hemisphere, b lies to the west of a, and therefore the right ascension is increased. Hence, if the right ascension be taken before and after the meridian, the whole change of parallax in right ascension between the two observations is the sum (s) of the two parts before and after the meridian; and the $hor.\ par. = \frac{s \times \cos.\ va}{\sin.ZP \times S}$, where $S =$ sum of sines of the two hour angles.

161. To apply this Rule, observe the right ascension of the planet when it passes the meridian, compared with that of a fixed star, at which time there is no parallax in right ascension; about 6 hours after, take the difference of their right ascensions again, and observe how much the difference (d) between the apparent right ascensions of the planet and fixed star has changed in that time. Next observe the right ascension of the planet for 3 or 4 days when it passes the meridian, in order to get its true motion in right ascension; then if its motion in right ascension in the above interval of time between the taking of the right ascensions of the fixed star and planet on and off the meridian be equal to d, the planet has no parallax in right ascension; but if it be not equal to d, the difference is the parallax in right ascension, and hence, by the last Article, the horizontal parallax will be known. Or one observation may be made as long before the planet comes to the meridian, by which a greater difference will be obtained.

Ex. On August 15, 1719, *Mars* was very near a star of the 5th magnitude in the eastern shoulder of aquarius, and at 9*h*. 18′ in the evening, Mars fol-

lowed the star in 10′. 17″, and on the 16th at 4*h*. 21′ in the morning it followed it in 10′. 1″, therefore in that interval, the apparent right ascension of Mars had increased 16″ in time. But according to observations made in the meridian for several days after, it appeared, that Mars approached the star only 14″ in that time, from its proper motion, therefore 2″ in time, or 30″ in motion, is the effect of parallax in the interval of the observations. Now the declination of Mars was 15°, the co-latitude 41°. 10′, and the two hour angles 49°. 15′ and 56°. 39′; therefore the *hor. par.* $= \frac{30'' \times \cos. 15°}{\sin. 41°. 10' \times \sin. 49°. 15' + \sin. 56°. 39'} = 27\frac{1}{2}''$. But at that time, the distance of the earth from Mars was to its distance from the sun as 37 to 100, and therefore the sun's horizontal parallax comes out 10″,17.

162. When Dr. Maskelyne was at St. Helena and Barbadoes, he made several observations of this kind on the moon, in order to determine her horizontal parallax; and he further observes, "that if the like observations were repeated in different parts of the earth, it would probably afford the best means, yet proposed, for ascertaining the true figure of the earth, as they would determine the ratio of the diameters of the parallels of latitude to each other, the horary parallaxes being in proportion thereto: For though the earth affords but a small base at the moon, yet, by repeating these trials, and comparing the results, we may hope to attain that degree of exactness, which we could never expect from fewer observations."

163. But besides the effect of parallax in right ascension and declination, it is manifest that the latitude and longitude of the moon and planets must also be affected by it; and as the determination of this, in respect to the moon, is in many cases, particularly in solar eclipses, of great importance, we shall proceed to show how to compute it, supposing that we have given the latitude of the place, the time, and consequently the sun's right ascension, the moon's true latitude and longitude, with her horizontal parallax.

164. Let HZR be the meridian, ΥEQ the equator, p its pole; ΥC the ecliptic, P its pole; Υ the first point of aries, HQR the horizon, Z the zenith, FIG. 30.
ZL a secondary to the horizon passing through the true place r and apparent place t of the moon; draw Pt, Pr, which produce to s, drawing the small circle ts parallel to ov; let rn be perpendicular to Pt, and draw the small circle ra parallel to ov; then rs, or ta, is the parallax in latitude, and ov the parallax in longitude. Draw the great circles $P\Upsilon$, $PZAB$, $Ppde$, and ZW perpendicular to pe; then as $\Upsilon P = 90°$, and $\Upsilon p = 90°$, Υ must (4) be the pole of $Ppde$, and therefore $d\Upsilon = 90°$; consequently d is one of the solsticial points ♋ or ♑; also, draw Zx perpendicular to Pr, and join $Z\Upsilon$, $p\Upsilon$. Now ΥE, or the angle ΥpE, or $Zp\Upsilon$, is the right ascension of the mid-heaven, which is known (106);

$PZ = AB$ (because AZ is the complement of both) the altitude of the highest point A of the ecliptic above the horizon, called the nonagesimal degree, and ♈A, or the angle ♈PA is its longitude. Now in the right angled triangle ZpW, we have Zp the co-latitude of the place, and the angle ZpW, the difference between the right ascension of the mid-heaven ♈pE and ♈d; hence, (Trig. Art. 212.) cot. $p.Z$: rad :: cos. p : tan. pW; therefore $PW = pW \pm pP$, where the upper sign is to be taken when the right ascension of the mid-heaven is less than 180°, and the under, when greater. Also, in the triangles WZp, WZP, (Trig. Art. 231.) sin Wp : sin WP :: cot. WpZ : cot. WPZ, or tan. AP♈; and as we know ♈o, or ♈Po, the true longitude of the moon, we know APo, or ZPx. Also (Trig. Art. 219.) cos. WPZ, or sin. APZ, : rad. :: tan. WP : tan. ZP. Hence, in the triangle ZrP, we know ZP, Pr and the angle P, from which the angle ZrP or trs, and Zr may be found; for in the right angled triangle ZPx, we know ZP and the angle P, to find Px; therefore we know rx; and hence (Trig. Art. 231.) we may find the angle Zrx, with which, and rx, we may find Zr the *true* zenith distance; to which, as if it were the *apparent* zenith distance, find the parallax (154) and add to it, and you will get very nearly the apparent zenith distance, corresponding to which, find the parallax rt; then in the right angled triangle rst, which may be considered as plane, we know rt and the angle r, to find rs the parallax in *latitude*; find also ts, which multiplied (108) by the secant of tv, the apparent latitude, gives the arc ov, the parallax in *longitude*.

Ex. On January 1, 1771, at 9*h*. apparent time, in lat. 53°N. the moon's true longitude was 3*s*. 18°. 27′. 35″, and latitude 4°. 5′. 30″S. and its horizontal parallax 61′. 9″; to find its parallax in latitude and longitude.

The sun's right ascension was 282°. 22′. 2″ by the Tables, and its distance from the meridian 135°; also (106) the right ascension ♈E of the mid-heaven was 57°. 22′. 2″; hence, the whole operation for the solution of the triangles may stand thus.

Tri. ZpW	ZpW =	32°. 37′. 58″		cos.	9.9253864
	Zp =	37 . 0 . 0		tan.	9.8871144
	pW =	32 . 23 . 57		tan.	9.8025008
	Pp =	23 . 28 . 0			
	PW =	55 . 51 . 57			
Tri. WpZ, WPZ	pW =	32 . 23 . 57	A.C.	sin.	0.2709855
	PW =	55 . 51 . 57		sin.	9.9178865
	ZpW =	32 . 37 . 58		cot.	10.1935941
	AP♈ =	67 . 29 . 8		tan.	10.3824661

	$oP\Upsilon$	$= 108°. 27'. 35''$		
	oPA	$= 40. 58. 27$		
Tri. WPZ	APZ	$= 67. 29. 8$	sin.	9.9655700
	WP	$= 55. 51. 57$	tan. + 10	20.1688210
	ZP	$= 57. 56. 36$	tan.	10.2032510
Tri. ZPx	ZP	$= 57. 56. 36$	tan.	10.2032555
	ZPx	$= 40. 58. 27$	cos.	9.8779500
	Px	$= 50. 19. 33$	tan.	10.0812055
	Pr	$= 94. 5. 30$		
Tri. ZPx, Zrx	rx	$= 43. 45. 57$	*A.C.* sin.	0.1600743
	Px	$= 50. 19. 33$	sin.	9.8863144
	ZPx	$= 40. 58. 27$	tan.	9.9387676
	Zrx	$= 44. 1. 16$	tan.	9.9851563
Tri. Zrx	Zrx	$= 44. 1. 16$	cos. + 10	19.8567795
	rx	$= 43. 45. 57$	tan.	9.9812846
	Zr	$= 53. 6. 10$	cot.	9.8754949
	Zr	$= 53. 6. 10$	sin.	9.9029362
	Hor. par. $= 61'. 9'' = 3669''$.		log.	3.5645477
	rt uncorrected $= 2934'' = 48'. 54''$		log.	3.4674839
	App. zen. dist. $Zt = 53°. 55'. 4''$ nearly		sin.	9.9075042
	Hor. par. $= 61'. 9'' = 3699''$.		log.	3.5645477
Tri. trs	Par. rt cor. $= 2965'' = 49'. 25''$		log.	3.4720519
	$trs = 44°. 1'. 16''$		cos.	9.8567795
	rs par. in *lat.* $= 2132'' = 35'. 32''$		log.	3.3288314
Tri. trs	rt cor. $= 2965''$		log.	3.4720519
	$trs = 44°. 1'. 16''$		sin.	9.8419369
	$ts = 2061'' = 34'. 21''$		log.	3.3139888
	True lat. $ro = 4°. 5'. 30''$.			
	App. lat. $tv = ro - rs = 4°. 41'. 2''$		sec.	10.0014528
	ov par. in *long.* $= 2067'' = 34'. 27''$		log.	3.3154416

The value of tv is $ro -$ or $+ rs$, according as the moon has N. or S. latitude. The Figure is drawn for north latitude, but the Example is for south latitude.

This is the direct method of solving the problem from the triangles; but the

operation may be rendered easier by the following Rule (the most convenient of any yet given) discovered by Dr. MASKELYNE, but communicated without the demonstration. The investigation here given, is by the Rev. Dr. BRINKLEY, Professor of Astronomy at Dublin.

Let the height H of the nonagesimal degree, or PZ, and the angle ZPr (n), the moon's true distance from the nonagesimal, be computed as before. Put $P=$ the parallax ov in longitude, $Q=$ the parallax at in latitude, depressing the moon southwards, $L=$ the true latitude, l the apparent latitude, h the horizontal parallax. Now

$$\left.\begin{array}{l} P : rn :: \text{rad.} : \sin. Pr \\ rn : rt :: \sin. ntr : \text{rad.} \\ rt : h :: \sin. Zt : \text{rad.} \end{array}\right\} \therefore P : h :: \sin. ntr \times \sin. Zt : \sin. Pr, \text{ radius being unity;}$$

hence, $P=\dfrac{h \times \sin. ntr \times \sin. Zt}{\sin. Pr}=$ (as $\sin. ntr \times \sin. Zt = \sin. ZPt \times \sin. PZ$) $\dfrac{h \times \sin. PZ \times \sin. ZPt}{\sin. Pr}=\dfrac{h \times \sin. H \times \sin. \overline{n+P}}{\cos. L}$, the parallax in *Longitude*.

Also, $tn : tr :: \cos. rtn : \text{rad.} :: \sin. rtn : \tan. rtn$
$tr : h :: \sin. Zt : \text{rad.}$
$\therefore tn : h :: \sin. rtn \times \sin. Zt : \tan. rtn \times \text{rad.} :: \sin. PZ \times \sin. ZPt : \dfrac{\sin. ZPt}{\sin. Pt \times \cot. ZP - \cos. Pt \times \cos. ZPt}$, substituting for the third and fourth terms their values; hence, $tn = h \times \sin. PZ \times \sin. Pt \times \cot. ZP - h \times \sin. PZ \times \cos. Pt \times \cos. ZPt = h \times \cos. H \times \cos. l - h \times \sin. H \times \sin. l \times \cos. \overline{n+P}$.

Now as the angle rPn is very small, we have $an=\dfrac{rn^2}{2 \tan. Pr}$* $=$ (from the first proportion above) $\dfrac{P^2 \times \sin. Pr^2}{2 \tan. Pr}=\frac{1}{2} P^2 \times \sin. Pr \times \cos. Pr = \frac{1}{2} P \times P \times \sin. Pr \times \cos. Pr =$ (as, from above, $P \times \sin. Pr = h \times \sin. PZ \times \sin. ZPt$) $\frac{1}{2} P \times h \times \sin. H \times \sin. \overline{n+P} \times \sin. L$, or $\sin. l$ nearly; hence, $Q = ta = tn - an = h \times \cos. H \times \cos. l - h \times \sin. H \times \sin. l \times \cos. \overline{n+P} - h \times \sin. H \times \frac{1}{2} P \times \sin. \overline{n+P} \times \sin. l$. But as P is very small, we may call $\frac{1}{2} P$ the sine of $\frac{1}{2} P$, and its cosine we may put $=$ rad. $=1$; hence, for $\cos. \overline{n+P}$ we may substitute $\cos. \overline{n+P} \times \cos. \frac{1}{2} P$, and for $\frac{1}{2} P \times \sin. \overline{n+P}$ we may put $\sin. \overline{n+P} \times \sin. \frac{1}{2} P$; hence, $Q = h \times \cos. H \times \cos. l - h \times \sin. H \times \sin. l \times \overline{\cos. \overline{n+P} \times \cos. \frac{1}{2} P + \sin. \overline{n+P} \times \sin. \frac{1}{2} P} =$ (because by plane Trig. Art. 103. $\cos. \overline{n+P} \times \cos. \frac{1}{2} P + \sin. \overline{n+P} \times \sin. \frac{1}{2} P = \cos. \overline{n+\frac{1}{2}P}$) $h \times \cos. H \times \cos. l - h \times \sin. H \times \sin. l \times \cos. \overline{n+\frac{1}{2}P}$, the parallax in *Latitude*.

Now P enters into the expression for the value of P, and as P is very small,

* If we conceive two tangents to be drawn to Pr and Pa at r and a, and to meet, then rn may be considered as the sine of ra to the length of these tangents as a radius, and therefore, by the property of the circle, $an = rn^2$ divided by twice the tangent.

we must first suppose $P = \frac{h \times \sin. H \times \sin. n}{\cos. L}$, which will give a near value of P; then put that value into the numerator, and you will get a very accurate value of P. Also, in the expression for Q, we have the apparent latitude, which cannot be known without knowing Q; hence we must first get a near value of Q and apply it to the true latitude to get the apparent nearly; to do this, we may omit the second part as being small, on account of sin. l being small for the moon, and suppose $Q = h \times \cos. H \times \cos. l = h \times \cos. H \times \cos. L$ nearly; or when the latitude is very small, as is the case of the moon in solar eclipses, we may suppose $Q = h \times \cos. H$, from which we shall get the apparent latitude with sufficient accuracy.

In the application of this Rule, regard must be had to the signs of the quantities; if $n + \frac{1}{2}P$ be greater than 90° its cosine becomes negative, in which case Q will be the *sum* of the quantities, unless the apparent latitude l is south, in which case, its cosine will be negative, which makes the first term negative. In general, Q will be the *sum* of the two parts, when $n + \frac{1}{2}P$ and the moon's apparent distance from P are, one greater and the other less than 90°; otherwise Q will be the *difference*. The parallax in longitude *increases* the longitude, if the body be to the *east* of the nonagesimal degree, and *decreases* it, if it be to the *west*. This Rule is more correct than the other, because in that we took the small circle ts, instead of a great circle from t, as the perpendicular from t upon Pr produced. This error, for the moon, may sometimes amount to about 2″. It may be corrected by applying an found above.

To apply this Rule to the last case, we have $H = 57°. 56'. 36''$, $n = 40°. 58'. 27''$, $L = 4°. 5'. 30''$ south, $h = 61'. 9'' = 3669''$; hence,

Log. h		3.5645477
Sin. H		9.9281518
Cos. L.	A. C.	0.0011084
		3.4938079
Sin. n		9.8167176
Log. $2044'' = 34'. 4'' = P$ nearly		3.3105255
Therefore $n + P = 41°. 32'. 31''$; hence,		
		3.4938079
Sin. $\overline{n+P}$		9.8216237
Log. $2067'' = 34'. 27''$ par. in *Longitude*		3.3154316
Log. h		3.5645477
Cos. H		9.7248963
Log. $2133'' = 35'. 33''$ par. lat. nearly		3.2894440

$$\begin{array}{r}
4^\circ.\ \ 5'.\ 30'' \\
\hline
4\ .\ 41\ .\ \ 3
\end{array}\ \text{app. lat. nearly.}$$

Log. h - - - - - -	3.5645477	first part of Q.
Cos. H - - - - - -	9.7248963	
Cos. $l = 4^\circ.\ 41'.\ 3''$ nearly - -	9.9985470	
Log. $1941'' = 32'.\ 21''$ - - -	3.2879910	

Log. h - - - - - -	3.5645477	second part of Q.
Sin. H - - - - - -	9.9281518	
Sin. l - - - - - -	8.9120258	
Cos. $\overline{n+\frac{1}{2}P}$ - - - -	9.8759399	
Log. $191'' = 3'.\ 11''$ - - -	2.2806552	

$$\begin{array}{r}
32\ .\ 21 \\
\hline
35\ .\ 32
\end{array}\ \text{par. in } \textit{Latitude}.$$

The *sum* of the two parts is here taken, because Pt is greater than 90°, and $n+\frac{1}{2}P$ less than 90°.

165. Hitherto we have considered the effect of parallax, upon supposition that the earth is a sphere; but as the earth is a spheroid, having the polar diameter shorter than the equatorial, it will be necessary to show how the computations are to be made for this case. The following method is given by Clairaut.

166. Let $EPQp$ be the earth, EQ the equatorial and Pp the polar diameters, O the place of the spectator, HCR the rational horizon, to which draw $ZONK$ perpendicular; L the moon, join LO, LC, LK, and draw CV perpendicular to LK. Now to compare the apparent places seen from O and C, let us compare the places seen from O and K, and from K and C. Put h = the horizontal parallax to the radius OC, or ON which is very nearly equal to it, on account of the smallness of the angle CON. Let $CO = 1$, and CN (the sine of CON to that radius) $= a$, t = tan. of the angle KCN the latitude of the place; then rad. $= 1 : t :: a : ta = NK$; hence, as h = the angle under which ON (which we may consider as equal to unity) appears when seen directly at the moon, we have $h \times ta$ = the angle under which NK would appear; therefore $h \times \overline{1+ta}$ = the horizontal parallax of OK; considering therefore K as the center of a sphere and KO the radius, compute the parallax as before. Now as the planes of all the circles of declination pass through Pp, in estimating the parallax either from K or O, the parallax in right ascension must be the same, because K and O lie in the plane of the same circle of declination; the only

difference therefore between the effect of parallax at K and O must be in declination. Now at K, the angular distance of the moon from the pole P is LKP, and the angular distance from C is LCP; the difference of these two angles therefore, or CLK, is the difference between the parallax in declination at K and at C, and this angle CLK is *always* to be *added* to the polar distance seen from K to get the polar distance from C. Now $CLK = h \times CV$; but the angle VCK $(= LCE)$ is the moon's declination, therefore $CV = CK \times \cos.\ \text{dec.}$ also, $CK = \frac{CN}{\cos.\ KCN} = \frac{a}{\cos.\ \text{lat.}}$; hence, $CLK = \frac{h \times a \times \cos.\ \text{dec.}}{\cos.\ \text{lat.}}$. This therefore is the equation of declination for the spheroid, to be applied to find the parallax in declination seen from C, after having calculated the effect of parallax in declination for a sphere whose center is K and radius KO. There is no equation for the parallax in right ascension. To find how this equation in declination will affect the latitude, let P be the pole of the equator, p the pole of the ecliptic, L the place of the moon seen from K, and b seen from C; then bL is the equation in declination; draw La perpendicular to pb, and ba is the equation in latitude, and the angle apL the equation in longitude. Now considering bL and ba as the variations of the two sides Pb, pb, whilst Pp and the angle P remain constant, we have $bL : ba ::$ (Trig. Art. 262.) rad. : cos. b, or cos. $L =$ (Trig. Art. 243.) $\frac{\cos.\ Pp - \cos.\ Pp \times \cos.\ pb}{\sin.\ Pp \times \sin.\ pb}$; hence, $ba = bL \times$

FIG. 32.

$$\overline{\frac{\cos.\ Pp - \cos.\ Pb \times \cos.\ pb}{\sin.\ Pb \times \sin.\ pb}} = \frac{h \times a}{\cos.\ \text{lat.}} \times \overline{\frac{\cos.\ Pb}{\sin.\ pb} - \frac{\cos.\ Pb \times \cos.\ pb}{\sin.\ pb}} = \frac{h \times a}{\cos.\ \text{lat.}} \times$$

$$\overline{\frac{\cos.\ Pb}{\sin.\ pb} - \cos.\ Pb \times \text{cotan.}\ pb} = \frac{h \times a}{\cos.\ \text{lat.}} \times \overline{\frac{\cos.\ 23^\circ.\ 28'}{\cos.\ \text{moon's lat.}} - \sin.\ \text{dec.} \times \tan.\ \text{moon's lat.}}$$

But if CP be to CE as $1 : 1 + m$, and $x, y, =$ the sine and cosine of the latitude of the place, then $a = 2m \times xy$, as shown in the Chapter on the Figure of the Earth; hence, $ba = 2hmx \times \overline{\frac{\cos.\ 23^\circ.\ 28'}{\cos.\ \text{moon's lat.}} - \sin.\ \text{dec.} \times \tan.\ \text{moon's lat.}}$ The sign $-$ becomes $+$ if the declination and latitude of the moon be of different affections, that is, one south and the other north. The latitude here used, is that seen from the center of the earth. This correction increases the moon's distance from the pole p of the ecliptic.

167. To find the correction of the longitude, or the angle Lpa, we have (13) $La = Lpa \times \sin.\ pL$, hence, $Lpa = \frac{La}{\sin.\ pL}$; but $aL = bL \times \sin.\ b$, and by spher. trig. $\sin.\ Pb : \sin.\ p :: \sin.\ Pp : \sin.\ b = \frac{\sin.\ p \times \sin.\ Pp}{\sin.\ Pb}$; also, $Lb = 2hmx$; hence, $Lpa = 2hmx \times \frac{\sin.\ p \times \sin.\ Pp}{\sin.\ Pb \times \sin.\ pL} = 2hmx \times \frac{\cos.\text{lon.}\ ☾ \times \sin.\ 23^\circ.28'}{\cos.\ \text{dec.}\ ☾ \times \cos.\text{lat.}\ ☾}$

=(as the cos. of the moon's latitude may be considered equal to unity) $2hmx \times \frac{\sin. 23°. 28'}{\cos. \text{dec.} ☾} \times \cos. \text{lon.} ☾$. In north latitude, we must *add* this correction to the longitude seen from K, when the moon is in the descending signs 3, 4, 5, 6, 7, 8, but *subtract* it, when in the ascending signs 0, 1, 2, 9, 10, 11, to have the longitude seen from C; and the contrary when the latitude of the place is south.

168. According to the Tables of MAYER, the greatest parallax of the moon, (or when she is in her perigee and in opposition) is 61′. 32″; the least parallax (or when in her apogee and conjunction) is 53′. 52″, in the latitude of Paris; the arithmetical mean of these is 57′. 42″; but this is not the parallax at the mean distance, because the parallax varies inversely as the distance, and therefore the parallax at the mean distance is 57′. 24″, an harmonic mean between the two. M. de LAMBRE recalculated the parallax from the same observations from which MAYER calculated it, and found it did not exactly agree with MAYER's. He made the equatorial parallax 57′. 11″,4. M. de la LANDE makes it 57′. 5″ at the equator, 56′. 53″,2 at the pole, and 57′. 1″ for the mean radius of the earth, supposing the difference of the equatorial and polar diameters to be $\frac{1}{300}$ of the whole. From the formula of MAYER, the equatorial parallax is 57′. 11″,4 with the following equations, according to M. de la LANDE.

57′. 11″,4 − 3′. 7″,7 cos. ano. ☾
+ 10,0 cos. 2 ano. ☾
− 0,3 cos. 3 ano. ☾
− 37,3 cos. arg. evection
+ 0,3 cos. 2 arg. evect.
+ 26,0 cos. 2 dist. ☾ à ☉
− 1,0 cos. dist. ☾ à ☉
+ 0,2 cos. 4 dist. ☾ à ☉
+ 2,0 cos. 2 (apo. ☾ − ☉)
+ 0,2 cos. 3 (apo. ☾ − ☉)
+ 1,0 cos. (arg. evect. + ano. ☉)
+ 0,8 cos. (2 arg. lat. − ano. ☾ cor.)
− 0,8 cos. (2 dist. ☾ à ☉ − ano. ☉)
− 0,7 cos. (2 dist. ☾ à ☉ + ano. ☉)
+ 0,6 cos. (arg. evect. − mean ano. ☾)
+ 0,4 cos. 2 (☊ − ☉), or 2 (☉ + sup. ☊)
+ 0,3 cos. mean ano. ☉
+ 0,2 cos. (mean ano. ☾ − mean ano. ☉)
+ 0,1 cos. (2 dist. ☉ à ☾ + mean ano. ☾)

169. Let $r = \frac{1}{2}$ the semiaxis major, $\rho = \frac{1}{2}$ the semiaxis minor, $n =$ the sine, m the cosine of the angle OCE; then, from conics, the sine of the horizontal polar parallax : sine of the hor. parallax at $O :: \sqrt{r^2n^2 + \rho^2m^2} : r\rho$; hence the sine of the hor. par. at $O = \frac{r\rho}{\sqrt{r^2n^2+\rho^2m^2}} \times$ the sine of the hor. polar parallax. If $r : \rho :: 230 : 229$, we have the following Table for the horizontal parallax for every degree of latitude, that at the pole being unity.

Lat.	Hor. Par.	Lat.	Hor. Par.	Lat.	Hor. Par.
0°	100438	31°	100321	61°	100103
1	100438	32	100314	62	100097
2	100437	33	100307	63	100091
3	100436	34	100300	64	100085
4	100435	35	100293	65	100079
5	100434	36	100286	66	100073
6	100432	37	100279	67	100067
7	100430	38	100272	68	100062
8	100428	39	100265	69	100057
9	100426	40	100257	70	100052
10	100424	41	100250	71	100047
11	100421	42	100243	72	100042
12	100418	43	100235	73	100038
13	100415	44	100227	74	100034
14	100412	45	100219	75	100030
15	100408	46	100211	76	100026
16	100404	47	100203	77	100023
17	100400	48	100195	78	100020
18	100396	49	100187	79	100017
19	100391	50	100180	80	100014
20	100386	51	100173	81	100012
21	100381	52	100166	82	100010
22	100376	53	100159	83	100008
23	100371	54	100152	84	100006
24	100365	55	100145	85	100004
25	100359	56	100138	86	100003
26	100353	57	100131	87	100002
27	100347	58	100124	88	100001
28	100341	59	100117	89	100000
29	100335	60	100110	90	100000
30	100328				

Hence, by multiplying the polar parallax by the number corresponding to any latitude, it gives the horizontal parallax at that latitude. From the

Theorem, the parallax may be very easily calculated for any other ratio of the diameters of the earth.

170. To find the mean distance Cs of the moon, we have AC, the mean radius (r) of the earth, : Cs, the mean distance (D) of the moon from the earth, :: sin. $57'. 1'' = AsC$ (168) : radius :: 1 : 60,3; consequently $D = 60,3r$; but $r = 3964$ miles; hence, $D = 239029$ miles.

171. According to M. de la LANDE, the horizontal semidiameter of the moon : its horizontal parallax for the mean radius (r) of the earth :: $15'$: $54'. 57'',4$, or very nearly as 3 : 11; hence, the semidiameter of the moon is $\frac{3}{11}r = \frac{3}{11} \times 3964 = 1081$ miles; and as the magnitudes of spherical bodies are as the cubes of their radii, we have the magnitudes of the moon and earth as $3^3 : 11^3 :: 1 : 49$.

172. In the spheroid, besides the parallax in right ascension and declination, latitude and longitude, there is also a parallax in azimuth, and also a correction of the parallax in altitude. For the plane which is perpendicular to the surface at O, always passes through ON, and therefore the azimuth seen from O or N and from C must be different, except when the body is on the meridian, in which case the plane also passes through C; and the altitude seen from N must also be different from that seen from C. Hence, having compared the parallax between O and N in altitude, we shall want a correction for the difference between the altitudes and azimuths seen from N and C. Let therefore CN represent CN in FIG. 31. L the moon, LCR a plane perpendicular to the horizon, and then will NCR be the azimuth seen from C; draw NM perpendicular to CR, MS perpendicular to CL, and LR perpendicular to the horizon; and let m and n be the sine and cosine of NCM, r the sine of MCS, $a = CN$, the sine of CON in FIG. 31. and c the cosine of LNR, and let $d =$ the distance of the moon; then $cd = RN$, $ma = MN$. Now the line CO in FIG. 31. or unity, at the distance d appears under an angle h when seen directly; hence, $\frac{1}{d} : h :: \frac{ma}{cd}$: the angle $NRC = \frac{hma}{c}$ the difference of the azimuths seen from C and N. Also, as the arc parallel to the horizon between any two secondaries to it varies (13) as the cosine of the altitude, the arc of the difference of the azimuths at the altitude of the moon $= hma = h \times MN$. Now as the plane NML is perpendicular to CLM, and NM is extremely small, the altitudes seen from N and M will not sensibly differ; hence, the difference between the altitudes at N and C is the angle $CLM = h \times SM = h \times r \times CM = h \times r \times n \times a$. If the moon be to the *south* of the prime vertical, we must *subtract* this correction from the altitude at N to get the altitude at C; if it be to the *north*, we must *add* the correction.

173. But the most elegant and simple method of finding the parallax in latitude and longitude on a spheroid, is the following, given by MAYER.

The parallax at any place O in the spheroid is the same as on a sphere whose radius is CO, and latitude OCE; subtract therefore the angle COK (found from the following Table) from the latitude OvE on the spheroid, and you get the angle OCE the latitude of the point O reduced to a sphere. Also, the horizontal parallax which is made use of, must be adapted to the radius OC, by diminishing the equatorial horizontal parallax by a quantity corresponding to the difference between CE and CO. This diminution is also found in the same Table. The latitude thus reduced, and the horizontal parallax thus found, are to be employed in computing the moon's parallaxes in longitude, latitude, right ascension and declination, which will now be performed by the Rule (164) founded on the hypothesis of the earth being a sphere; for by means of the Table, both the base of the parallax and the latitude of the place are referred to the earth's center. FIG. 31.

ARGUMENT.

Elevation of the Pole, and Equatorial Parallax.

Elev. of Pole.	Equatorial Parallax. 54′	57′	60′	Reduct. of Elevat. of Pole.
	Reduction of Parallax.			
0°	−0″,0	−0″,0	−0″,0	−0′. 0″
6	0, 2	0, 2	0, 2	3. 6
12	0, 6	0, 7	0, 7	6. 4
18	1, 4	1, 4	1, 5	8. 57
24	2, 3	2, 5	2, 6	11. 6
30	3, 5	3, 7	3, 9	12. 56
36	4, 9	5, 1	5, 4	14. 12
42	6, 3	6, 7	7, 0	14. 51
48	7, 7	8, 2	8, 6	14. 51
54	9, 2	9, 7	10, 2	14. 12
60	10, 5	11, 1	11, 7	12. 56
66	11, 7	12, 4	13, 0	11. 6
72	12, 7	13, 4	14, 1	8. 57
78	13, 4	14, 2	14, 9	6. 4
84	13, 9	14, 6	15, 4	3. 6
90	14, 1	14, 8	15, 6	0. 0

Ex. If the latitude on the spheroid be 63°, and the equatorial parallax be 56′; what are the reductions?

The reduction of the parallax is 11″,5, and of the elevation of the pole it is 55″; hence, the reduced latitude is 62°. 59′. 5″, and the parallax 55′. 48″,5.

CHAP. VII.

ON REFRACTION.

Art. 174. WHEN a ray of light passes out of a vacuum into any medium, or out of any medium into one of greater density, it is found to deviate from its rectilinear course towards a perpendicular to the surface of the medium into which it enters. Hence, light passing out of a vacuum into the atmosphere will, where it enters, be bent towards a radius drawn to the earth's center, the top of the atmosphere being supposed to be spherical and concentric with the center of the earth; and as, in approaching the earth's surface, the density of the atmosphere continually increases, the rays of light, as they descend, are constantly entering into a denser medium, and therefore the course of the rays will continually deviate from a right line and describe a curve; hence, at the surface of the earth, the rays of light enter the eye of the spectator in a different direction from what they would have entered, if there had been no atmosphere; consequently the apparent place of the body from which the light comes must be different from the true place. Also, the refracted ray must move in a plane perpendicular to the surface of the earth; for conceiving a ray to come in that plane before it is refracted, then the attraction being always towards the perpendicular which lies in that plane, the ray must continue to move in that plane. Hence, the refraction is always in a vertical circle. The ancients were not unacquainted with this effect. PTOLEMY mentions a difference in the rising and setting of the stars in different states of the atmosphere; he makes however no allowance for it in his computations from his observations; this correction therefore must be applied, where great accuracy is required. ARCHIMEDES observed the same in water, and thought the quantity of refraction was in proportion to the angle of incidence. ALHAZEN, an Arabian Optician, in the eleventh century, by observing the distance of a circumpolar star from the pole, both above and below, found them to be different, and such as ought to arise from refraction. SNELLIUS, who first observed the relation between the angles of incidence and refraction, says, that WALTHERUS in his computation allowed for refraction; but TYCHO was the first person who constructed a Table for that purpose, which however was very incorrect, as he supposed the refraction at 45° to be nothing. About the year 1660, CASSINI published a new Table of Refractions, much more correct than that of TYCHO; and since his time, Astronomers have employed much attention in constructing more correct Tables, the niceties of modern Astronomy requiring their utmost accuracy. We shall treat this subject, by first showing the practical methods by which the quantity of refraction is determined at some certain

altitudes, and then give the investigation of the rules for the variation at different altitudes, from which a Table for the Refraction at all altitudes may be constructed.

175. *First method.* Take the altitude of the sun, or a star whose right ascension and declination are known, and note the time by the clock; observe also the times of their transits over the meridian; then find (92) the hour angle; hence in the triangle PZx, we know PZ and Px the complements of latitude and declination, and the angle xPZ, to find the side Zx, the complement of which is the altitude, the difference between which and the observed altitude is the refraction of that altitude.

Ex. On May 1, 1738, at $5h.\ 20'$ in the morning, CASSINI observed the altitude of the sun's center at Paris to be $5°.\ 0'.\ 14''$, and the sun passed the meridian at $12h.\ 0'.\ 0''$, to find the refraction, the latitude being $48°.\ 50'.\ 10''$, and the declination was $15°.\ 0'.\ 25''$. The sun's distance from the meridian was $6h.\ 40'$, which gives $100°$ for the hour angle xPZ; also, $PZ=41°.\ 9'.\ 50''$ and $Px=74°.\ 59'.\ 35''$; hence, $Zx=85°.\ 10'.\ 8''$, consequently the true altitude was $4°.\ 49'.\ 52''$. Now to $5°.\ 0'.\ 14''$, the apparent altitude, add $9''$ for the parallax, and we have $5°.\ 0'.\ 23''$ the apparent altitude corrected for parallax; hence, $5°.\ 0'.\ 23''-4°.\ 49'.\ 52''=10'.\ 31''$ the refraction at the apparent altitude $5°.\ 0'.\ 14''$.

176. *Second method.* Take the greatest and least altitude of a circumpolar star which passes through, or very near, the zenith, when it passes the meridian above the pole; then the refraction being nothing in the zenith, we shall have the true distance of the star from the pole at that observation, the altitude of the pole above the horizon being previously determined; but when the star passes the meridian under the pole, we shall have its distance affected by refraction, and the difference of the two observed distances above and below the pole gives the refraction at the apparent altitude below the pole.

Ex. M. de la CAILLE observed at Paris a star to pass the meridian within $6'$ of the zenith, and consequently at the distance of $41°.\ 4'$ from the pole; hence it must pass the meridian under the pole at the same distance, or at the altitude $7°.\ 46'$; but the observed altitude at that time was $7°.\ 52'.\ 25''$; hence the refraction was $6'.\ 25''$ at that apparent altitude.

177. *Third method.* M. de la CAILLE also employed observations made at Paris and at the Cape of Good Hope, in order to ascertain the refraction. The method he made use of was this: The distance of the parallels of Paris and the Cape was found to be about $82°.\ 46'$, the half of which is $41°.\ 23'$; therefore a star vertical to a parallel in the middle between Paris and the Cape, must

be at the zenith distance of 41°. 23′ from each. Now the sum of the apparent zenith distances of such a star was found to be 82°. 44′. 46″, which therefore is the distance of the two parallels, diminished by the sum of the two refractions at the zenith distance 41° 23′, for refraction elevating a star, must make the apparent zenith distance of each star less than the true distance. Next, the apparent altitude of the pole at the Cape was observed to be 33°. 56′. 49″,1, and the altitude at Paris to be 48°. 52′. 27″,5, the sum of these two apparent altitudes is 82°. 49′. 16″,6 the distance of the parallels increased by the sum of the two refractions corresponding to the altitude of each pole. The difference of these two determinations is 4′. 30″,6 for the sum of the four refractions. Now taking the refraction to be as the tangent of the zenith distance, (182), he found the tangents of 41°. 23′, and of the complement of the altitudes of the two poles, and divided 4′. 30″,6 into four parts in the ratio of these tangents, making the refraction a fortieth part less at the Cape than at Paris, as he had observed it; hence, he got 1′. 36″,5 for the refraction at the altitude 33°, 56′. 49″,1 at the Cape, and 58″,2 at the altitude 48°. 52′. 27″,5 at Paris; also 57″,2 for the refraction at the zenith distance 41°. 23′ at the Cape, and 58″,7 for the refraction at the zenith distance 41°. 23′ at Paris. The altitudes and zenith distances corrected by these refractions give 82°. 46′. 42″ for the true distance of the parallels of Paris and the Cape.

178. Having determined the refraction at the altitude 48°. 52′. 27″,5 at Paris, he calculated the refractions from that altitude up to the zenith, upon supposition that they were as the tangents of the zenith distances, and hence he knew the refractions at these altitudes at the Cape. Therefore, by taking the meridian altitudes of stars from 7° to 48° at Paris, and the corresponding meridian altitudes at the Cape, and correcting these latter for refraction, he got the refraction from 7° to 48° at Paris; for the sum of the two true zenith distances was 82°. 46′. 42″, therefore knowing the true zenith distance at the Cape, the true zenith distance at Paris was known, the difference between which and the apparent zenith distance was the refraction. Thus M. de la Caille formed his Table of refractions. His method was very ingenious; but from more accurate observations since his time, it appears, that his refractions are a little too great. This Dr. Maskelyne has clearly shown in the *Phil. Trans.* 1787. By comparing the sum of the two apparent zenith distances of stars observed at a low altitude at Paris, and consequently at an high altitude at the Cape, and at an high altitude at Paris, and therefore at a low altitude at the Cape, he found the refraction at the Cape to be a fortieth part less than at Paris.

179. *Fourth method.* Boscovich proposes to find the refraction by the circumpolar stars, only by knowing its variation at different altitudes. Let a and a' be the apparent meridian zenith distances of a star below and above the pole, x and x' the respective refractions; b and b' the apparent meridian zenith distances of another star below and above the pole, z and z' the corresponding

refractions; then the true distance will be $a+x$, $a'+x'$, and $b+z$, $b'+z'$; and as the distance of the pole from the zenith is equal to half the sum of the greatest and least true zenith distances, $a+x+a'+x'=b+z+b'+z'$; hence, (A) $x+x'-z-z'=b+b'-a-a'$. Now taking, at first, the refractions to be as the tangent of the zenith distances, (182), we have tan. a : tan. a' :: x : $x'=\frac{x \tan. a'}{\tan. a}$; for the same reason $z=\frac{x \tan. b}{\tan. a}$, $z'=\frac{x \times \tan. b'}{\tan. a}$; substitute these into the equation (A), and we get $x=\frac{\overline{b+b'-a-a'} \times \tan. a}{\tan. a+\tan. a'-\tan. b-\tan. b'}$; hence the other refractions are known. But as the refractions vary more accurately as the tangent of the zenith distance diminished by three times the refraction, put $a-3x=m$, $a'-3x'=m'$, $b-3z=n$, $b'-3z'=n'$, and we have $x=\frac{\overline{b+b'-a-a'} \times \tan. m}{\tan. m+\tan. m'-\tan. n-\tan. n'}$, the correct refraction at the apparent altitude a; hence we know $x'=\frac{x \tan. m'}{\tan. m}$, $z=\frac{x \times \tan. n}{\tan. m}$ and $z'=\frac{x \tan. n'}{\tan. m}$. The operation may be shortened, by taking $3x$, $3x'$, $3z$, $3z'$ from the common Tables. As $a+x$, $a'+x'$, are the true zenith distances of one of the stars below and above the pole, the true zenith distance of the pole will be one half of $a+x+a'+x'$, which is the complement of the latitude of the place.

Ex. The apparent zenith distance of η *Draconis* below and above the pole was observed to be 69°. 5′. 2″,4 and 13°. 8′. 27″,2; and of θ *Ursæ minoris* 53°. 2′. 57″,2 and 29°. 11′. 23″,2; to find the corresponding refractions, and the latitude of the place.

$a = 69°.\ 5'.\ 2'',4$	$\tan. a = 2,616$	$\frac{c}{c'} = 52'',807$
$a' = 13.\ 8.\ 27,2$	$\tan. a' = 0,233$	$x = 52,807 \times 2,616 = 138'',2$
$b = 53.\ 2.\ 57,2$	$2,849$	$x' = 52,807 \times 0,233 = 12,3$
$b' = 29.\ 11.\ 23,2$	$\tan. b = 1,329$	$z = 52,807 \times 1,329 = 70,2$
$a + a' = 82.\ 13.\ 29,6$	$\tan. b' = 0,558$	$z' = 52,807 \times 0,558 = 29,5$
$b + b' = 82.\ 14.\ 20,4$	$1,887$	
$c = 0.\ 0.\ 50,8$	$c' = 0,962$	

$a = 69°.\ 5'.\ 2'',4$	$a' = 13°.\ 8'.\ 27'',2$	$b = 53°.\ 2'.\ 57'',2$	$b' = 29°.\ 11'.\ 23'',2$
$3x = 0.\ 6.\ 54,6$	$3x' = 0.\ 0.\ 36,9$	$3z = 0.\ 3.\ 30,6$	$3z' = 0.\ 1.\ 28,5$
$m = 68.\ 58.\ 7,8$	$m' = 13.\ 7.\ 50,3$	$n = 52.\ 59.\ 26,6$	$n' = 29.\ 9.\ 54,7$

$\tan. m = 2,6009$	$\frac{c}{c'} = 53'',505$	$a + a' \quad - \quad = 82°.\ 13'.\ 29'',6$
$\tan. m' = 0,2333$	$x = 53,505 \times 2,6009 = 139'',2$	$x + x' \quad - \quad = 0.\ 2.\ 31,7$
$2,8342$	$x' = 53,505 \times 0,2333 = 12,5$	$a + a' + x + x' = 82.\ 16.\ 1,3$
$\tan. n = 1,3266$	Refraction at zenith dist. 69°. 5'. 2'',4 is 139'',2; at zenith dist. 13°. 8'. 27'',2 is 12'',5.	$41.\ 8.\ 0,6$
$\tan. n' = 0,5581$		$90.\ 0.\ 0$
$1,8847$		Lat. of Place $48.\ 51.\ 59,4$
$c' = 0,9495$		We may get the correct refractions z, z' in like manner.

180. *Fifth method.* Dr. MASKELYNE informs us in the *Phil. Trans.* 1787, that Dr. BRADLEY found his refractions in the following manner. He observed the pole star, and other circumpolar stars, above and below the pole, and from thence deduced the apparent zenith distance of the pole. By the apparent and equal zenith distances of the sun at the two equinoxes, having at the same time opposite right ascensions, as found by comparing (118) its observed transits over the meridian with those of fixed stars, he found the apparent zenith distance of the equator, which diminished by parallax and added to the apparent zenith distance of the pole, gave a sum less than 90° by the sum of the two refractions belonging to the pole and meridian altitude of the equator*. Now he observed, that the difference of the refractions at these altitudes came out within 2'' or 3'', from the best Tables then extant, whether deduced solely from observations, or partly from observation and partly from theory. Hence, knowing the sum and difference of the refractions, he knew the refraction at each altitude. He afterwards more accurately divided the sum of the two refractions, by taking the parts in proportion to the tangents of the zenith

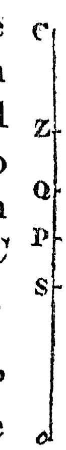
* For the sum of the two *true* zenith distances=90°; but the true distance of each is diminished by refraction, and therefore the sum (after the correction for parallax) must be less than 90° by the sum of the two refractions.

distances. The apparent zenith distance of the equator, by the mean of 20 observations in 1746-47 he found to be 51°. 27'. 28"; and the mean apparent zenith distance of the pole, by observations made between 1750–52, was 38°. 30'. 35"; the sum of which being 89°. 58'. 3" the sum of the two refractions is 1'. 57"; consequently the polar refraction is $45\frac{1}{2}$", and the equatorial 1'. $11\frac{1}{2}$"; therefore the latitude of Greenwich Observatory is 51°. 28'. $39\frac{1}{2}$". Dr. BRADLEY here supposed the sun's horizontal parallax to be $10\frac{1}{2}$"; but Dr. MASKELYNE observes, that had he taken it $8\frac{3}{4}$", as determined from the two last transits of *Venus* over the sun, the refraction at 45°, which he fixed at 57", would have come out $56\frac{1}{2}$", and the latitude of the Observatory 51°. 28'. 40". Dr. BRADLEY having thus settled the refraction at the altitude of the equator and pole, could calculate the refraction at all higher altitudes, or for all stars between the equator and pole, by taking it as the tangent of the zenith distances, which would be very accurate for all such altitudes. Hence, by taking the altitudes of the circumpolar stars above and below the pole, and knowing the refraction above, he immediately got the refraction at the lower altitudes; for knowing the refraction at the altitude above the pole, he knew the true altitude above, and knowing the altitude of the pole he got the true distance of the star from the pole, which subtracted from the altitude of the pole, gave the true altitude below, the difference between which and the apparent altitude was the refraction. When the weight and temperature of the air remain the same, the Dr. found that the refraction varied as the tangent of the zenith distance diminished by three times the refraction found by the common Rule; and having fixed the refraction at 45° (whose tangent, if radius = 1, is unity) to be 57", if $r =$ the refraction in the Tables, $z =$ the apparent zenith distance, he got this proportion, $r : 57'' :: \tan. \overline{z - 3r} : 1$.* And by comparing the refractions in different temperatures of the air, and at different altitudes of the barometer, he inferred the following elegant Rule for determining the refraction at all altitudes: Put $a =$ the altitude of the barometer in inches, $h° =$ the altitude of FAHRENHEIT'S thermometer, then the *true* refraction : $57'' :: \frac{a}{29{,}6} \times \tan. \overline{z - 3r} : \frac{h° + 350°}{400°}$. The very near agreement of this Rule with that given by MAYER, and their agreement with observations, are a strong confirmation of the accuracy of each.

* The application of this Rule to find the refraction at all altitudes is thus: Let the apparent zenith distance be z, then the refraction will be nearly $57'' \times \tan. z$, which put $= r$; and the correct mean refraction will be $57'' \times \tan. \overline{z - 3r}$. If at very low altitudes it should be required to have the refraction more correctly, put $57'' \times \tan. \overline{z - 3r} = r'$, and the refraction becomes $57'' \times \tan. \overline{z - 3r'}$. Let the refraction at the apparent zenith distance 70° be required. The tangent of 70° is 2,747; hence $57'' \times 2{,}747 = 2'. 36'{,}6$, which multiplied by 3 and subtracted from 70° gives 69°. 52'. 10", the tangent of which is 2,728; therefore $57'' \times 2{,}728 = 2'. 35''{,}5$ the *mean* refraction at the apparent zenith distance 70°. In this manner Table XI. was calculated.

This correction for the barometer and thermometer may be immediately found from Table XII.—The Instrument invented by Mr. RAMSDEN, called a *Circular Instrument* (for a description of which see my *Treatise on Practical Astronomy*), is admirably calculated to determine the quantity of refraction at all altitudes; for by taking the altitude and azimuth of a body whose declination is known, the true altitude may be immediately computed from the latitude of the place, declination of the body, and observed azimuth; hence, the difference between the observed and computed altitudes gives the refraction at that apparent altitude.

181. *Sixth method.* From Dr. BRADLEY's observations of the zenith distances of the polar star above and below the pole, and the zenith distance of Capella south of the zenith and below the pole, to find the mean refraction at 45°, the barometer being at 29,6 inches, and the thermometer at 50°; also, the mean declinations of the pole star and Capella, and the latitude of the place.



Let Z be the zenith, P the apparent place of the pole, C the apparent place of Capella south of the zenith, c that below the pole. Let the refraction at C (computed by Dr. BRADLEY's Rule) $=C$, at $P=P$, and at $c=c$; and let the true refractions at these places be respectively $n\,C$, $n\,P$, nc, or to those computed by Dr. BRADLEY's Rule, in the ratio of $n:1$. Then the true polar distance of Capella from the observation above the pole $=ZC+n\,C+ZP+nP$, and below the pole $=Zc+nc-ZP-nP$; hence, $n=\frac{ZC+2ZP-Zc}{c-2P-C}$. But as ZP, the apparent zenith distance of the pole, cannot be observed directly, let ZQ be the apparent zenith distance of the pole star above the pole, and ZS that below, and $n\,Q$, $n\,S$, the respective refractions; then $\frac{1}{2}(ZQ+ZS)+\frac{1}{2}(nQ+nS)=$ co-latitude; but this quantity added to the true zenith distance of Capella south of the zenith $=$ true distance of Capella below the pole, lessened by the same quantity; hence, $\frac{1}{2}(ZQ+ZS)+\frac{1}{2}(nQ+nS)+ZC+nC=Zc+nc-\frac{1}{2}(ZQ+ZS)-\frac{1}{2}(nQ+nS)$, and $n=\frac{ZC+ZQ+ZS-Zc}{c-Q-S-C}$ the ratio of the refractions to Dr. BRADLEY's refraction. If a number of zenith distances of the pole star above and below the pole be observed, and also of Capella south of the zenith and below the pole, and their refractions be computed by Dr. BRADLEY's Rule, the mean of each being taken, we shall obtain n more accurately. For example:

ZC mean of	25 =	5°. 45′. 38″,4	C mean of	25 =	0′. 5″,78
ZQ	94 =	36. 28. 22,23	Q	94 =	0. 42,6
ZS	109 =	40. 32. 50,65	S	109 =	0. 48,64
Sum -	=	82. 48. 51,28	Sum -	- =	1. 37,02
Zc mean of	44 =	82. 41. 25,14	c mean of	44 =	6. 58,48
Dif. -	=	5. 26,14	Dif. -	=	5. 21,46

Hence, $n=\frac{5.\ 26,14}{5.\ 21,46}=1.01456$, which multiplied by $57''$ Dr. BRADLEY's refraction at $45°$ gives $57'',83$ the corrected refraction.

Or n may be found thus: Let the observed zenith distances of two circumpolar stars above and below the pole, when corrected for the equations of the stars to reduce them to their mean place, and reduced by precession to the same epoch, be A, B, and C, D, the former, that nearest the pole; and the corresponding computed refractions by Dr. BRADLEY's Rule, be a, b, and c, d; then double the co-latitude will be $A+a+B+b$ and $C+c+D+d$; but calling the corrected refractions na, nb, nc, nd, we then have $A+na+B+nb=C+nc+D+nd$, and $n=\frac{A+B-C-D.}{c+d-a-b}$

Let one of the stars be the sun, and C, D its observed zenith distance, at the summer and winter solstice, corrected by its parallax, equation of obliquity, and reduced by its gradual diminution to the same epoch as for the star; then the double latitude for the sun $=C+nc+D+nd$, and co-latitude for the star $=A+na+B+nb$; hence, $A+na+B+nb+C+nc+D+nd=180°$, and $n=\frac{180°-(A+B+C+D)}{a+b+c+d}$: these methods were given by Dr. MASKELYNE.

Having thus explained the practical methods of finding the refraction, we proceed to investigate its laws.

182. Let ACn be the angle of incidence, ACm the angle of refraction, and consequently mCn the quantity of refraction; let AT be the tangent of Am, mv its sine, nw the sine of An, and draw rm parallel to vw; then as the refraction in air is very small, we may consider mrn as a rectilinear triangle, and hence, by similar triangles, $Cv : Cm :: rn : mn=\frac{Cm \times rn}{Cv}$; but Cm is constant, and as the ratio of mv to nw is constant by the laws of refraction, their difference rn must vary as mv; hence, mn varies as $\frac{mv}{Cv}$; but $AT=\frac{Cm \times mv}{Cv}$ which varies as $\frac{mv}{Cv}$, because Cm is constant; hence, the refraction mn varies as AT, the tangent of the apparent zenith distance of the star, because the angle of refraction ACm is the angle between the refracted ray and the perpendicular to the surface of the medium, which perpendicular is directed to the zenith. Whilst therefore the refraction is very small, so that rmn may be considered as a rectilinear triangle, this Rule will be sufficiently accurate; otherwise we must use Dr. BRADLEY's Rule, the demonstration of which is given by BOSCOVICH in his Works, Vol. II. but one of the principles, that the force with which the ray is attracted in passing through the air may be considered as uniform, is taken from Mr. SIMPSON's Solution in his Mathematical Dissertations. We shall therefore first give his reasons for this supposition. After constructing his

Table of refraction, he observes, that the only material objection which it is liable to is, its being founded upon supposition, that the density of the air decreases uniformly, which appears contrary to experiment, whereby it is proved, that the density of the air decreases as the compressing force decreases: But though this is true in air of the same temperature, yet it cannot be supposed to hold true in the earth's atmosphere, since the upper region thereof is known to be much colder, and consequently the elasticity there is much less than at the earth's surface: But a convincing proof that this law of density cannot obtain in our atmosphere is, that the mean horizontal refraction computed from it, according to the known refractive power and specific gravity of the air, will be found to come out no less than $52'$, which is greater by about $\frac{1}{3}$ of a degree than it ought to be, it being only $33'$; whereas, if the same refraction be calculated upon the hypothesis of the density decreasing uniformly, and compared with observations, the difference will be much less. This latter hypothesis will therefore best correspond to the state of our atmosphere.

183. Let us therefore suppose the atmosphere to be divided into an infinite number of lamina concentric with the center of the earth, and of an equal thickness, then the density of these lamina is supposed to decrease uniformly, for the reasons above given, and therefore the difference of the densities is constant. But when a ray of light passes out of one medium into another, it is attracted by a force which depends on the difference of their densities, and therefore when the difference is constant the force is constant. Hence, a ray of light descending through the atmosphere may be supposed to be attracted by it in a direction perpendicular to the surface of the earth by a constant force.

184. Let C be the center of the earth, AM its surface, ZF the top of the atmosphere, FA the passage of the ray; draw the tangents SFH, IAG cutting each other in I, and let CH, CG be drawn perpendicular to them, and AL parallel to CF. Now the state of the atmosphere remaining the same, the sine of incidence is to the sine of refraction for each lamina in a given ratio, therefore by composition, the sine of incidence CFH at F is to the sine of refraction CAG at A in a given ratio. Hence, if radius $=1$, $\frac{CH}{CF}$ and $\frac{CG}{CA}$ will be these respective sines; but the velocities at F and A are as CG to CH, which assume as 1 to $1+b$; and if $MF=e$, $CM=1$, $\frac{CH}{CF}:\frac{CG}{CA}::\frac{1+b}{1+e}:1$; put $m=\frac{1+b}{1+e}$, $a=$ angle CAG, and then $1:m::\sin. a:\sin. CFH=m\times\sin. a$. Let $x=$ angle ACF, $r=$ angle GIH of refraction. In the quadrilateral figure $CAIF$, the angle $ACF+IFC=$ the sum of the external angles $GIH+CAG$, because $FIA+CAI$ added to each would make the sum equal to four right FIG. 35.

angles; hence, IFC or $CFH = CAG - ACF + GIH$, that is, $m \times \sin. a = \sin. \overline{a - x - r}$, therefore $1 : m :: \sin. a : \sin. \overline{a - x - r}$; but by plain trigonometry, the sum of the sines of two angles : their difference :: tan. of half the sum of the angles : tan. of half their difference; hence, $1 + m : 1 - m :: \tan. \overline{a - \frac{1}{2}.x - r}$: $\tan. \frac{1}{2}.\overline{x - r}$, and as this ratio is constant, the tan. $\overline{a - \frac{1}{2}.x - r}$ varies as the tan. $\frac{1}{2}.\overline{x - r}$; but as the difference between x and r must be very small, the tangent of $\frac{1}{2}.\overline{x - r}$ may be considered as equal to the angle itself $\frac{1}{2}.\overline{x - r}$; also, a is the apparent zenith distance; hence, the angle $\frac{1}{2}.\overline{x - r}$ varies as the tangent of the apparent zenith distance diminished by $\frac{1}{2}.\overline{x - r}$. If therefore the ratio of x to r be constant, then $x - r$, and consequently r itself, will vary as the tangent of the zenith distance diminished by some multiple of r; for if $dr = x$, then $x - r = dr - r = \overline{d - 1} \times r$; let therefore $1 + m : 1 - m :: \tan. \overline{a - \frac{1}{2}nr} : \tan. \frac{1}{2} nr$, and then the refraction r varies as $\tan. \overline{a - \frac{1}{2} nr}$. On this supposition $\frac{1}{2}.\overline{x - r} = \frac{1}{2} nr$, or $x - r = nr$. That x is to r in a constant ratio may be thus proved.

185. Let us conceive AF to be an indefinitely small part of the whole curve, taken any where, and AL (which is drawn parallel to FC) is the sagitta of the curve. Put $v =$ the velocity through FA, $\dot{t} =$ the time, $z = CF$, $\dot{z} = FM$, $\dot{x} =$ the angle FCA, $\dot{r}$ = the angle GIH, $f =$ the force in the direction FC. Now from the principles of Mechanics, $AF = v\dot{t}$, and the sagitta $LA = \frac{1}{2}\ddot{z} = f\dot{t}^2$; hence, the tangent AI (which $= \frac{1}{2}AF) = \frac{1}{2}v\dot{t}$; also, as the arc varies as the angle multiplied into the radius, $AM = z\dot{x}$, and the sine of ALI or CFL $= \frac{AM}{AF} = \frac{z\dot{x}}{v\dot{t}}$; but $AI : AL :: \sin. ALI : \sin. AIL$, that is, $\frac{1}{2}v\dot{t} : f\dot{t}^2 :: \frac{z\dot{x}}{v\dot{t}} : \sin. \dot{r}$ or $\dot{r}$, hence, $\frac{\dot{r}}{\dot{x}} = \frac{2fz}{v^2}$. Now if we consider the velocity and distance from the center as having but a very small variation, and f to be constant (183), we may consider $\frac{\dot{r}}{\dot{x}}$ as constant, and consequently $\dot{r}$ varies as $\dot{x}$, therefore r varies as x when AF is finite. Hence (184), r varies as the tan. $\overline{a - \frac{1}{2}nr}$.

186. Because $1 + m : 1 - m :: \tan. \overline{a - \frac{1}{2} nr} : \tan. \frac{1}{2} nr ::$ (by trig.) $\sin. a + \sin. \overline{a - nr} : \sin. a - \sin. \overline{a - nr}$, hence, $m \times \sin. a = \sin. \overline{a - nr} =$ (by trig.) $\sin. a \times \cos. nr - \sin. nr \times \cos. a =$ (because nr being a very small arc its $\cos. = \sqrt{1 - n^2r^2}, = 1 - \frac{1}{2}n^2r^2$, and the sine = arc very nearly) $\sin. a - \sin. a \times \frac{1}{2}n^2r^2 - nr \times \cos. a$, and by dividing by $\sin. a$, we have $m = 1 - \frac{1}{2} n^2r^2 - nr \times \cot. a$. Now let a' be any other apparent zenith distance, and r' the refraction, then, for the same reason, $m = 1 - \frac{1}{2} n^2r'^2 - nr' \times \cot. a'$; make these values of m equal, and we get $\frac{1}{2} n = \frac{r' \times \cot. a' - r \times \cot. a}{r^2 - r'^2}$. Now by Dr. BRADLEY's observations, if $a = 60°$, $r = 1'. 38'',4$; and if $a = 90°$, $r = 33'$; hence, $\frac{1}{2} n = 2,996$; he therefore

assumes $\frac{1}{2}n=3$; the refraction therefore varies as the tang. $a-3r$, that is, *the refraction varies as the tangent of the apparent zenith distance diminished by three times the refraction.* SIMPSON makes $n=5,5$, CASSINI $=6,452$ and BOUGUER $=6,645$. But Dr. BRADLEY's value is most to be depended upon, as best agreeing with observations, which we shall therefore follow.

187. Because $m=1-\frac{1}{2}n^2r^2-nr\times\cot. a$, therefore, as $\frac{1}{2}n^2r^2$ is very small in respect to the other terms, $m=1-nr\times\cot. a$; hence, $1-m=nr\times\cot. a$. For the horizontal refraction, $a=90^\circ$, $r=33'$; therefore $m=1-\frac{1}{2}n^2r^2=\cos. nr$; hence, if $n=6$, we have $m=\cos. 6r=\cos. 3'. 18''=0,9983$. Hence also (184), $x-r=nr=6r$, according to Dr. BRADLEY, therefore $x=7r$, or the angle which the refracted ray subtends at the center of the earth $=7$ times the refraction.

188. Join CI, and let the angle $ACI=y$, then CIA or $CIG=a-y$, $CIH=a-y+r$, and their sines are as the perpendiculars CG, CH, which are inversely as the velocities at A and F, or as $1:1+b$; hence, $\overline{1+b}\times\sin.\overline{a-y}=\sin.\overline{a-y+r}=\sin. a-y\times\cos. r+\sin. r\times\cos.\overline{a-y}=$(because r being very small its $\cos.=1$, and it's sine$=r$) $\sin.\overline{a-y}+r\times\cos.\overline{a-y}$; hence, $1+b=1+r\times\cot.\overline{a-y}$, and $b=r\times\cot.\overline{a-y}$. But if we make a approach to 90°, y will be very small when compared with a, therefore $b=r\times\cot. a$. If $a=60^\circ$, then $r=1'. 38'',4$ according to Dr. BRADLEY; hence, $b=r\times\cot. a=\sin. r\times\cot. a=0,0002755$; therefore the sine of incidence out of a vacuum into air at the mean density at the earth's surface is to the sine of refraction as $1,0002755:1$. Mr. HAUKSBEE makes it as $1.000264:1$ by experiment. As $b=r\times\cot. a$, therefore $6b=6r\times\cot. a=1-m$ from the last Article; hence, $b=\frac{1-m}{6}$.

189. Having determined the values of b and m, we get, from the equation $\frac{1+b}{1+e}=m$, the value of $e=\frac{1-m+b}{m}=\left(\text{as } b=\frac{1-m}{6}\right)\frac{7-7m}{6m}=0,001942$ parts of the earth's radius $=77,25$ miles, the altitude above the earth's surface at which the air begins to have any sensible effect on the rays of light to refract them.

190. The refraction varies as the tan. $\overline{a-3r}$ at any altitude above the earth's surface; for the proof remains the same for whatever part of the curve you take from the top of the atmosphere. Hence we may find the refraction at any altitude, by making e denote its distance from the top of the atmosphere; for by the last Article $m=\frac{7}{7+6e}=$(by division, and neglecting all the powers of e above the first on account of their smallness) $1-\frac{6e}{7}=$(187) $\cos. 6r$; hence, the cos. of $6r$ being known, $6r$, and consequently r itself, the horizontal refraction in this case, will be known, and hence the refraction at any other altitude.

4

191. As (186) $m \times \sin. a = \sin. \overline{a - 6r}$, according to Dr. BRADLEY, put $p =$ the complement of a, and let $m \times \cos. p = \cos. q$, then $\cos. q = m \times \cos. p = m \times \sin. a = \sin. \overline{a - 6r} =$ (as $a = 90° - p$) $\sin. \overline{90° - p - 6r} = \cos. \overline{p + 6r}$, hence, $p + 6r = q$, therefore $r = \frac{q - p}{6}$. This expression is accommodated to find the refraction below the horizon, when the observer is elevated above it, by making p negative. Hence, the refraction below the horizon increases very fast, r being expressed by the sum of p and q.

192. In the horizon, $\cos. 6r = 1 - \frac{6e}{7}$, therefore $\frac{6e}{7} = 1 - \cos. 6r = \text{ver. sin. } 6r = 18r^2$ by the property of the circle; consequently the horizontal refraction r varies as the square root of e. Hence, if h be the altitude of the atmosphere, we know the horizontal refraction at any altitude $h - e$ above the horizon, for it will be to the horizontal refraction on the earth's surface as $\sqrt{e} : \sqrt{h}$. The horizontal refraction therefore being known, the refraction at any other altitude will be known.

193. Upon the same principles, we have a very elegant method of finding the radius of curvature to the curve which the ray describes. Let AF be an indefinitely small part of the curve adjacent to A the surface of the earth, and conceive AZV to be a circle of curvature, O its center, and QOK perpendicular to AV, which therefore must bisect AV. Then the angle $AIE = FOA = 2FVA = FKA$; but (187) $7AIE = FCA$, therefore $7FKA = FCA$; hence, $AK = 7AC$ the radius of the earth, and therefore is a constant quantity for all angles IAE. Hence, the center of the circle of curvature is always in the line QK. By trig. $AO : AK ::$ rad. $= 1 : \sin. AOK$ or IAE; hence, $AO = \frac{AK}{s.IAE} = \frac{7AC}{s.IAE}$, and as AC is constant, the radius of curvature varies inversely as the sine of the apparent zenith distance. Hence, for horizontal refractions, the radius of curvature is equal to 7 times the radius of the earth. This agrees with the conclusions deduced by J. H. LAMBERT in his very elegant Treatise entitled, *Les Proprietés remarquables de la Route de la Lumière par les Airs*, which he has applied with so much success to terrestrial refractions, and which we shall now proceed to consider.

194. Suppose MF to be any object, and FA the curve described by a ray of light coming from F to A; then for so small a distance we may suppose FA to be circular. Let $m = \sin. FAE$, then $AO = \frac{7AC}{m}$ is known. Now the effect of refraction in altering the apparent altitude is the angle between AI and the chord drawn to the arc FA; for the latter is the direction in which F would be seen if there were no refraction, and the former if seen by refraction; but this

angle between the chord and tangent must be equal to $\frac{1}{2}FOA=\frac{FA}{2AO}$; but FA $=\frac{AM}{m}$, and $AO=\frac{7AC}{m}$; hence, the refraction $=\frac{AM}{14AC}=\frac{1}{14}$ of the angle ACM. Hence, any point situated in the line MF, and seen at A, has the same refraction, for it is independent of the altitude MF; consequently any object situated in a line perpendicular to the earth will not have its apparent length altered by refraction, because each end will appear equally elevated by it. Hence also, the terrestrial refraction varies as the distance AM. If therefore MF be a mountain, and we want to find the altitude from the given distance AM, and the apparent angle of elevation MAI, we must first correct this angle by subtracting from it $\frac{1}{14}$ of ACM.

195. Hence we may readily find the distance at which an object of a given altitude whose top is depressed below the horizon, may be seen by refraction. For take $AK=7AC$, and with the center K describe the circle Ar, and the FIG.
point r will be seen by refraction; draw $srvC$, and Av is the distance at which 37.
an object vr is visible; draw also the tangent Ax. Now the angles ACv, AKr being very small, and the arcs Av, Ar very nearly equal, $sr : sv :: AC : AK ::$ $1 : 7$, and $vr : sv :: 6 : 7$, therefore $sv=\frac{7vr}{6}$; but $sv=\frac{Av^2}{2}$, the radius of the earth being unity; therefore $\frac{Av^2}{2}=\frac{7vr}{6}$, consequently $Av=\sqrt{\frac{14vr}{6}}=\sqrt{\frac{7vr}{3}}$. Hence, the distance at which an object can be seen, varies as the square root of its altitude.

196. If yw be perpendicular to the surface of the earth and equal to vr, the object vr can be seen at y without refraction; but yw or $vr=\frac{Ay^2}{2}$; hence, Ay $=\sqrt{2vr}$, therefore the distance at which an object can be seen by refraction : distance at which it could be seen without refraction $:: \sqrt{\frac{7vr}{3}} : \sqrt{2vr} :: \sqrt{7}$ $: \sqrt{6}$, which is nearly as $14 : 13$.

197. An eye at r sees A in the direction of the tangent at r, and therefore it appears below the horizon at v by the angle formed by the two tangents to r and v, or by the angle CrK. Now (195) Av, or the angle ACv, $=\sqrt{\frac{7vr}{3}}$, and $Kr : CK (:: 7 : 6) ::$ sin. rCK or rCA : sin. $CrK ::$ (on account of the smallness of these angles) rCA, or $\sqrt{\frac{7vr}{3}}$, : $CrK=\sqrt{\frac{12vr}{7}}$ the depression of the point A below the horizon. Hence, the depression below the horizon varies as the square root of the altitude.

198. Considering the arcs Av, Ar as equal on account of the smallness of the angle ACv, the sagittas sv, sr will be inversely as the radii; hence, $sv : sr :: 7 : 1$, therefore $rv : sv :: 6 : 7$, and consequently the point r appears to be elevated by a quantity equal to $\frac{1}{6}\ vr$ or $\frac{1}{7}\ sv$; but $sv = \frac{Av^2}{2}$, therefore $sr (= \frac{1}{7} sv) = \frac{Av^2}{14}$. Hence, as the refraction remains nearly the same for all objects near the horizon, this correction must be made in calculating the altitudes of such objects from the apparent angles of elevation. All the above numbers are for the mean state of the air.

199. Hence, we may find the altitude vr of a cloud at r, by observing the instant when it ceases to be enlightened by the sun; for at that time calculate the depression of the sun below the horizon, and from it subtract the horizontal refraction and you will have the true depression below the horizon, or the angle between As and a tangent to v, or the angle ACv; hence we know Av, and consequently vr. This supposes that the ray coming to the cloud is a tangent to the surface of the sea, or to an horizontal plane at land.

200. Let SB be a ray of light falling on the atmosphere at B and refracted in the curve BAE touching the earth at A, and emerging in the direction EF, meeting DC parallel to SB in F; to find CF. As Cv is a perpendicular upon the incident ray, and CA upon the refracted ray, they will be as the sine of incidence to the sine of refraction out of a vacuum into air of the same density as that at the earth's surface, or as $1{,}0002755 : 1$; hence, put $m = 1{,}0002755 = Cv = Cr$, $n =$ the angle $CFr = Fsx = rCv = 2ACv$, or twice the horizontal refraction, and $CF = Cv \times \text{cosec. } n =$ (if $n = 66'$) $53{,}1$ radii of the earth. If the direction of the ray of light be not parallel to DC but to dCf, and the angle dCD be put$=x$, then the angle $rCf = n + x$, and $Cf = v \times \text{cosec. } \overline{n + x}$.

201. If the line dC be supposed to join the centers of the sun and earth, and the ray SB to come from the limb of the sun, Cf will be the length of the total shadow of the earth, as all the umbra beyond f will have some rays of the sun by refraction. Now let $x = 16'$ the sun's semidiameter, and $Cf = v \times \text{cosec. } 82' = 41{,}94$ semidiameters of the earth, which being very little more than $\frac{2}{3}$ of the distance of the moon, it appears, that in a total eclipse of the moon, some rays from the sun must fall upon it, which is the cause of its being visible in that situation.

202. Having thus fully explained the principles of refraction, and the methods of constructing the Tables for the mean refraction, it will be proper to give some account of the variations to which the air is subject, from a change of temperature and density, for which proper corrections are given, except when the observations are very near to the horizon, where changes frequently take place which cannot be altogether accounted for, and for which therefore

no correction can be applied; they probably arise from exhalations of various kinds which are suddenly raised and suspended in the air near to the earth's surface, the causes of which do not sensibly affect the barometer and thermometer. Hence, all observations made very near to the horizon must be subject to a very considerable degree of uncertainty, and therefore Astronomers never use them when great accuracy is required.

203. TYCHO, when he constructed his Table of refraction, knew that it was subject to variation; but CASSINI and PICARD were the first who measured accurately the change. PICARD found, from the meridian altitudes of the sun, that the refraction was greater in winter than in summer; he observed also, that it was greater in the night than in the day. And from observing the horizontal refraction of the upper limb of the sun when it first appeared in the horizon, and then that of the lower limb, he found that in the time in which the sun was rising, the refraction was diminished 25″. BOUGUER observed in America, that the refractions in the night were greater than in the day, by about $\frac{1}{6}$ or $\frac{1}{7}$. Dr. NETTLETON measured the altitude of an hill in a clear day; and repeating the observations in a cloudy day when the air was somewhat gross and heavy, he found the angle considerably greater. He also observed, that the altitudes of some of the hills which he measured appeared greater in the morning before sun rise and late in the evening, than at noon in a clear day. At the time of the great frost at Paris in 1740, MONNIER observed, when the thermometer was 10° below the freezing point, that at the apparent altitude 4°. 44½′ the refraction was 11′. 15″; but when the mercury stood at 24° above the freezing point, the refraction at the same altitude was found to be only 9′. 20″; hence there was a difference of 1′. 55″ for 36°. of the thermometer. The barometer was at 28 inches. From these differences of refractions in summer and winter, in the day and night, it might be conjectured that the refractions would be greater towards the north, where it is colder. But the French Academicians in the year 1737, at Tornea on the borders of Lapland, where they were sent to measure a base in order to determine the length of a degree of latitude, found that the refractions agreed with those at Paris. M. de la CAILLE however found that the refractions at the Cape of Good Hope, were about $\frac{1}{40}$ less than at Paris; from which small difference, he concluded that a Table of refractions might be constructed which would answer very accurately for every part of the temperate zone. In the torrid zone M. BOUGUER found the horizontal refraction to be 27′; at 6° high, 7′. 4″; and at 45° high, 44″. Admitting therefore the refraction to be less in climates warmer than at Paris, we may conclude that it must be greater in those which are colder, and that it was from want of a sufficient number of observations, or from their inaccuracy, that the Academicians in Lapland did not find it so.

204. The refraction being thus found to vary in different states of the air, the next enquiry is, what allowance must be made for any variation of the temperature and weight of the air, from any standard which we may make the mean. Dr. BRADLEY made 29,6 inches the mean standard for the barometer, and as Mr. HAUKSBEE had determined from experiment that the refraction was in proportion to the density of the air, it must also be as the altitude of the mercury in the barometer. Now in the mean state of the air, that is, when the barometer is at 29,6 inches, and FAHRENHEIT's thermometer at 50°, the refraction (180) : $57'' :: \tan. \overline{z-3r} : 1$; hence, at any altitude (a) of the mercury, the refraction : $57'' :: a \times \tan. \overline{z-3r} : 29,6$. The refraction, thus corrected for the variation of the weight of the air, agrees very well with observations. The next thing to be done is, to find how the refraction varies in different temperatures. M. de la CAILLE found that the refraction was diminished $\frac{1}{27}$ part from an increase of 10° in the altitude of the mercury in the thermometer of REAUMUR. MAYER observed that the refraction varied about $\frac{1}{22}$ part for 10° of variation. M. BONNER made some experiments in order to determine the variation of refraction arising from that of the temperature; calling the refraction unity for the altitude 10° of the thermometer, he found the refraction to be 0,92 at the altitude 30°, or diminished $\frac{1}{25}$ for a variation of 10°; and at 8° below 0° he found the refraction to be 1,085 or $\frac{1}{21,18}$ for a variation of 10°. The mean of these differ but very little from the determination of MAYER. The observations upon which Dr. BRADLEY formed his rate of variation, have never been published. He used FAHRENHEIT's thermometer, and fixed the mean temperature at 50°; and if $h°$ be any other altitude, he found that the refraction varied in the ratio of $400° : h° + 350°$, or $1 : \frac{h° + 350°}{400°}$. Hence, allowing for the variation of temperature and weight, he found, the *true* refraction : $57'' :: \frac{a}{29,6} \times \tan. \overline{z-3r} : \frac{h° + 350°}{400°}$. And this agrees very accurately with the Rule deduced by MAYER.

205. When the sun is in the horizon, the rays in passing very obliquely through the atmosphere are so far separated, that M. BOUGUER, in a Work entitled *Traité d'Optique sur la Gradation de la Lumière*, has concluded from experiment, that the intensity of light is 1354 times less than when the sun is in the zenith. M. de MAIRAN thinks that the weakness of the sun's rays in the former case is principally to be attributed to the quantity of vapours with which the lower parts of the atmosphere are always filled.

206. It is owing to the atmosphere that we have any twilight in the morning and evening, which arises both from refraction and reflection of the sun's rays. It may be explained thus. Let AB be the surface of the earth, Sm a

ray of light coming from the sun, and beginning to be refracted at m; let it describe the curve mBn touching the earth at B, and at n let it be reflected into the curve nA, touching the earth at A, the place of the spectator; in this position therefore of the sun, the twilight just appears; draw the tangents Avz, mz, Bv, and join vnC. Then AO (the radius of curvature to the arc An) $=7AC$ (193), considering An as a circle, from which it will differ but very little. Now suppose twilight to begin when the sun is 18° below the horizon, that being about the quantity found by computing the sun's depression from the observed time at which the twilight begins; it varies however in different seasons; hence, the angle $z=162°$; but the difference between the angle z and the angle BvA is the refraction through mB, or 33′; therefore the angle $AvB=$ 162°. 33′, and $AvC=81°.\ 16\frac{1}{2}'$, consequently $ACv=8°.\ 43\frac{1}{2}'$, and hence nCO $=171°.\ 16\frac{1}{2}'$; also, $On : Oc :: 7 : 6$; hence, $On=7 : Oc=6 ::$ sin. $nCO=171°.\ 16\frac{1}{2}'$: sin. $CnO=7°.\ 28\frac{1}{4}'$, therefore $nOC=1°.\ 15\frac{1}{4}'$; hence, sin. nCO : sin. $nOC :: On=7 : Cn=1{,}01$, from which take $Cx=1$, and we have $nx=0{,}01=$ 39,64 miles. But (189) the ray begins to be refracted at the altitude of 77,25 miles; hence the reflection takes place at about half the altitude at which the refraction begins. This is upon supposition that the rays come to the spectator after one reflection. If we suppose them to come after 2, 3 or 4 reflections, the altitudes nx will be about 12, 5,4 and 3 miles respectively, and the densities of the air 10,75, 2,9 and 1,8 less than at the earth's surface. Which of these is most probable, may admit of some doubt. That air at the altitude of 39,64 miles, where it is 2700 less dense than at the earth's surface, should have the power of reflecting rays so copiously, is almost incredible. And why should that particular density reflect, when it is not the boundary of the atmosphere, it having been shown that light is refracted at twice that altitude? It appears more probable that the reflection arises from the vapours and exhalations of various kinds with which the lower parts of the atmosphere are charged; for the twilight lasts till the sun is further below the horizon in the evening, than it is in the morning when it begins; and it is longer in summer than in winter. Now in the *former* case, the heat of the day has raised the vapours and exhalations; and in the *latter*, they will be more elevated from the heat of the season; therefore, upon supposition that the reflection is made by them, the twilight ought to be longer in the evening than in the morning, and longer in summer than in winter.

207. Another effect of refraction is that of giving the sun and moon an oval appearance, by the refraction of the lower limb being greater than that of the upper, whereby the vertical diameter is diminished. For suppose the diameter of the sun to be 32′, and the lower limb to touch the horizon, then the mean refraction at that limb is 33′, but the altitude of the upper limb being then 32′, its refraction is only 28′. 6″, the difference of which is 4′. 54″, the quantity by

which the vertical diameter appears shorter than that parallel to the horizon. When the body is not very near the horizon, the refraction diminishing nearly uniformly, the figure of the body is very nearly that of an ellipse. Now it is proved in that article where the diminution of weight of a body upon the surface of a spheroid is investigated, that the diameter (D) of an ellipse, which is nearly a circle, is diminished, in going from the major to the minor axis, as the square of the sine (s) of the angle which it makes with the major axis; hence, if d = the diminution of the vertical diameter, rad.$^2 : s^2 :: d$: the diminution of the diameter D. Thus we may find the diameter in any position; and in cases where extreme accuracy is required, such as measuring with a micrometer the distance of Venus or Mercury on the sun's disc from its limb, this circumstance may be considered.

CHAP. VIII.

ON THE SYSTEM OF THE WORLD.

Art. 208. WHEN any effect or phænomenon is discovered by experiment or observation, it is the business of Philosophy to investigate its cause. But there are very few, if any, enquiries of this kind, where we can be led from the effect to the cause by a train of mathematical reasoning, so as to pronounce with certainty upon the cause. Sir I. Newton therefore, in his Principia, before he treats on the System of the World, has laid down the following Rules to direct us in our researches into the constitution of the universe.

Rule I. No more causes are to be admitted than what are sufficient to explain the phænomenon.

Rule II. Of effects of the same kind, the same causes are to be assigned, as far as it can be done.

Rule III. Those qualities which are found in all bodies upon which experiments can be made, and which can neither be increased nor diminished, may be looked upon as belonging to all bodies.

Rule IV. In Experimental Philosophy, propositions collected from phænomena by induction, are to be admitted as accurately or nearly true, until some reason appears to the contrary.

The principles, upon which the application of these Rules is admitted, are, the supposition that the operations of nature are performed in the most simple manner, and regulated by general laws. And although their application may, in many cases, be very unsatisfactory, yet in the instances to which we shall here want to apply them, their force is little inferior to that of direct demonstration, and the mind rests equally satisfied as if the matter could be strictly proved.

209. The diurnal motion of all the heavenly bodies may be accounted for, either by supposing the earth to be at rest, and all the bodies daily to perform their revolutions in circles parallel to each other; or by supposing the earth to revolve about one of its diameters as an axis, and the bodies themselves to be fixed, in which case their apparent diurnal motions would be the same. If we suppose the earth to be at rest, all the fixed stars must make a complete revolution, in parallel circles, every day. But it will be shown in a future part of this Work, that the nearest of the fixed stars cannot be less than 400000 times further from us than the sun is, and that the sun's distance from the earth is not less than 93 millions of miles. Also from the discoveries which are every day

making by the improvement of telescopes, it appears that the heavens are filled with an almost infinite number of stars, to which the number visible to the naked eye bears no proportion, and whose distances are, probably, incomparably greater than what we have stated above. But that an almost infinite number of bodies, most of them invisible except by the best telescopes, at almost infinite distances from us and from each other, should have their motions so exactly adjusted, as to revolve in the same time, and in parallel circles, and all this without their having any central body, which is a physical impossibility, is an hypothesis, which, by the Rules we have here laid down, is not to be admitted, when we consider, that all the phænomena may be solved simply by the rotation of the earth about one of its diameters. If therefore we had no other reason, we might rest satisfied that the apparent diurnal motions of the heavenly bodies are produced by the earth's rotation. But we have other reasons for this supposition. Experiments prove that all the parts of the earth have a gravitation towards each other. Such a body therefore, the greatest part of whose surface is a fluid, must, from the equal gravitation of its parts, form itself into a perfect sphere. But it appears from mensuration, that the earth is not a perfect sphere, but a spheroid, having the equatorial longer than its polar diameter. Now if we suppose the earth to revolve, the parts most distant from the axis must, from their greater velocity, have a greater tendency to fly off, and therefore that diameter which is perpendicular to the axis must be increased. That this must be the consequence appears from taking an iron hoop and making it revolve swiftly about one of its diameters, and that diameter will be diminished and the diameter perpendicular to it increased. The figure of the earth must therefore have arisen from its rotation, which is further confirmed from the following consideration. There can be but one diameter about which the earth can revolve, which can solve all the phænomena of the apparent revolution of the heavenly bodies; for if the diameter about which the earth is supposed to revolve were changed, it would change the situation of all the bodies in respect to the horizon and zenith; now *that* diameter about which the earth must revolve, in order to satisfy all the phænomena, is the diameter which, from mensuration, is found to be the shortest. Another reason for the earth's rotation is from analogy. The planets are opaque and spherical bodies like to our earth; now all the planets, on which sufficient observations have been made to determine the matter, are found to revolve about an axis, and the equatorial diameters of some of them are visibly greater than their polar. When these reasons, all upon different principles, are considered, they amount to a proof of the earth's rotation about its axis, which is as satisfactory to the mind as the most direct demonstration could be. These however are not all the proofs which might be offered; the situations and motions of the bodies in our system necessarily require this motion of the earth.

210. Besides this apparent diurnal motion, the sun, moon, and planets have another motion; for they are observed to make a complete revolution amongst the fixed stars, in different periods. But whilst they are performing these motions in respect to the fixed stars, they do not always appear to move in the same direction, or in that direction in which their complete revolutions are made, but sometimes appear stationary, and sometimes to move in a contrary direction. We will here briefly describe and consider the different systems which have been invented, in order to solve these appearances. PTOLEMY supposed the earth to be perfectly at rest, and all the other bodies, that is, the sun, moon, planets, comets and fixed stars, to revolve about it every day; but that, besides this diurnal motion, the sun, moon, planets and comets had a motion in respect to the fixed stars, and were situated, in respect to the earth, in the following order; the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn. These revolutions he first supposed to be made in circles about the earth placed a little out of the center, in order to account for some irregularities of their motions; but as their retrograde motions and stationary appearances could not thus be solved, he supposed them to revolve in epicycloids, in the following manner. Let ABC be a circle, S the center, E the earth, $abcd$ another circle whose center v is in the circumference of the circle ABC. Conceive the circumference of the circle ABC to be carried round the earth every 24 hours according to the order of the letters, and at the same time let the center v of the circle $abcd$ have a slow motion in the opposite direction, and let a body revolve in this circle in the direction $abcd$; then it is manifest, that by the motion of the body in this circle and the motion of the circle itself, the body may describe such a curve as is represented by $klmnop$; and if we draw the tangents El, Em, the body would appear stationary at the points l and m, and its motion would be *retrograde* through lm, and then *direct* again. Now to make Venus and Mercury always accompany the Sun, the center v of the circle $abcd$ was supposed to be always very nearly in a right line between the earth and sun, but more nearly so for Venus than for Mercury, in order to give each its proper elongation. This system, although it will account for all the apparent motions of the bodies, yet it will not solve the phases of Venus and Mercury; for in this case, in both conjunctions with the sun they ought to appear dark bodies, and to lose their light both ways from their greatest elongations; whereas it appears from observation, that in one of their conjunctions they shine with a full face. This system therefore cannot be true. FIG. 40.

211. The system received by the Egyptians was this: The Earth is immoveable in the center, about which revolve, in order, the Moon, Sun, Mars, Jupiter and Saturn; and about the Sun revolve Mercury and Venus. This disposition will account for the phases of Mercury and Venus, but not for the apparent motions of Mars, Jupiter and Saturn.

212. The next system which we shall mention, though posterior in time to the true, or *Copernican System*, as it is usually called, is that of TYCHO BRAHE, a Polish Nobleman. He was pleased with the Copernican system, as solving all the appearances in the most simple manner; but conceiving, from taking the literal meaning of some passages in Scripture, that it was necessary to suppose the earth to be absolutely at rest, he altered the system, but kept as near to it as possible. And he further objected to the earth's motion, because it did not, as he conceived, affect the motion of comets observed in opposition, as it ought; whereas, if he had made observations on some of them, he would have found that their motions could not otherwise have been accounted for. In his system, the earth is placed immoveable in the center of the orbits of the sun and moon, without any rotation about an axis; but he made the sun the center of the orbits of the other planets, which therefore revolved with the sun about the earth. By this system, the different motions and phases of the planets may be solved, the latter of which could not be, by the Ptolemaic system; and he was not obliged to retain the epicycloids in order to account for their retrograde motions and stationary appearances. One obvious objection to this system is, the want of that simplicity by which all the apparent motions may be solved, and the necessity that all the heavenly bodies should revolve about the earth every day; also, it is physically impossible that a large body, as the sun, should revolve about a much smaller body, as the earth, at rest; if one body be much larger than another, the center about which they revolve must be very near to the large body; this will be proved when we come to the principles of *physical* Astronomy. And this argument holds also against the Ptolemaic system. It appears also from observation, that the plane in which the sun must, upon this supposition, diurnally move, passes through the earth only twice in a year. It cannot therefore be any force in the earth which can retain the sun in its orbit, for it would move in a spiral continually changing its plane. In short, the complex manner in which all the motions are accounted for, and the physical impossibility of such motions being performed, is a sufficient reason for rejecting this system; especially when we consider, in how simple a manner all these motions may be accounted for, and demonstrated from the common principles of motion. Some of TYCHO's followers, seeing the absurdity of supposing all the heavenly bodies daily to revolve about the earth, gave a rotatory motion to the earth, in order to account for their diurnal motion; and this was called the *Semi-Tychonic System;* but the objections to this system are, otherwise, just the same.

213. The system which is now universally received is called the *Copernican*. It was formerly taught by PYTHAGORAS, who lived about 500 years before J. C. and PHILOLAÜS, his disciple, maintained the same; but it was afterwards rejected till revived by COPERNICUS. Here the Sun is placed in the center of the

System, about which the other bodies revolve in the following order; Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian Planet, which was lately discovered by Dr. HERSCHEL; beyond which, at immense distances, are placed the fixed stars; the moon revolves about the earth, and the earth revolves about an axis. This disposition, and these motions of the bodies, solve, in the most simple manner, not only all the phases, and the direct and retrograde motions, but also every other irregularity belonging to them, and which motions may also be accounted for upon physical principles. We may also further observe, that the supposition of the earth's motion is necessary, in order to account for a small apparent motion which every fixed star is found to have, and which cannot otherwise be accounted for. The harmony of the whole will be as satisfactory a proof of the truth of this System, as the most direct demonstration could be; this System therefore we shall assume.

CHAP. IX.

ON KEPLER'S DISCOVERIES.

Art. 214. KEPLER was the first who discovered the figures of the orbits of the planets to be ellipses, having the sun in one of the foci. PTOLEMY supposed that the orbits of the planets were circles, having the earth, not in the center C, but at some other point S; and taking $CB = CS$, he supposed that they revolved with an uniform angular velocity about B, called the *Punctum æquantis*. This was his supposition to account for the equation of the planet's orbit, or the *first* inequality of its motion; but it was supported neither by observation nor demonstration. TYCHO altered this hypothesis, by placing B at a different distance from C, by which he found his computations would agree with his observations within a few minutes. Notwithstanding which, KEPLER suspected the hypothesis could not be true; for, from the goodness of TYCHO's observations, he believed that there could not have been so great a difference between the computations and observations, if it were true. But in respect to the orbit of the sun, or rather of the earth, the ancients, and also TYCHO, believed the motion was equable about the center C. From the equation of the orbit, TYCHO computed the excentricity SB, which taken from AS gave a quantity BA different from the radius AC at first supposed; whence he concluded that the sun was not always at the same distance from C. This induced KEPLER* to suspect that the center was not the point about which the motion was equal, but that it bisected the excentricity. To determine this point he proceeded thus.

215. Let B be the point about which the motion is equable, S the sun, take $BC = SC$, and let D and E be the places of the earth when the planet *Mars* is at the same point M of its orbit. On May 18, 1585, and January 22, 1591, he took the two places of Mars, found from observation, and by calculation reduced its places to May 30, and January 20, in the same respective years, at which times the longitude of Mars seen from B, as calculated by TYCHO, was $6^s.\ 13°.\ 28'$, and therefore he knew that Mars was in the same point of its orbit; and the angles MBD, MBE were, each $64°.\ 23\frac{1}{2}'$. Now the longitudes of Mars on May 30, and January 20, were, by observation, $5^s.\ 6°,\ 37'$ and $7^s.\ 21°.\ 34'$, the differences between which and $6^s.\ 13°.\ 28'$, the heliocentric longitude before calculated, are $36°.\ 51'$ and $38°.\ 6'$ for the angles BMD, BME; consequently BD is less than BE, and therefore B is not the center of the circle. KEPLER next calculated the value of BC, and found it to be 1837, AC being 100000. Now TYCHO had found from his observations, that the whole distance BS from the sun to the center of equality was 3584, therefore its half was 1792, which being so nearly equal to 1837, KEPLER immediately concluded that C bisected the excentricity.

* See his Work, *De motibus Stellæ Martis.*

216. Having found that the center of the earth's orbit bisected the excentricity, he proceeded to examine the same in the orbit of Mars, in the following manner. Let S be the sun, C the center of the circle, B the point about which the motion is equable; and let D, E, F, G be 4 places of Mars observed in opposition; he then proposed the following Problem. To find the angles FBA, FSA such, that the four points D, E, F, G may be in the circumference of the circle, and C in the center between B and S. He resolved this by assuming the distance SB and the angles FBA, FSA, and thence calculated all the other parts, to find whether all the angles formed about S were together equal to four right ones. He made 70 suppositions before he got one to agree with observation, the calculation of every one of which was extremely long and tedious: *Si te hujus laboriosæ methodi pertæsum fuerit, jure mei te misereat, qui eam ad minimum septuagies ivi cum plurima temporis jactura, et mirari desines hunc quintum jam annum abire, ex quo Martem aggressus sum, quamvis annus* 1603 *pene totus opticis inquisitionibus fuit traductus;* pag. 95. Having thus determined the excentricity of the orbit of Mars, he calculated 12 oppositions observed by TYCHO, none of which differed more than 1'. 47"; but he found that the hypothesis agreed neither with the latitude observed in opposition, nor with the longitude out of opposition, which differed sometimes 8' from observation. The circle which so well represented the 12 oppositions had its excentricity $SB = 18564$, but he found $SC = 11332$ and $CB = 7232$, the mean distance of the earth from the sun being 100000. From the want of agreement between the observed and computed latitudes in opposition, and the longitudes out of opposition, and from SB not being bisected in C, KEPLER was persuaded that the orbit of Mars was not a circle. He therefore computed, in the following manner, three distances of Mars from the sun, with the corresponding heliocentric longitudes, by which he could determine both the figure and magnitude of its orbit. FIG. 43.

217. Let S be the sun, M Mars, D, E, two places of the earth when Mars was in the same point M of its orbit. When the earth was at D, he observed the difference between the longitudes of the sun and Mars, or the angle MDS; in like manner he observed the angle MES. Now the places D, E of the earth in its orbit being known, the distances DS, ES and the angle DSE will be known; hence, in the triangle DSE, we know DS, SE, and the angle DSE, to find DE and the angles SDE, SED; hence we know the angles MDE, MED; therefore in the triangle MDE, we know DE, and the angles MDE, MED, to find MD; and lastly, in the triangle MDS, we know MD, DS, and the angle MDS, to find MS, the distance of Mars from the sun. He also found the angle MSD, the difference of the heliocentric longitudes of Mars and the earth. By this method, KEPLER, from observations made on Mars when in aphelion and perihelion (for he had determined the position of the line of FIG. 44.

the apsides, by a method which we shall afterwards explain, independent of the form of the orbit), determined the former distance from the sun to be 166780, and the latter 138500, the mean distance of the earth from the sun being 100000; hence, the mean distance of Mars was 152640 and the excentricity of its orbit 14140. He then determined, in like manner, three other distances, and found them to be 147750, 163100, 166255. He next calculated the same three distances, upon supposition that the orbit was a circle, and found them to be 148539, 163883, 166605; the errors therefore of the circular hypothesis were 789, 783, 350. But he had too good an opinion of TYCHO's observations to suppose that these differences might arise from their inaccuracy; and as the distance between the aphelion and perihelion was too great, upon supposition that the orbit was a circle, he knew that the form of the orbit must be an oval; *Itaque planè hoc est: Orbita planetæ non est circulus, sed ingrediens ad latera utraque paulatim, iterumque ad circuli amplitudinem in perigæo exiens, cujusmodi figuram itineris ovalem appellitant*, pag. 213. And as of all ovals, the ellipse appeared to be the most simple, he first supposed the orbit to be an ellipse, and placed the sun in one of the foci; and upon calculating the above observed distances, he found they agreed together. He did the same for other points of the orbit, and found that they all agreed; and thus he pronounced the orbit of Mars to be an ellipse, having the sun in one of its foci. Having determined this for the orbit of Mars, he conjectured the same to be true for all the other planets, and upon trial he found it to be so. Hence he concluded, *That the six primary Planets revolve about the Sun in ellipses, having the Sun in one of the foci.*

A TABLE

Of the relative *mean* distances of the Planets from the Sun, according to different Authors.

Planets	KEPLER	STREET	HALLEY	M. de la LANDE	*Log. Dist.*
Mercury	38806	38710	38710	38710	9,5878221
Venus	72413	72333	72333	72333,24	9,8593379
Earth	100000	100000	100000	100000	0,0000000
Mars	152349,5	152369	152369	152369,27	0,1828973
Jupiter	520000	520110	520098	520279,2	0,7162364
Saturn	951003,5	953800	954007,4	954072,4	0,9795813

The relative mean distance of the *Georgian* Planet from the sun is 1918352, according to M. de la PLACE.

The logarithms are here put down upon supposition that the mean distance of the earth from the sun is unity, this being the case in all Astronomical Tables. The mean distances are nearly as 4, 7, 10, 15, 52, 95, 192.

218. Having thus discovered the relative mean distances of the planets from the sun, and knowing their periodic times, he next endeavoured to find if there was any relation between them, having had a strong passion for finding analogies in nature. He saw that the more distant a planet was from the sun the slower it moved, so that on a double account the periodic times of the more distant planets would be increased. Saturn, for example, is $9\frac{1}{2}$ times further from the sun than the earth is, and the circle described by Saturn is so much greater in proportion; and as the earth revolves in 1 year, if their velocities were equal, the periodic time of Saturn would be $9\frac{1}{2}$ years; whereas its periodic time is near 30 years. The periodic times therefore of the planets increase in a greater ratio than their distances, but in a less ratio than the squares of their distances; for upon that supposition the periodic time of Saturn would be about $90\frac{1}{4}$ years. On March 8, 1618, he began to compare the powers of these quantities, and at that time he took the squares of the periodic times and compared them with the cubes of the mean distances, but, from some error in the calculation, they did not agree. But on May 15, having made the last computations again, he discovered his error, and found an exact agreement between them. Thus he discovered the famous Law, *That the squares of the periodic times of all the planets are as the cubes of their mean distances from the sun.* Sir I. NEWTON afterwards proved that this is a necessary consequence of the motion of a body in an ellipse about the focus. *Prin. Phil. Lib.* I. *Sec.* 2. *Pr.* 15.

219. KEPLER also discovered from observation, that the velocities of the planets, when in their apsides, are inversely as their distances from the sun; whence it followed, that they describe, in these points, equal areas about the sun in equal times. And although he could not prove, from observation, that the same was true in every point of the orbit, yet he had no doubt but that it was so. He therefore applied this principle to find the equation of the orbit (as will be explained in the next Chapter), and finding that his calculations agreed with observations, he concluded it was true in general, *That the planets describe about the sun equal areas in equal times.* This discovery was, perhaps, the foundation of the PRINCIPIA, as it probably might suggest to Sir I. NEWTON the idea, that the proposition was true in general, which he afterwards proved it to be. These important discoveries are the foundation of all Astronomy.

220. He also speaks of *Gravity* as a power which is mutual between all bodies; and tells us, that the earth and moon would move towards each other, and meet at a point as much nearer to the earth than the moon, as the earth is greater than the moon, if their motions did not hinder it. He further adds, that the tides arise from the gravity of the waters towards the moon. That the reader may have a better conception of his ideas on this subject, we shall here give his own words.

Vera doctrina de gravitate his innititur axiomatibus.

Omnis substantia corporea, quatenus corporea, apta nata est quiescere omni loco, in quo solitaria ponitur, extra orbem virtutis cognati corporis.

Gravitas est affectio corporea, mutua inter cognata corpora ad unitionem seu conjunctionem (quo rerum ordine est et facultas magnetica) ut multo magis terra trahat lapidem, quàm lapis petit terram.

Gravia (si maximè terram in centro mundi collocemus) non feruntur ad centrum mundi, ut ad centrum mundi, sed ut ad centrum rotundi cognati corporis, telluris scilicet. Itaque ubicunque collocetur seu quocunque transportetur tellus facultate suâ animali, semper ad illam feruntur gravia.

Si terra non esset rotunda, gravia non undiquaque ferrentur recta ad medium terræ punctum, sed ferrentur ad puncta diversa à lateribus diversis.

Si duo lapides in aliquo loco mundi collocarentur propinqui invicem, extra orbem virtutis tertii cognati corporis; illi lapides ad similitudinem duorum magneticorum corporum coirent loco intermedio, quilibet accedens ad alterum tanto intervallo, quanta est alterius moles in comparatione.

Si luna et terra non retinerentur vi animali, aut aliâ aliquâ æquipollenti, quælibet in suo circuitu; terra ascenderet ad lunam quinquagesimâ quartâ parte intervalli, luna descenderet ad terram quinquaginta tribus circiter partibus intervalli: ibique jungerentur: posito tamen, quòd substantia utriusque sit unius et ejusdem densitatis.

Si terra cessaret attrahere ad se aquas suas; aquæ marinæ omnes elevarentur, et in corpus lunæ influerent.

Orbis virtutis tractoriæ, quæ est in lunâ, porrigitur usque ad terras, et prolectat aquas sub zonam torridam, quippe in occursum suum quacunque in verticem loci incidit, insensibiliter in maribus inclusis, sensibiliter ibi ubi sunt latissimi alvei oceani, aquisque spatiosa reciprocationis libertas, quo facto nudantur littora zonarum et climatum lateralium, et si qua etiam sub torrida sinus efficiunt reductiores oceani propinqui. Itaque aquis in latiori alveo oceani assurgentibus, fieri potest, ut in angustioribus ejus sinubus, modo non nimis arctè conclusis, aquæ præsente lunâ etiam aufugere ab eâ videantur: quippe subsidunt, foris subtractâ copiâ aquarum. *See the Introduction to the abovementioned Work.*

CHAP. X.

ON THE MOTION OF A BODY IN AN ELLIPSE ABOUT THE FOCUS.

Art. 221. AS the orbits which are described by the primary planets revolving about the sun are ellipses having the sun in one of the foci, and each describes about the sun equal areas in equal times, we next proceed to deduce, from these principles, such consequences as will be found necessary in our enquiries respecting their motions. From the equal description of areas about the sun in equal times, it appears* that the planets move with unequal angular velocities about the sun. The proposition therefore, which we here propose to solve, is, given the periodic time of a planet, the time of its motion from its aphelion, and the excentricity of its orbit, to find its angular distance from the aphelion, or its *true* anomaly, and its distance from the sun. This was first proposed by KEPLER, and therefore goes by the name of KEPLER'S PROBLEM. He knew no direct method of solving it, and therefore did it by very long and tedious tentative operations.

222. Let $AGQB$ be the ellipse described by the body about the sun at S in one of its foci, AQ the major, GB the minor axis, A the aphelion, Q the perihelion, P the place of the body, $AVGE$ a circle, C its center; draw NPI perpendicular to AQ, join PS, NS and NC, on which produced let fall the perpendicular ST. Let a body move uniformly in the circle from A to D with the *mean* angular velocity of the body in the ellipse, whilst the body moves in the ellipse from A to P; then the angle ACD is the *mean*, and the angle ASP the *true* anomaly; and the difference of these two angles is called the *Equation of the planet's center*, or *Prosthapheresis*. Let p=the periodic time in the ellipse or circle (the periodic times being equal by supposition), and t= the time of describing AP or AD; then, as the bodies in the ellipse and circle describe equal areas in equal times about S and C respectively, we have FIG. 46.

area ADC : area of the circle :: $t : p$,

area of the ellipse : area ASP :: $p : t$,

* For if $AP\mathfrak{Q}$ be an ellipse described by a planet about the sun at S in the focus, the indefinitely small area PSp described in a given time will be constant; draw Pr perpendicular to Sp; and, as the area SPp is constant for the same time, Pr varies as $\frac{1}{Sp}$; but the angle pSP varies as $\frac{Pr}{Sp}$, and therefore it varies as $\frac{1}{Sp^2}$; that is, in the *same* orbit, the angular velocity of a planet varies inversely as the square of its distance from the sun. For *different* planets, the areas described in the same time are not equal, and therefore Pr varies as $\frac{\text{area } SPp}{Sp}$, consequently the angle pSP varies as $\frac{\text{area } SPp}{Sp^2}$; that is, the angular velocities of *different* planets are as the areas described in the same time directly and the squares of their distances from the sun inversely. FIG. 45.

also, area of the circle : area of the ellipse :: area ASN : area ASP $\therefore$ area ADC : area ASP :: area ASN : area ASP; hence, $ADC = ASN$; take away the area ACN which is common to both, and the area $DCN = SNC$; but $DCN = \frac{1}{2}DN \times CN$, and $SNC = \frac{1}{2}ST \times CN$; therefore $ST = DN$. Now if t be given, the arc AD will be given; for as the body in the circle moves uniformly, we have $p : t :: 360° : AD$. Thus we always find the mean anomaly at any given time, knowing the time when the body was in the aphelion; hence if we can find ST, or ND, we shall know the angle NCA, called the *excentric* anomaly, from whence, by one proportion (223), we shall be able to find the angle ASP the *true* anomaly. The Problem is therefore reduced to this; to find a triangle CST, such that the angle $C +$ the degrees of an arc equal to ST may be equal to the given angle ACD. This may be expeditiously done by trial in the following manner, given by M. de la CAILLE in his Astronomy. Find what arc of the circumference of the circle $ADQE$ is equal to CA, by saying, $355 : 113 :: 180°. : 57°. 17'. 44'',8$ the number of degrees of an arc equal in length to the radius CA; hence $CA : CS :: 57°. 17'. 44'',8$: the degrees of an arc equal to CS. Assume therefore the angle SCT, multiply its sine into the degrees in CS, and add it to the angle SCT, and if it equal the given angle ACD, the supposition was right; if not, add or subtract the difference to or from the first supposition, according as the result is less or greater than ACD, and repeat the operation, and in a very few trials you will get the accurate value of the angle SCT. The degrees in ST may be most readily obtained by adding the logarithm of CS to the logarithm of the sine of the angle SCT and subtracting 10 from the index, and the remainder will be the logarithm of the degrees of ST. Having found the value of AN, or the angle ACN, we proceed next to find the angle ASP.

223. Let v be the other focus, and put $AC = 1$; then by Eucl. B. II. P. 12. $SP^2 - Pv^2 = vS^2 + 2vS \times vI = \overline{vS + 2vI} \times vS = \overline{2Cv + 2vI} \times 2SC = 2CI \times 2SC$; hence, $SP + Pv : 2CI :: 2SC : SP - Pv$, or $2 : 2CI :: 2SC : SP - \overline{2 - SP}$, or $1 : CI :: SC : SP - 1$, and $SP = 1 + CS \times CI = 1 + CS \times \cos. \angle ACN$. By my Trigon. Art. 94. $\frac{1 - \cos. ASP}{1 + \cos. ASP} = \overline{\tan. \frac{1}{2} ASP}^2$. But SP, or $1 + CS \times \cos. ACN : \text{rad.} = 1 :: SI$, or $CS + CI$, or $CS + \cos. ACN : \cos. ASP = \frac{CS + \cos. ACN}{1 + CS \times \cos. ACN}$. Hence, $\overline{\tan. \frac{1}{2} ASP}^2 \left(= \frac{1 - \cos. ASP}{1 + \cos. ASP}\right) = \frac{1 + CS \times \cos. ACN - CS - \cos. ACN}{1 + CS \times \cos. ACN + CS + \cos. ACN} = \frac{1 - CS + \cos. ACN \times \overline{CS - 1}}{1 + CS + \cos. ACN \times \overline{CS + 1}} = \frac{SQ - \cos. ACN \times SQ}{SA + \cos. ACN \times SA} = \frac{1 - \cos. ACN}{1 + \cos. ACN} \times \frac{SQ}{SA} =$ (by the above theorem in trig). $\overline{\tan. \frac{1}{2} ACN}^2 \times \frac{SQ}{SA}$; therefore $\sqrt{SA} : \sqrt{SQ} :: \tan. \frac{1}{2} ACN : \tan. \frac{1}{2} ASP$, consequently we get ASP the *true* anomaly.

Ex. Required the true place of *Mercury* on August 26, 1740, at noon, the equation of the center, and its distance from the sun.

By M. de la CAILLE's Astronomy, Mercury was in its aphelion on August 9, at 6*h*. 37. Hence on August 26, it had passed its aphelion 16*d*. 17*h*. 23′; therefore 87*d*. 23*h*. 15′. 32″ (the time of one revolution) : 16*d*. 17*h*. 23′:: 360° : 68°. 26′. 28″ the arc AD, or mean anomaly. Now (according to this Author) $CA : CS$:: 1011276 : 211165 (222):: 57°. 17′. 44″,8 : 11°. 57′. 50″ = 43070″, the value of CS reduced to the arc of a circle, the log. of which is 4,6341749. Also, 68°. 26′. 28″ = 246388″. Assume the angle SCT to be 60° = 216000″, and the operation (222) to find the angle ACN will stand thus:

4,6341749		
9,9375306	log. of - -	216000 = a
4,5717055	- -	37300
		253300
		246388
		6912 = b
4,6341749		
9,9287987	- - -	209088 = $a - b$ = 58°. 4′. 48″ = c
4,5629736	- - -	36557
		245645
		246388
		743 = d
4,6341749		
9,9297694	- - -	209831 = $c + d$ = 58°. 17′. 11″ = e
4,5639443	- - -	36639
		246470
		246388
		82 = f
4,6341749		
9,9296626	- - -	209749 = $e - f$ = 58°. 15′. 49″ = g
4,5638375	- - -	36630
		246379
		246388
		9 = h; hence, as the difference

between the value deduced from the assumption and the true value is now diminished about 9 times every operation, the next difference would be 1″; if therefore we add h to g, and then subtract 1″, we get 58°. 15′. 57″ for the true value of the angle ACN, the *excentric* anomaly. Hence (223), find the *true* anomaly ASP, from the proportion there given, by logarithms thus:

Log. tang. 29°. 7′. 58″$\frac{1}{2}$ - - -	9,7461246
$\frac{1}{2}$ Log. $SQ = 800111$ - - -	2,9515751
	12,6976997
$\frac{1}{2}$ Log. $SA = 1222441$ - - -	3,0436141
Log. tang. 24°. 16′. 15″ - -	9,6540856

Hence, the *true* anomaly is 48°. 32′. 30″. Now the aphelion A was in 8ˢ. 13°. 54′. 30″; therefore the true place of Mercury was 10ˢ. 2°. 27′. Hence, 68°. 26′. 28″ − 48°. 32′. 30″ = 19°. 53′. 58″ the *equation of the center*. Also, $SP = 1 + CS \times \cos. \angle ACN = 1,10983$ the distance of Mercury from the sun, the radius of the circle, or the mean distance of the planet, being unity. Thus we are able to compute, at any time, the place of a planet in its orbit, and its distance from the sun; and this method of computing the *excentric* anomaly appears to be the most simple and easy of application of all others, and capable of any degree of accuracy.

224. As the bodies at D and P departed from A at the same time, and will coincide again at Q, ADQ, APQ being performed in half the time of a revolution; and as at A the planet moves with its least angular velocity (by the Note to Art. 221.), therefore from A to Q, or in the *first* 6 signs of anomaly, the angle ACD will be greater than ASP, or the *mean* will be greater than the *true* anomaly; but from Q to A, or in the *last* 6 signs, as the planet at Q moves with its greatest angular velocity, the *true* will be greater than the *mean* anomaly.

225. When the excentricity, and consequently the angle NCD, is very small, as in the orbits of Venus and the Earth, ND, considered as very nearly a straight line, will be equal and parallel to ST, therefore SD is parallel to CN and consequently the angle $NCD = CDS$. Now in the triangle DCS, we know the two sides DC, CS, and the included angle DCS, the supplement of DCA; hence we can find the angle CDS or DCN. If the angle DCN do not exceed $1\frac{1}{2}°$ the conclusion will be accurate to a second; and if it be greater, this method will give a near value of it, and consequently we shall get a near value of the angle ACN to begin the operation with in the method already explained, which will be better, perhaps, than guessing at first. In our Example, the angle

DSC, or *SCT* nearly, would, by this calculation, have been found 58°. 13′. 1″, whence *ST* would first have been found 10°. 10′. 12″, and after two more operations the accurate value would have been obtained. When the angle *DCN* is not very small, M. Cassini, in his Elements of Astronomy, page 144, has given the following method of finding it.

226. Draw *Dz* perpendicular to *ST*, and *Tz* is the sine of the arc *DN*, consequently *Sz* is the difference between the arc *DN* and its sine, or it may be considered as the difference between the arc of the angle *CDS* and its sine; compute therefore the angle *CDS* (225), and by the following Table take out the difference between the arc and its sine, and say *SD* : *Sz* :: rad. : sin. *SDz*, which subtract from the angle *SDC* and you have the angle *zDC*, or the alternate angle *DCN*. The rest of the operation is the same as before.

A TABLE

Showing the Difference between the Arcs of a Circle and their Sines, Radius being 10000000.

Arc		Dif.	Arc		Dif.	Arc		Dif.	Arc		Dif.
1°.	00′	9	4°.	00′	567	7°.	00′	3037	10°.	00′	8848
	10	15		10	641		10	3259		10	9299
	20	23		20	720		20	3492		20	9755
	30	31		30	807		30	3734		30	10235
	40	42		40	900		40	3989		40	10730
	50	56		50	1000		50	4255		50	11241
2.	00	71	5.	00	1108	8.	00	4532	11.	00	11767
	10	90		10	1222		10	4822		10	12312
	20	113		20	1344		20	5122		20	12873
	30	139		30	1474		30	5435		30	13450
	40	169		40	1613		40	5761		40	14042
	50	203		50	1759		50	6100		50	14654
3.	00	239	6.	00	1913	9.	00	6450	12.	00	15278
	10	281		10	2077		10	6815		10	15921
	20	328		20	2255		20	7194		20	16585
	30	380		30	2432		30	7585		30	17266
	40	437		40	2625		40	7985		40	17964
	50	499		50	2827		50	8404		50	18680

Ex. To find the *true* anomaly of Mercury, the *mean* being 60°.

Let the mean distance of Mercury be 100000, and the excentricity CS will be 20878, according to CASSINI; hence, in the triangle DCS, $DC=100000$, $CS=20878$, and the angle $DCS=120°$, therefore $DC=111905$, and the angle $SDC=9°.\ 17'.\ 52''$, corresponding to which we find, in the Table, the value of $Sz=7120$; hence, $111905 : 71$* :: rad. : sin. $\angle SDz=2'.\ 11''$, which subtracted from $9°.\ 17'.\ 52''$ leaves $9°.\ 15'.\ 41''$ † for the angle DCN, which subtracted from 60° gives $50°.\ 44'.\ 19''$ for the angle NCA. Hence,

Log. tan. $25°.\ 22'.\ 9''$ - - - - -	9,6759392
$\frac{1}{2}$ Log. $SQ=79122$ - - - - - -	2,4491486
	12,1250878
$\frac{1}{2}$ Log. $SA=120878$ - - - - -	2,5411738
Log. tan. $20°.\ 59'.\ 18''$ - - - - -	9,5839140

Therefore the true anomaly is $41°.\ 58'.\ 36''$. Hence, the equation of the center is $18°.\ 1'.\ 24''$.

These indirect methods of finding the equation of the planet's center are, in general, more ready for practice than any of the direct methods.

227. The method ascribed by some Writers to SETH WARD, Professor of Astronomy at Oxford, and published in 1654, although, as M. de la LANDE observes, it is given both by WARD and MERCATOR to BULLIALDUS, is less accurate than these we have already given; yet as it may, in many cases, serve as a useful approximation, it deserves to be mentioned. He assumed the angular velocity about the other focus v to be uniform‡, and therefore made it

* By the Table 7120 is the difference, if the radius be 10000000, but as the radius here is 100000 the difference will be only 71.

† For the utmost exactness, we should take Sz corresponding to $9°.\ 15'.\ 41''$ instead of $9°.\ 17'.\ 52''$, but the difference is too small to be worth notice.

‡ That this is not true may be thus shown. With the center S and radius $SW=\sqrt{AC \times CE}$ describe the circle zW, then the area of this circle = area of the ellipse; let a body, moving uniformly in it, make one revolution in the same time the body does in the ellipse; and let the bodies set off at the same time from A and z, and describe AP, zv, in the same time; then the $\angle zSv$ is the *mean*, and ASP the *true* anomaly. Draw pS indefinitely near to PS, and Pr, po perpendicular to Sp, FP; then $Pr=Po$. Now the $\angle PFp$ varies as $\frac{po}{PF}=\frac{Pr}{PF}$; but in a given time the area PSp is given, $\therefore Pr$ varies as $\frac{1}{PS}$; hence, the $\angle PFp$, described in a given time, varies as $\frac{1}{PF \times PS}$, which is not a constant quantity. Also, $PS : PF :: \angle PFp \left(\frac{1}{PF \times PS}\right) : \angle PSp \left(\frac{1}{PS^2}\right)$. And by the Note to Art. (221) as equal areas are described in equal times in the circle and ellipse about S, the angular velocity about S in the circle becomes

represent the *mean* anomaly. Produce vP to r, and take $Pr=PS$; then in the triangle Svr, $rv+vS : rv-vS :: \tan. \frac{1}{2}.\overline{\angle vSr+vrS} : \tan. \frac{1}{2}.\overline{\angle vSr-vrS}$; now $\frac{1}{2}.\overline{rv+vS}=\frac{1}{2}AQ+\frac{1}{2}vS=AS$, and $\frac{1}{2}.\overline{rv-vS}=\frac{1}{2}AQ-\frac{1}{2}vS=SQ$; also, $\tan. \frac{1}{2}.\overline{\angle vSr+vrS}=\tan. \frac{1}{2}\angle AvP$, and $\frac{1}{2}.\overline{\angle vSr-vrS}=$ (as $Pr=PS$) $\frac{1}{2}.\overline{\angle vSr-PSr}=\frac{1}{2}\angle ASP$; hence, *the aphelion distance : perihelion distance :: tan. of $\frac{1}{2}$ the mean anomaly : tan. $\frac{1}{2}$ true anomaly*. This is called the *simple elliptic hypothesis*, and was used by Dr. HALLEY in constructing his *Tabula pro expediendo calculo æquationis centri Lunæ*. In the orbit of the earth, the error is never greater than 17″; in the orbit of the moon, it may be 1′. 35″. By this hypothesis, for 90° from aphelion and perihelion, the computed place is *backwarder* than the true; and for the other part it is *forwarder*.

228. Although the indirect methods above explained are, in general, the best for practice, yet as the Reader may wish to see the direct method of solving the Problem, we shall give that of Dr. KEILL; as being the most simple, and which may frequently be applied with advantage. Let the arc $ND=y$, $e=$the sine of AD, $f=$the cosine, $SC=g$. Then by trigonometry, the sine of $NA=y-\frac{y^3}{2.3}+$&c. and cosine$=1-\frac{y^2}{2}+\frac{y^4}{2.3.4}-$&c. hence, the sine of $AN=e-fy-\frac{ey^2}{2}+\frac{fy^3}{2.3}+\frac{ey^4}{2.3.4}-$&c. Also, rad.$=1$: sin. AN or $\angle SCT :: SC=g : ST$ or ND or $y=ge-gfy-\frac{gey^2}{2}+\frac{gfy^3}{2.3}+\frac{gey^4}{2.3.4}-$&c. hence, $ge=y+gfy+\frac{gey^2}{2}-\frac{gfy^3}{2.3}-\frac{gey^4}{2.3.4}+$&c. Put $ge=z$, $1+gf=a$, $\frac{ge}{2}=b$, $\frac{gf}{2.3}=c$, $\frac{ge}{2.3.4}=d$, &c. hence, $z=ay+by^2-cy^3-dy^4+$ &c. and by the reversion of series, $y=\frac{z}{a}-\frac{bz^2}{a^3}+\frac{2b^2+ac}{a^5}\times z^3-\frac{5abc-5b^3+a^2d}{a^7}\times z^4$&c. but $b=\frac{ge}{2}=\frac{z}{2}$, $d=\frac{z}{2.3.4}$, &c. therefore $y=\frac{z}{a}-\frac{z^3}{2a^3}+\frac{cz^3}{a^4}-\frac{5cz^5}{2a^6}$ &c. If the arc AN be greater than 90° and less than 270°, f becomes negative, and therefore gf or c will be negative; hence, $y=\frac{z}{a}-\frac{z^3}{2a^3}-\frac{cz^3}{a^4}+\frac{5cz^5}{2a^6}$ &c. Now to reduce the value of y into degrees, we know that an arc equal to radius, or unity, is equal to 57,29578 degrees $=r$; hence, $1 : r :: \frac{z}{a}-\frac{z^3}{2a^3}+\frac{cz^3}{a^4}-$&c. : the degrees of the arc $y=\frac{rz}{a}-\frac{rz^3}{2a^3}+\frac{rcz^3}{a^4}-$&c. For the orbit of the earth, the first term will be sufficient, not dif- FIG. 46.

$\frac{1}{SW^2}$. Hence, the angular velocity about F is greater or less than the mean angular velocity, according as $PF\times PS$ is less or greater than SW^2, or than $AC\times CE$. Also, the angular velocity about F is the same in similar points of the ellipse in respect to the center, or at equal distances from the center.

fering from the truth the ten thousandth part of a degree. In other cases it may be necessary to take more terms.

Ex. Let the excentricity of the earth's orbit be 0,01691, the mean distance being $=1$, and the mean anomaly $30°$; to find the true anomaly.

Log. of g - - - -	8,2281436
Log. sin. of $e=30°$ - - -	9,6989700
Log. of r - - - - -	1,7581226
Log. of rge, or rz - - -	9,6852362
Log. of a - - - - -	0,0063137
Log. of $\frac{rz}{a}$ - - - - -	9,6789225 . . the natural

number corresponding to which, being a decimal, is 0°, $47744=28'.\ 38''=y$, which is true to a second; therefore $AN=29°.\ 31'.\ 22''$; hence,

Log. tan. 14°. 45'. 41″ - -	9,4207651
$\frac{1}{2}$ Log. $SQ=98309$ - - -	2,4962966
	11,9170617
$\frac{1}{2}$ Log. $SA=101691$ - - -	2,5031412
Log. tan. 14°. 32'. 25″ - -	9,4139205

Hence, the *true* anomaly is 29°. 4'. 50″; consequently the equation of the center is 55'. 10″.

229. When P and D are very near A, the variation of PS will be very small; now by the Note to Art. 221. the angular velocity of P at A about S : angular velocity of D about C:: $\frac{\text{area des. by } P}{SP^2} : \frac{\text{area des. by } D}{DC^2}$, and therefore the angular velocities will be nearly in a given ratio so long as P is near to A; hence, the difference of the angular velocities must vary nearly as the angular velocities themselves; that is, the equation of the center varies nearly as the angular velocity of P about S, or as the true anomaly. The same is true at the perihelion Q.

230. The *greatest* equation of the center may be easily found from the Note Art. 227, giving the dimensions of the orbit. For as long as the angular velocity of the body in the circle is greater than that in the ellipse about S, the equation will keep increasing, the bodies setting out from A and z; and when they become equal, the equation must be the greatest; this therefore happens when $\frac{1}{SP^2}=\frac{1}{SW^2}=\frac{1}{AC\times CE}$, or when $AC\times CE=SP^2$; hence, SP is known. Let SW represent the value of SP; then as we know SW, FW $(=2AC-SW)$

will be known, and as SF is known, we can find the angle FSW the *true* anomaly*. Hence (223), $\sqrt{SQ} : \sqrt{SA} ::$ tan. $\frac{1}{2}$ *true* anom. : tan. $\frac{1}{2}$ *excen.* anom. ACN or tan. $\frac{1}{2}$ SCT; and as we know SC, we can find ST or ND; and to convert that into degrees, say, rad. $=1 : ND :: 57°. 17'. 44'',8$: the degrees in ND, which added to, or subtracted from, the angle ACN gives ACD the *mean* anomaly, the difference between which and the *true* anomaly is the *greatest* equation. Thus we may find the equation at any other time, given SP. Or the cos. ACN may in general be found thus. By Art. 223. $SP = 1 + CS \times$ cos. ACN; hence, cos. $ACN = \frac{SP-1}{CS}$; consequently *log. cos.* $ACN =$ *log.* $\overline{SP-1}$ $-$ *log.* CS. FIG. 46.

231. The excentricity, and consequently the dimensions of the orbit, may be found from knowing the greatest equation. For (230) the greatest equation is when the distance is a mean between the semi-axis major and minor, and therefore in orbits nearly circular, the body must be nearly at the extremity of the minor axis, and consequently the angle NCA or SCT will be nearly a right angle, therefore ST is nearly equal to SC; also NSA will be very nearly equal to PSA. Now the angle $NCA - NSA$ or $PSA = SNC$, and $DCA - NCA = DCN$; add these together and $DCA - PSA = DCN + SNC$, which (as NC is nearly parallel to DS) is nearly equal to $2DCN$; that is, the difference between the *true* and *mean* anomaly, or the equation, is nearly equal to twice the arc DN, or twice ST, or very nearly twice SC. Hence, $57°. 17'. 44'',8$: half the greatest equation :: rad. $=1 : SC$ the excentricity. But if the orbit be considerably excentric, to this excentricity compute the greatest equation; and then, as the equation varies very nearly as SC, say, as the computed equation : excentricity found :: given greatest equation : true excentricity.

Ex. If we suppose, with M. de la CAILLE, that Mercury's greatest equation is $24°. 3'. 5''$; then $57°. 17'. 44'',8 : 12°. 1'. 32'',5 :: 1 : ,209888$ the excentricity very nearly. Now the greatest equation computed from this excentricity is $23°. 54'. 28'',5$; hence, $23°. 54'. 28'',5 : 24°. 3'. 5'' :: ,209888 : ,211165$ the true excentricity. M. de la LANDE makes the greatest equation $23°. 40'$, and the excentricity ,207745.

232. The converse of this Problem, that is, given the excentricity and true anomaly to find the mean, may be very readily and directly solved. The excentricity being given, the ratio of the major to the minor axis is known†, which is the ratio of NI to PI; hence, the angle ASP being given, we have

* Let $D = WS - SF$, then (Trig. Art. 131), sin. $\frac{1}{2}FSW = \sqrt{\frac{r^2 \times \frac{1}{2}(FW+D) \times \frac{1}{2}(FW-D)}{WS \times SF}}$, and log. sin. $\frac{1}{2}FSW = \frac{1}{2}$ (log. $\frac{1}{2}.(FW+D)$ + log. $\frac{1}{2}(FW-D)$ + ar. co. log. WS + ar. co. log. SF).

† For as AC, CS are known, we have $GC = \sqrt{SG^2 - SC^2} = \sqrt{AC^2 - SC^2}$.

$PI : NI :: \text{tan. } ASP : \text{tan. } ASN$; therefore in the triangle NCS, we know NC, CS and the angle CSN, to find the angle SCN, the supplement of which is the angle ACN or SCT; hence, in the right angled triangle STC, we know SC and the angle SCT, to find ST, which is equal to ND the arc measuring the equation, which may be found by saying, Radius : $ST :: 57°.\ 17'.\ 44'',8$: the degrees in ND, which added to ACN gives ACD the mean anomaly.

To find the hourly Motion of a Planet in its Orbit, having given the mean *hourly Motion.*

233. The hourly motion of a planet in its orbit is found immediately from the Note to Art. 227; for it appears from thence, that the angles PSp, WSw, described by the body at P in the ellipse and the body W in the circle in the same time are as $SW^2 : SP^2$, or as $AC \times CE : SP^2$; hence, $PSp = WSw \times \frac{AC \times CE}{SP^2}$ the hourly motion of a planet in its *orbit*, the angle WSw being the *mean* motion of the planet in an hour. For extreme accuracy, SP must be taken at the middle of the hour. Thus we may easily compute a Table of the hourly motions of the planets in their orbits.

To find the hourly Motion of a Planet in Latitude and Longitude.

234. Let AD be the ecliptic, AE the orbit of the planet; and let Bm represent the hourly motion in the orbit; draw the great circles BC, mo perpendicular to AD, and the small circle Bn parallel to AD. Now by plane trigonom.

$Bm : Bn :: \text{rad.} : \text{sin. } Bmn$ or ABC, and

$Bn : Co :: \text{cos. } BC : \text{rad.}$ (13)

$\therefore Bm : Co :: \text{cos. } BC : \text{sin. } ABC$; but $\text{sin. } ABC = \frac{\text{cos. } A}{\text{cos. } BC}$; and (233) if $a =$ the semi-axis major, b the semi-axis minor of the orbit, $x =$ the distance of the planet from the sun, $v =$ the *mean* hourly motion, then $Bm = v \times \frac{ab}{x^2}$; hence, $v \times \frac{ab}{x^2} : Co :: \text{cos. } BC : \frac{\text{cos. } A}{\text{cos. } BC}$, therefore $Co = v \times \frac{ab}{x^2} \times \frac{\text{cos. } A}{\text{cos. } BC^2} = v \times \frac{ab}{x^2} \times \frac{\textit{cos. incl. orb. to ecl.}}{\textit{cos. lat.}^2}$ the hourly motion in *Longitude*.

Also, $Bm : mn :: \text{rad.} : \text{cos. } Bmn$ or $ABC ::$ (because $\text{cos. } B = \text{cotan. } AB \times \text{tan. } BC$ divided by rad.) $\text{rad}^2. : \text{cotan. } AB \times \text{tan. } BC$; hence (radius being unity), $mn = v \times \frac{ab}{x^2} \times \text{cotan. } AB \times \text{tan. } BC = v \times \frac{ab}{x^2} \times$ *cot. plan. dist. node* $\times$ *tan. lat.* the hourly motion in *Latitude*. Hence we may construct Tables of the hourly motions of the planets both in longitude and latitude.

CHAP. XI.

ON THE OPPOSITIONS AND CONJUNCTIONS OF THE PLANETS.

Art. 235. THE place and time of the opposition of a superior planet, or conjunction of an inferior, are the most important observations for determining the elements of the orbit, because at that time the observed is the same as the true longitude, or that seen from the sun; whereas if observations be made at any other time, we must reduce the observed to the true longitude, which requires the knowledge of their relative distances, and which, at that time, are supposed not to be known. They also furnish the best means of examining and correcting the Tables of the planets motions, by comparing the computed with the observed places.

236. To determine the time of opposition, observe, when the planet comes very near to that situation, the time at which it passes the meridian, and also its right ascension (118 or 122); take also its meridian altitude; do the same for the sun, and repeat the observations for several days. From the observed meridian altitudes find the declinations, and from the right ascensions and declinations compute (124) the latitudes and longitudes of the planet, and the longitudes of the sun. Then take a day when the difference of their longitudes is nearly 180°, and on that day reduce the sun's longitude, found from observation when it passed the meridian, to the longitude found at the time (t) the planet passed, by finding from observation, or computation, at what rate the longitude then increases. Now in opposition the planet is retrograde, and therefore the difference between the longitudes of the planet and sun increase by the sum of their motions. Hence the following Rule; As the sum of their daily motions in longitude : the difference between 180° and the difference of their longitudes reduced to the same time (t), (subtracting the sun's longitude from that of the planet to get the difference reckoned from the sun according to the order of the signs) :: 24*h*. : interval between that time (t) and the time of opposition. This interval added to or subtracted from the time (t), according as the difference of their longitudes at that time was greater or less than 180°, gives the time of opposition. If this be repeated for several days and the mean of the whole taken, the time will be had more accurately. And if the time of opposition found from observation be compared with the time by computation from the Tables, the difference will be the error of the Tables, which may serve as a means of correcting them.

Ex. On October 24, 1763, M. de la LANDE observed the difference between the right ascension of β *Aries* and *Saturn*, which passed the meridian at 12*h*.

17′. 17″ apparent time, to be 8°. 5′. 7″, the star passing first. Now the apparent right ascension of the star at that time was 25°. 24′. 33″,6, hence, the apparent right ascension of Saturn was 1ˢ. 3°. 29′. 40″,6 at 12*h*. 17′. 17″ apparent time, or 12*h*. 1′. 37″ mean time. On the same day he found, from observation of the meridian altitude of Saturn, that its declination was 10°. 35′. 20″ N. Hence, from the right ascension and declination of Saturn, its longitude is found to be 1ˢ. 4°. 50′. 56″, and latitude 2°. 43′. 25″ south. At the same time the sun's longitude was found by calculation to be 7ˢ. 1°. 19′. 22″, which subtracted from 1ˢ. 4°. 50′. 56″ gives 6ˢ. 3°. 31′. 34″; hence, Saturn was 3°. 31′. 34″ beyond opposition, but being retrograde must afterwards come into opposition. Now, from the observations made on several days at that time, Saturn's longitude was found to decrease 4′. 50″ in 24 hours, and by computation the sun's longitude increased 59′. 59″ in the same time, the sum of which is 64′. 49″; hence, 64′. 49″ : 3°. 31′. 34″ :: 24*h*. : 78*h*. 20′. 20″, which added to October 24, 12*h*. 1′. 37″ gives 27*d*. 18*h*. 21′. 57″ for the time of opposition. Hence we may find the longitude of Saturn at the time of opposition, by saying, 24*h* : 78*h*. 20′. 20″ :: 4′. 50″ : 15′. 47″ the retrograde motion of Saturn in 78*h*. 20′. 20″, which subtracted from 1ˢ. 4°. 50′. 56″ leaves 1ˢ. 4°. 35′. 9″ the longitude of Saturn at the time of opposition. In like manner we may find the sun's longitude at the same time, in order to prove the opposition; hence, 24*h*. : 78*h*. 20′. 20″ :: 59′. 59″ : 3°. 15′. 47″, which added to 7ˢ. 1°. 19′. 22″, the sun's longitude at the time of observation, gives 7ˢ. 4°. 35′. 9″ for the sun's longitude at the time of opposition, which is exactly opposite to that of Saturn. Hence also we may find the latitude of Saturn at the same time, by observing in like manner the daily variation, or by computation from the Tables after the elements of its motions are known and the Tables constructed; by which it appears, that in the interval between the time of observation and opposition the latitude had increased 6″, and consequently the latitude was 2°. 43′. 31″.

237. This is the method which is now made use of to determine the time of opposition of the planets. The method used by TYCHO, HEVELIUS and FLAMSTEAD was the same, except that they determined the latitude and longitude of the planet from observing its distance from two known fixed stars, in the following manner. Let *P* be the pole of the ecliptic, *a* and *b* the two stars, *m* the planet; then observe *ma*, *mb*. Also, *Pa*, *Pb* the complements of the latitudes of *a* and *b*, and the angle *aPb* the difference of their longitudes, are known, from which find *ab* and the angle *Pab*; then in the triangle *amb* we know all the sides to find the angle *mab*, which added to or subtracted from the angle *Pab*, according to the position of *m*, gives the angle *Pam*; hence, in the triangle *Pam*, we know *Pa*, *am* and the angle *Pam*, to find *Pm* the complement of the planet's latitude, and the angle *aPm* the difference between the longitudes of the planet and the star *a*. Thus also may the place of any new

phænomenon, as a comet, be determined, if you have not an opportunity of observing its right ascension and declination, which however is the most accurate method.

238. The place and time of conjunction of an inferior planet may be found in like manner, when the elongation of the planet from the sun, near the time of conjunction, is sufficient to render it visible; the most favourable time therefore must necessarily be when the geocentric latitude of the planet at the time of conjunction is the greatest. In the year 1689, Venus was in its inferior conjunction on June 25, and it was observed on 21, 22, and 28; from which observations its conjunction was found to be at 13*h*. 46′ apparent time at Paris, in longitude ♋ 4°. 53′. 40″, and latitude 3°. 1′. 40″ north. The time and place of the superior conjunction may be also thus observed, when the state of the air is very favourable; for as Venus is then about six times as far from the earth as at its inferior conjunction, its apparent diameter and the quantity of light which we receive from it are so small, as to render it difficult to be perceived. But the most accurate method of observing the time of an inferior conjunction both of Venus and Mercury is from observations made upon them in their transits over the sun's disc. This we shall explain, when we come to treat on that subject.

CHAP. XII.

ON THE MEAN MOTIONS OF THE PLANETS.

Art. 239. THE determination of the *mean* motions of the planets, from their conjunctions and oppositions, would very readily follow, if we knew the place of the aphelia and excentricity of their orbits; for then we could (223) find the equation of the orbit, and reduce the *true* to the *mean* place. The mean places being determined at two points of time give the mean motion corresponding to the interval between the times. But the place of the aphelion is best fixed from the mean motion. To determine therefore the mean motion, independent of the place of the aphelion, we must seek for such oppositions or conjunctions as fall very nearly in the same point of the Heavens; for then the planet being nearly in the same point of its orbit, the equation will be very nearly the same at each observation, and therefore the comparison between the true places will be nearly a comparison of their mean places. If the equation should differ much in the two observations, it must be considered. Now by comparing the modern observations, we shall be able to get nearly the time of a revolution; and then by comparing the modern with the ancient observations, the mean motion may be very accurately determined; for any error, by dividing it amongst a great number of revolutions, will become very small in respect to one revolution. As this will be best explained by an example, we shall give one from M. CASSINI (*Elem. d'Astron.* pag. 362), with the proper explanations as we proceed.

Ex. On September 16, 1701, *Saturn* was in opposition at 2*h*. when the place of the sun was ♍ 23°. 21′. 16″, and consequently Saturn in ♓ 23°. 21′. 16″, with 2°. 27′. 45″ south latitude. On September 10, 1730, the opposition was at 12*h*. 27′ and Saturn in ♓ 17°. 53′. 57″, with 2°. 19′. 6″ south latitude. On September 23, 1731, the opposition was at 15*h*. 51′ in ♈ 0°. 30′. 50″, with 2°. 36′. 55″ south latitude. Now the interval of the two first observations was 29 years (of which 7 were bissextiles) wanting 5*d*. 13*h*. 33′; and the interval of the two last was 1*y*. 13*d*. 3*h*. 24′. Also, the difference of the places of Saturn in the two first observations was 5°. 27′. 19″, and in the two last it was 12°. 36′. 53″. Hence, in 1*y*. 13*d*. 3*h*. 24′ Saturn had moved over 12°. 36′. 53″; therefore 12°. 36′. 53″ : 5°. 27′. 19″ :: 1*y*. 13*d*. 3*h*. 24′ : 163*d*. 12*h*. 41′ the time of moving over 5°. 27′. 19″ very nearly, because Saturn, being nearly in the same part of its orbit, will move nearly with the same velocity; this therefore *added* to the interval of time between the two first observations (because at the second obser-

vation Saturn wanted 5°. 27′. 19″ of being up to the place at the first observation) gives 29 common years 164*d.* 23*h.* 8′ for the time of one revolution. Hence say, 29*y.* 164*d.* 23*h.* 8′ : 365*d.* :: 360° : 12°. 13′. 23″. 50‴ the *mean* annual* motion of Saturn in a common year of 365 days, that is, the motion in a year if it had moved uniformly. If we divide this by 365 we shall get 2′. 0″. 28‴ for the mean daily motion of Saturn. If we had taken the mean annual motion of Saturn answering to 12°. 36′. 53″ in 1*y.* 13*d.* 3*h.* 24′, it would have been found 12°. 10′. 35″, which differing only about 3′ from the true motion, it follows that Saturn was then moving with its mean velocity, very nearly, and consequently was very near its mean distance. The mean motion thus determined will be sufficiently accurate to determine the number of revolutions which the planet must have made when we compare the modern with the ancient observations, in order to determine the mean motion more accurately.

The most ancient observation which we have of the opposition of Saturn was on March 2, in the year 228 before J. C. at one o'clock in the afternoon in the meridian of Paris, Saturn being then in ♍ 8°. 23′, with 2°. 50′ north lat. On February 26, 1714, at 8*h.* 15′, Saturn was found in opposition in ♍ 7°. 56′. 46″, with 2°. 3′ north lat. From this time we must subtract 11 days, in order to reduce it to the same style as at the first observation, and consequently this opposition happened on February 15, at 8*h.* 15′. Hence, the difference between these two places was only 26′. 14″. Also, the opposition in 1715 was

* If *a* be the mean place of a planet in its orbit, and *b* the mean place at the interval of a year (*ab* being the order of the signs), then *ab* is called the mean *annual* motion, the number of complete revolutions being rejected, if the planet have made one or more revolutions. Hence, if to the mean place of a planet at the beginning of any year we add the mean annual motion, it gives the mean place at the beginning of the next year, rejecting 360° if the sum be greater. The mean annual motion is that belonging to a common year of 365 days; therefore for a bissextile we must add the mean motion of 1 day in order to get the mean annual motion for that year. In like manner, if *a* and *b* be the mean places at the interval of 100 years containing 25 bissextiles, *ab* is called the mean *secular* motion; which added to the mean place of a planet at the beginning of any year, gives the mean place at the end of the 100th year from that time. For instance, the mean annual motion of *Mars* is 6ˢ. 11°. 17′. 10″; and its mean place at the beginning of 1789, was 9ˢ. 17°. 22′. 29″; to this therefore add 6ˢ. 11°. 17′. 10″ and (rejecting 360°) we get 3ˢ. 28°. 39′. 39″; the mean place at the beginning of 1790. As the mean daily motion of Mars is 31′. 27″, the mean annual motion in a year of 366 days is 6ˢ. 11°. 48′. 37″. Now in a bissextile, the year begins on January 1, at noon, but in the common years it begins on December 31, at noon, by the civil account; therefore the year *preceding* the bissextile has 366 days in the Astronomical Tables. Hence, at the beginning of 1787, the mean place of Mars being 8ˢ. 24°. 16′. 43″, and the next year being bissextile, if we add 6ˢ. 11°. 48′. 37″ it gives 3ˢ. 6°. 5′. 20″ for the mean place at the beginning of 1788. The mean secular motion of Mars is 2ˢ. 1°. 42′. 10″, which added to 11ˢ. 22°. 2′. 49″ the mean place of Mars at the beginning of the year 1400, will give 1ˢ. 23°. 44′. 59″ the mean place at the beginning of the year 1500. If the 100 years contain only 24 bissextiles, as may sometimes happen, the mean secular motion will be 2ˢ. 1°. 10′. 43″. But of this we shall have to say more, when we treat of the Construction of the Tables of the Planets motions.

FIG. 51.

on March 11, at 16*h*. 55′, Saturn being then in ♍ 21°. 3′. 14″, with 2°. 25 north lat. Now between the two first oppositions there were 1942 years (of which 485 were bissextiles) wanting 14*d*. 16*h*. 45′, that is, 1943 common years and 105*d*. 7*h*. 15′ over. Also the interval between the times of the two last oppositions was 378*d*. 8*h*. 40′, during which time, Saturn had moved over 13°. 6′. 28″; hence, 13°. 6′. 28″ : 26′. 14″ :: 378*d*. 8*h*. 40′ : 13*d*. 14*h*. which added to the time of the opposition in 1714, gives the time when the planet had the same longitude as at the opposition in 228 before J. C. This quantity added to 1943 common years 105*d*. 7*h*. 15′ gives 1943*y*. 118*d*. 21*h*. 15′, in which interval of time Saturn must have made a certain complete number of revolutions. Now having found, from the modern observations, that the time of one revolution must be nearly 29 common years 164*d*. 23*h*. 8′, it follows that the number of revolutions in the above interval was 66 . dividing therefore that interval by 66 we get 29*y*. 162*d*. 4*h*. 27 for the time of one revolution. From comparing the oppositions in the years 1714 and 1715, the true movement of Saturn appears to be very nearly equal to the mean movement, which shows that the oppositions have been observed very near the mean distance; consequently the motion of aphelion cannot have caused any considerable error in the determination of the mean motion. Hence, the mean annual motion is 12°. 13′. 35″. 14‴, and the mean daily motion 2′. 0″. 35‴. Dr. Halley makes the annual motion to be 12°. 13′. 21″. M. de Place makes it 12°. 13′. 36″,8. As the revolution here determined is that in respect to the longitude of the planet, it must be a *tropical* revolution. Hence, to get the sidereal revolution, we must say, 2′. 0″. 35‴ : 24′. 42″. 20‴ (the precession in the time of a tropical revolution (148)) :: 1 day : 12*d*. 7*h*. 1′. 57″, which added to 29*y*. 162*d*. 4*h*. 27′ gives 29*y*. 174*d*. 11*h*. 28′. 57″ the length of a sidereal year of Saturn.

240. In the same manner that we have determined the time of a tropical revolution of Saturn from those oppositions which happen nearly in the same point of the heavens, we may determine the periodic time of *Jupiter* and *Mars*; we shall therefore select such observations from Cassini, as may be proper for this purpose.

In 1699, *Jupiter* was in opposition at Paris on June 14, at 10*h*. 8′ in ♐ 23°. 52′. 40″, with 0°. 23′. 7″ north lat. In 1710 the opposition happened on May 17, at 18*h*. 24′ in ♐ 26°. 47′. 47″, with 1°. 4′. 50″ north lat. In 1711 the opposition was on June 20, at 6*h*. 37′ in ♐ 28°. 36′ with 0°. 15′. 50″ north lat. From these observations, the time of a mean revolution comes out 11*y*. 313*d*. 16*h*. 54′. Now the most ancient opposition is that observed by Ptolemy on May 15, 133 years after J. C. at 23*h*. 3′, Jupiter being in ♏ 23°. 22′. 22″. On May 12, 1698, it happened at 5*h*. 46′ in ♏ 22°. 20′. 32″. On June 14,

1699, it happened at 10*h*. 8′ in ♐ 23°. 52′. 42″. From these observations, proceeding as for Saturn, the time of a *tropical* revolution comes out 11*y*. 315*d*. 10*h*. But from the mean of several observations CASSINI determined it to be 11*y*. 315*d*. 14*h*. 36′. Hence, its mean annual motion is 30°. 20′. 31″. 50‴. In his Tables he makes it 30°. 20′. 34″. Dr. HALLEY, in his Tables, makes it 30°. 20′. 38″. M. de la PLACE makes it 30°. 20′. 31″,7.

In 1715 *Mars* was in opposition on April 21, at 11*h*. in ♏ 1°. 9′. 30″. On June 11, 1717, the opposition happened at 9*h*. 11′ in ♐ 20°. 17′. 15″. Now in this time, which was 2 years (one of which was a bissextile) and 50*d*. 22*h*. 11′, Mars had made one revolution and 49°. 27′. 45″ over; hence, from these two observations, we shall get a sufficient approximation to the time of a revolution, by saying, 360° + 49°. 27′. 45″ : 360° :: 781*d*. 22*h*. 11′ : 687*d*. 11*h*. 15′ the time of a revolution. Now, from the observations of PTOLEMY, it appears that Mars was in opposition on December 13, at 11*h*. 48′ at Paris, 130 years after J. C. in ♊ 21°. 22′. 50″. In 1709 Mars was in opposition on January 4, at 5*h*. 48′ in ♋ 14°. 18′. 25″. Between these observations there was an interval of 1578*y*. 11*d*. 18*h*., and consequently the time of a *tropical* revolution comes out 686*d*. 22*h*. 16′. From the mean of several results CASSINI makes it 686*d*. 22*h*. 18′. Hence, the mean annual motion is 6ˢ. 11°. 17′. 9″,5. Dr. HALLEY makes it 6ˢ. 11°. 17′. 10″ in his Tables; and M. de la LANDE makes it the same. The mean motions thus found may be considered as sufficiently accurate to settle the place of the aphelion and excentricity of the orbit; after which the periodic time may be determined with greater accuracy. Taking therefore the place of the aphelion and excentricity of Jupiter and Mars as we shall afterwards settle it, we will proceed to show how we may correct the periodic time already found, by allowing for the difference of the equations at the different observations.

On May 15, 133 years after J. C. *Jupiter* was in opposition at 23*h*. 3′ in the meridian of Paris, in ♏ 23°. 22′. 22″; and the equation of the orbit being 5°. 12′. 46″, the *mean* place was ♏ 28°. 35′. 8″. On May 12, 1698, at 5*h*. 46′ in the evening, Jupiter was in opposition in ♏ 22°. 20′. 32″, and the equation being 3°. 51′. 21″, the *mean* place was ♏ 26°. 11′. 53″; hence, the difference between the mean places was 2°. 23′. 15″, the time of describing which was 28*d*. 17*h*. 15′ according to the mean motion already determined; this added to the time of opposition on May 12, 1698, gives June 10, 11*h*. 1′ at which time the mean place was the same as at the first observation. Hence, the interval of these observations divided by 132, the number of revolutions, gives 11*y*. 315*d*. 12*h*. 54′ for the time of a mean *tropical* revolution. From the mean of this and two other observations, CASSINI finds the time to be 11*y*. 315*d*. 12*h*. 33′; and consequently its mean annual motion 30°. 20′. 33″. 56‴. *Elem. d'Astron.* p. 431.

On December 13, 130 years after J. C. *Mars*, from the observations of Ptolemy, was in opposition at 11*h*. 48′ in ♊ 21°. 22′. 50″; and the equation being 7°. 2′. 44″, the mean place was ♊ 14°. 20′. 6″. On December 11, 1691, at 3*h*. 14′ the opposition happened in ♊ 19°. 55′. 16″; and the equation being 10°. 16′. 14″, the mean place was ♊ 9°. 39′. 2″. Now the difference of these mean places was 4°. 41′. 4″, the time of describing which is 8*d*. 22*h*. 31′, which added to December 11, 3*h*. 14′ gives December 20, 1*h*. 45′, when the mean place was the same as at the first observation. The interval of these times was 1561 years (of which 390 were bissextiles) wanting 3*d*. 10*h*. 3′; which divided by 830, the number of revolutions, gives 686*d*. 22*h*. 18′. 39″ for the time of a mean *tropical* revolution; consequently the mean annual motion is 6ˢ. 11°. 17′. 9″,5. This correction is not necessary to be applied to our determination of the periodic time of Saturn; for as it was observed near the mean distance, where the equation is a maximum, the variation of $\frac{1}{2}$° in the place would not cause any sensible variation in the equation.

241. In the same manner we may determine the time of a tropical revolution of *Venus*, by comparing the time of two conjunctions, first getting an approximation in order to be sure of the number of revolutions. Now in 1709, on June 22, at 6*h*. apparent time, Venus was in superior conjunction in ♋ 0°. 56′. 30″; and in 1705, on June 21, at 22*h*. an inferior conjunction happened in ♋ 0°. 36′. 52″. In this interval Venus must have made 6$\frac{1}{2}$ revolutions and 19′. 38″; therefore the time of one revolution is found to be 224$\frac{2}{3}$ days nearly. To get the time more accurately, we must take two conjunctions at a greater interval of time, and allow for the difference of the equations at the times of observation. Now in 1639 on December 4, at 6*h*. 11′ mean time, Venus was in conjunction in ♊ 12°. 31′. 44″ on the ecliptic, and ♊ 12°. 31′. 37″ on its orbit; and the equation being 40′. 26″, gives its mean place ♊ 13°. 12′. 3″. In the conjunction 1716, on August 28, at 16*h*. 37′ mean time, Venus was in ♓ 5°. 49′. 53″ on its orbit; and the equation being 25′. 11″, gives the mean place ♓ 6°. 15′. 4″. Now in this interval of time, which has been 76 common years and 286*d*. 10*h*. 26′, there have, from what has been shown above, been 125 revolutions and 8ˢ. 23°. 3′. 1″; hence, 125 revolutions 8ˢ. 23°. 3′. 1″ : 360°. :: 76*y*. 286*d*. 10*h*. 26′. : 224*d*. 16*h*. 41′. 40″ the time of a mean *tropical* revolution. Hence, the mean annual motion is 7ˢ. 14°. 47′. 28″. *Elem. d'Astron.* page 562.

242. Cassini proposes also to find the time of a revolution, by comparing the ancient observations with the modern ones made when Venus was not in conjunction, for the ancient Astronomers could make no observations in conjunction for want of telescopes. For example. On December 25, 136, at 4*h*. Venus seen from the earth appeared in 1ˢ. 20°. 13′. 45″; and on December 17, 1594, at 4*h*. 30′ it was seen in 1ˢ. 23°. 1′. 36″, advanced 2°. 47′. 51″ beyond

the first observation; and as Venus ran through this space in $1d.\ 17h.\ 54'$, CASSINI concluded that on December 15, 1594, at $10h.\ 36'$ Venus was in the same place as at the first observation, the interval of which times was 1458 common years $354d.\ 6h.\ 36'$, in which Venus had made 2370 revolutions; hence, the time of one revolution is $224d.\ 16h.\ 39'.\ 4''$. This method would be accurate, provided the earth was at the same point at both times, and the orbit of Venus was fixed. Hence, the mean annual motion is $7^s.\ 14°.\ 47'.\ 45''$. CASSINI in his Tables makes it $7^s.\ 14°.\ 47'.\ 29''$. Dr. HALLEY makes it $7^s.\ 14°.\ 47'.\ 28''$. M. de la LANDE makes it $7^s.\ 14°.\ 47'.\ 30''$.

243. The periodic time of *Mercury* may be very accurately determined from its transits over the sun's disc; for as they have frequently been observed, we have an opportunity of chusing such as will give us a very accurate conclusion. From the observations of the conjunction of Mercury on November 6, 1631, CASSINI found the time of the conjunction to be at $19h.\ 50'$, and the true place of Mercury $1^s.\ 14°.\ 41'.\ 35''$. On November 9, 1723, at $5h.\ 29'$, the conjunction was in $1^s.\ 16°.\ 47'.\ 20''$, only $2°.\ 5'.\ 45''$ beyond the place at the first observation. Now according to the Tables of CASSINI, this difference is just equal to the motion of the aphelion of Mars in the same time; consequently Mercury was in the same place in its orbit at each time, and therefore the equation was the same. Also, the conjunctions happening very nearly at the same time of the year, the equation of time was very nearly the same, and therefore the difference of the apparent times is the same as of the true. Hence in the interval of 92 years (of which 22 were bissextiles) and $2d.\ 9h.\ 39'$, Mercury (from first finding nearly the time of a revolution by 2 conjunctions near each other) is found to have made 382 revolutions $2°.\ 5'.\ 45''$; hence, by proportion, the time of a tropical revolution is $87d.\ 23h.\ 14'.\ 20'',9$; and the mean annual motion comes out $1^s.\ 23°.\ 43'.\ 11''.\ 39'''$. CASSINI, in his Tables, makes it $1^s.\ 23°.\ 43'.\ 11''$. Dr. HALLEY makes it $1^s.\ 23°.\ 43'.\ 2''$; and M. de la LANDE, $1^s.\ 23°.\ 43'.\ 3''$.

On the Secular Motions of Jupiter and Saturn.

244. The time of a revolution of Saturn deduced from the modern observations comes out greater than that deduced from a comparison of the modern with the ancient observations. If therefore the modern observations could be depended upon to give the time of a revolution nearer than that difference, it would prove that the length of Saturn's year is increasing. Now although observations made at a small interval of time, could not be sufficient to establish this point, yet from a comparison of our observations with those made by TYCHO, it appears that this is the case. The length of the year therefore when

ascertained for one time will afterwards want a correction, and the quantity of this correction is called the *Secular Equation.*

245. KEPLER first observed this circumstance, from examining the observations of REGIOMONTANUS and WALTHERUS; for he constantly found Jupiter forwarder and Saturn backwarder than they ought to have been from the mean motions determined from the observations of PTOLEMY and TYCHO. He said the same of Mars; but M. de la LANDE observes, that he cannot find there is any secular equation wanted for that planet. FLAMSTEAD also observed, that in all the best Astronomical Tables, the mean motions of Saturn were too swift, and of Jupiter too slow; whence it came to pass, that the computations gave those conjunctions which happened when the planets were *direct*, some days sooner, and when *retrograde*, some days later than the time from observation; *Phil. Trans.* N°. 149. HEVELIUS also observed the same. M. MARALDI perceived also that the mean motions of Saturn, if we suppose them uniform, would not agree both with the observations of TYCHO and those of this age. Dr. HALLEY, in his Astronomical Tables, applied a secular equation of $9^\circ\frac{1}{4}$ for 2000 years to Saturn, and 3°. 40′ to Jupiter, but he does not give the observations from which he deduced these conclusions. M. de la LANDE, from comparing the oppositions in the years 1594, 1595, 1596 and 1597 with those in 1713, 1714, 1715, 1716 and 1717, found the mean motion of Saturn to be 12°. 13′. 19″. 14‴ which is 16″ in a year less than that given by CASSINI; and the duration of the revolution greater by near 4 days. He chose those oppositions which happened near the mean distance (as CASSINI did also), because the true and mean motions being then equal, the conclusions would be more accurate. He also chose other oppositions at the distance of about 120 years, and when Jupiter and Saturn were in similar situations, so that no error was to be apprehended from their mutual attraction, this being the same in each case. Now if with the mean motion found in 120 years, the place of Saturn, from where it is now found to be, be computed for the time of the observation before mentioned in the year 228 before J. C. the longitude will be found to be too great by 7°; this therefore is the secular equation for 2000 years, according to this mean motion. But from other observations he concluded the mean motion to be 12°. 13′. 26″,558. With this mean motion he finds the secular equation to be 47″ in the first century from which this motion was deduced; for with this mean motion and secular equation, the calculations best agree with the ancient observations. From the theory of attraction it appears, that supposing the aphelion of Saturn and Jupiter to be fixed, the secular equation varies as the square of the time, which M. de la LANDE thinks may be deduced from this consideration, that the velocity lost by Saturn in consequence of the cause which produces the equation being so very small, may be consi-

dered equal in equal times; whence from the principle of the law of falling bodies, the space lost must vary as the square of the time. Now from five observations of PTOLEMY, he found the secular equation for the first 100 years to be $47''$; hence, $100^2 : t^2 :: 47''$: the secular equation for t years. Now the logarithm of 47 *minus* the logarithm of 100^2 is 7,6720979; hence, if to this constant logarithm we add twice the logarithm of t, we shall have the logarithm of the secular equation for t years from the commencement of the 100 years, to be subtracted from the mean longitude.

246. But besides the secular equation, the mean motion of Saturn is also subject to other irregularities, which are found to follow from the attraction of Jupiter. Dr. HALLEY, in his Astronomical Tables, observes that Jupiter from his opposition in 1677, to that in 1689, was found, from indubitable observations, to be $12'$ slower than in the preceding or subsequent revolutions. Also the periodic time of Saturn between the years 1668 and 1698 was nearly a week shorter than its mean revolution; and the periodic time between 1689 and 1719 was nearly as much greater, so that between the two revolutions there was a difference of more than 13 days. This Dr. HALLEY supposes to arise from the attraction of the greater bodies in the system being different in different positions. For he observes, that in 1683 there was a conjunction of Jupiter and Saturn, when from the position of the apsides, the planets approached nearest to each other, and Saturn was most urged towards the sun and Jupiter from it; so that Jupiter's velocity being increased and its force to the sun diminished, its orbit was increased and consequently its periodic time; on the contrary, Saturn's velocity being diminished and its force to the sun increased, its orbit, and consequently its periodic time, was diminished. Now, says he, if the same thing should happen again, that is, if a conjunction should take place again in the same point of the Heavens, and the same effects should follow, we may hope that it can be accounted for from the Laws of gravity; but if, in like circumstances, the same effects are not found to take place, other extraneous causes are to be sought for. But M. de la PLACE has discovered, that these inequalities, as well as the secular equations, may be represented by an equation, from Jupiter's attraction, of $48'$, which depends on 5 times the longitude of Saturn minus twice that of Jupiter, of which the period is 918 years. For this we must employ the mean annual motion of $12°. 13'. 36'',81$. Thus all the irregularities of Saturn's motion are confined to a certain period, after which they all return again. In the years 1701 and 1760 the errors of Dr. HALLEY's Tables were $8\frac{1}{2}'$ and $21\frac{1}{2}'$, according to M. de la LANDE, so that the motion of Saturn was greater by $13'$, and its periodic time was shorter by $6\frac{1}{2}$ days, than in its revolution between 1686 and 1745. Now the mean anomaly in 1701 and 1760 was $3°. 1'$, and the angle at the sun between Jupiter and Saturn was $19°$ in 1701 and $30°$ in 1760, so that the error in the mean motion

could not arise from any dissimilar situations of Saturn in its orbit, by which the elements of the motions might err; nor from the different situations of Jupiter, that difference not being sufficient to cause such an error.

247. The motion of Jupiter requires also a secular equation, as Dr. HALLEY observed, who made it 3°. 49′. 24″ for 2000 years, or 34″,4 for the first century, supposing it to increase as the square of the time. M. MARALDI also observed, that the modern observations gave the motion of Jupiter greater than the ancient. M. de la LANDE found by comparing the observations made 240 years before J. C. with those in the year 508, that Jupiter's secular motion in 83 years was 2′. 04″. And comparing the observations in 508 with those in 1503 and 1504, we find nearly the same result. But if we compare the conjunction of Jupiter with *Regulus* on October 12, 1623, with the like observation made in 1706, we find it 21′ for 83 years. Dr. HALLEY, in his Tables, fixed it at 12′. 26″ for 83 years, which makes the revolution 8 hours shorter than that deduced from the ancient observations. The oppositions from 1689 to 1698 compared with those in 1749, give a mean motion equal to that in the Tables of CASSINI; which Tables give the place of Jupiter 1′ too much in 508. These conclusions indicate a great irregularity in Jupiter's motions; and this irregularity is further confirmed, if we consider that M. WARGENTIN makes the secular equation for the first 100 years to be 18″; M. BAILLY makes it $12\frac{1}{3}''$; and M. de la LANDE fixes it at $30\frac{1}{2}''$ for the first 100 years, or 3°. 23′. 20″ for 2000 years, admitting it to increase as the square of the time, which agrees nearly with Dr. HALLEY's determination. M. de la GRANGE, from the theory of gravity, finds it to be 3′. 18″, which, as M. de la LANDE observes, agrees very well with the observations from 1590 to 1762, but not with the ancient observations. EULER determined it from theory to be 2′. 23″. M. de la LANDE says, that his own secular equation, with the mean secular motion of 5^s. 6°. 27′. 30″, agree as nearly as possible to all the observations. M. de la PLACE found in 1786 an inequality of 20′ from the attraction of Saturn, the period of which equation is 918 years, as in Saturn. Thus he made the secular equation disappear, it being only an irregularity whose period is 918 years. This supposes a secular motion of 5^s. 6°. 17′. 33″. The secular equation being determined for 100 years, it may be found for any other time, as it was for Saturn, by taking it in proportion to the square of the time.

The longitude of the sun requires a secular equation of 12′ for 2500 years, arising from the diminution of the precession of the equinoxes, according to M. de la LANDE.

According to Dr. HALLEY.

Planets	*Tropical Revolution*	*Sidereal Revolution*	*Sec. movement*
Mercury	87d. 23h. 14'. 34",4	87d. 23h. 15'. 45",5	2s. 14°. 2'. 23"
Venus	224. 16. 41. 30,6	224. 16. 49. 14,5	6. 19. 11. 52
Mars	686. 22. 18. 18,8 or 1y. 321.	686. 23. 30. 34,7 or 1y. 321.	2. 1. 42. 20
Jupiter	4330. 8. 35. 4 or 11. 315.	4332. 8. 28. 1,1 or 11. 315.	5. 6. 28. 11
Saturn	10750. 13. 14. 42,1 or 29. 165.	10762. 20. 38. 41,1 or 29. 177.	4. 23. 6. 0

According to M. de la LANDE.

Planets	*Secular motion*	*Tropical Revolution.*	*Sidereal Revolution.*	*Mot. diur. trop.*
Mercury	2s. 14°. 4'. 20"	87d. 23h. 14'. 32",7	87d. 23h. 15'. 43",6	4°. 5'. 32",57
Venus	6. 19. 12. 25	224. 16. 41. 27,5	224. 16. 49. 10,6	1. 36. 7,8
Earth	0. 0. 46. 0	365. 5. 48. 48	365. 6. 9. 11,6	0. 58. 8,33
Mars	2. 1. 42. 10	686. 22. 18. 27,4	686. 23. 30. 35,6	0. 31. 26,66
Jupiter	5. 6. 17. 33	4330. 14. 39. 2	4332. 14. 27. 10,8	0. 4. 59,26
Saturn	4. 23. 31. 36	10746. 19. 16. 15,5	10759. 1. 51. 11,2	0. 2. 0,6

The secular motion is in respect to the equinox.

The secular motion of the *Georgian* Planet in respect to the equinox is 2s. 13°. 16'. 55"; its tropical revolution is *83y. 52d. 4h.* its sidereal revolution is *83y. 150d. 18h;* and its tropical diurnal motion is 42",678026.

Dr. HALLEY made the length of a tropical year *365d. 5h. 48'. 55"*; FLAMSTEAD and Sir I. NEWTON made it 57",5; MAYER 51"; and M. de la CAILLE in his Tables 49". By our determination, 57".

CHAP. XIII.

ON THE GREATEST EQUATION, EXCENTRICITY AND PLACE OF THE APHELIA OF THE ORBITS OF THE PLANETS.

Art. 248. HAVING determined the mean motions of the planets, we proceed next to show the method of finding the greatest equation of their orbits, and from thence the excentricity and place of their aphelia. For although, in order to determine the mean motions very accurately, these things were supposed to be known, yet without them the mean motions may be so nearly ascertained, that these elements may from thence be very accurately settled.

249. Let A be the aphelion, S the focus; take SW a mean proportional between the semi-axis major and minor, then (230) when the planet comes to the points V and W the equation is the greatest; at which times let the mean places be at v and w, then the difference between the true and mean motions from V to W is the sum of the angles VSv, WSw, or $2WSw$, the half of which is the greatest equation. Now to find when this happens, observe the true places of the planet when at V and W, take the difference of the two places, and compute the mean motions for the same time, and half the difference is the greatest equation. But as it is impossible to fix upon the times when the planet is accurately at V and W, several observations must be made about each time, and comparing them two by two, find those where the difference between the true and mean motions is the greatest, and half the difference is the greatest equation. The observer will easily find when the planet is got near to the mean distance, by comparing his observations for several days, and observing whether the true motion be nearly equal to the mean motion. Hence, if we bisect the interval it will give the place A of the aphelion. Having found the greatest equation, the excentricity will be known (231). Or the greatest equation may be found thus. Having made two observations near to V and W, find the equation corresponding, and from thence the place of the aphelion and excentricity; then compute for the two times of observation the equations corresponding, and also the greatest equation; and the difference between half the sum of the computed equations for the times of observation and the computed greatest equation shows the error arising from the observation; which added to the equation found from observation gives the greatest equation.

Ex. To find the greatest equation of the sun. From the observations of M. de la CAILLE in 1751, on

October 7, sun's place observed was -	6^s. 13°. 47'. 13'',7
March 28, 1752 - - - - -	0. 8. 9. 25, 5
	5. 24. 22. 11, 8
Mean motion by calculation - -	5. 20. 31. 43, 2
	3. 50. 28, 6

The half of which 1°. 55'. 14'',3 is the greatest equation, if no correction be required. But if we take the place of the aphelion and excentricity from this equation, considered as the greatest, and calculate the equations for these two times, half the difference will be the supposed greatest equation; compute also the greatest equation, and we shall find that these differ by 18'',6, which shows that the greatest equation deduced from these two observations differs from the greatest equation itself by that quantity; this therefore added to 1°. 55'. 14'',3 gives 1°. 55'. 32'',9 for the greatest equation. From the mean of several observations M. de la CAILLE makes it 1°. 55'. 32''.

In the year 1717 on March 21, the sun's place on the meridian at Paris, by CASSINI'S Astronomy page 191, was in ♈ 0°. 47'. 28'' and on September 23, in ♎ 0°. 15'. 50''. Hence, the true motion in 185*d.* 23*h.* 45' was 5^s. 29°. 28'. 22'', and the mean motion in that time was 6^s. 3°. 19'. 12'', half the difference of which is 1°. 55'. 25''. By thus comparing the observation on September 23, 1717, with the observation on March 21, 1718, the equation comes out 1°. 55'. 16'',5. If we compare the observation on March 28, 1717, with that on September 27, following, the equation comes out 1°. 55'. 37'',5. And if we compare the observation on March 28, 1718, with that on September 27, 1717, the equation is found to be 1°. 56'. 3'',5. The mean of all these is 1°. 55'. 35'',5 for the greatest equation, differing only 3'',5 from the other; but CASSINI, in his Tables, makes it 1°. 55'. 51''. In the Tables of MAYER it is 1°. 55'. 31'',6. M. de LAMBRE, from the observations of Dr. MASKELYNE, makes it 1°. 55'. 30'',9 in 1780; for on account of the diminution of the excentricity of the earth's orbit, the greatest equation is subject to a diminution.

250. To find the place of the aphelion A, observe the interval of time from m to n, two opposite points in the orbit; and if that be equal to half the anomalistic revolution, or the time from A to Q, the points m and n must coincide with A and Q; for the whole area can only be bisected by the line ASQ passing through S, and consequently the time of half a revolution about S can never be equal to half the time of one whole revolution but from A to Q, the FIG. 53.

areas being in proportion to the times (219). Now the difference (d) between the times from A to Q, and from m to n must, by taking away the time from m to Q which is common to both, be equal to the difference between the times through Am and Qn. Put t=the time from A to m, and let m and n be the angular velocities about S in 24 hours at A and Q; then $n : m :: t : \frac{mt}{n}$ = the times through Qn, the time of describing equal angles being inversely as the angular velocities; hence, $t - \frac{mt}{n} = d$, consequently $n - m : n :: d : t$. Now if the observation be made at m when the sun is past A, the time through mQn must be less than the time from A to Q, because the area ASm being greater than QSn, the area AmQ described about S must be greater than that of mQn; and the contrary if m be on the other side of A.

Ex. On December 30, 1743, at $0h.\ 3'.\ 7''$ mean time, M. de la Caille found the sun's longitude to be ♑ 8°. 29'. 12'',5; and on June 30, 1744, at $0h.\ 3'$ it was ♋ 8°. 51'. 1'',5; the interval of these two places is 180°. 21'. 49''. Now reckoning, with M. de la Caille, the annual progressive motion of the apogee of the earth's orbit to be 1'. 3'', the distance of the apogee from the perigee is 180°. 0'. 31'',5; but the sun had described 180°. 21'. 49'', which exceeds 180°. 0'. 31'',5, half an anomalistic revolution, by 21'. 17'',5; and the sun's motion on June 30, being 57'. 12'' in 24 hours, 57'. 12'' : 21'. 17'',5 :: $24h : 8h.\ 56'$ the time of describing 21'. 17'',5, which subtracted from June 30, $0h.\ 3'$ gives June 29, $15h.\ 7'$ when the sun was in ♋ 8°. 29'. 43'' at the distance of 180°. 0'. 31'',5 from the place where it was on December 30, at $0h.\ 3'.\ 7''$; the interval of these two times is $182d.\ 15h.\ 3'.\ 53''$, which being less than $182d.\ 15h.\ 7'.\ 1''$, half the time of an anomalistic revolution (150), by 3'. 8'' ($=d$), the sun was not come to its apogee on June 29, $15h.\ 7'$. Now the sun's motion on June 30, was 57'. 12'' in a day $=m$, and on December 30, 61'. 12'' $=n$; hence 4' : 57'. 12'' :: 3'. 8'' : 44'. 48'', which added to June 29, $15h.\ 7'$ gives June 29, $15h.\ 44'.\ 48''$ when the sun was in its apogee, at which time the sun's place was in ♋ 8°. 31'. 21'', which therefore was the place of the apogee.

251. To find the excentricity, we have (231) 57°. 17'. 44'',8 : 57'. 45'',5 (the half of 1°. 55'. 30'',9 the greatest equation according to M. de Lambre) :: 1 : ,01681 the excentricity, the mean distance being unity. As the orbit is very nearly a circle, the correction is unnecessary.

252. The above method of finding the place of the aphelion from the greatest equation is very applicable to the case of the sun and moon, but it cannot be applied with the same success to the planets, because they do not revolve about

the earth, and therefore their velocities near the apsides, in respect to the sun, cannot be obtained in like manner. M. CASSINI (*Elem. d'Astron.* pag. 366.) therefore proposes the following method. Having found the greatest equation, by observing the angle described between the mean distances B and D through the aphelion A, observe the planet at r near to A, and the angle BSr will be the true angle described between B and r; then from the time of describing this angle compute the mean motion; and if the difference between the true and mean motions be *equal* to the greatest equation, then r is the aphelion; if it be *less*, the planet is not got to its aphelion. Make then another observation at m, and if the difference between the true and mean motions be now *greater* than the equation, the planet is got beyond A. Hence say, as the sum of the equations at r and m : the equation at r :: the angle rSm : the angle rSA the distance of the point r from the aphelion; for (229) when the distance from the aphelion is small, the equation varies very nearly as the true anomaly. This may be corrected, if necessary, by calculating, from the place of the aphelion, whether the body be found at r and B when it ought. And to find the time of coming to the aphelion, say, as the sum of the equations at r and m : the equation at r :: time of describing rm : time of describing rA.

Ex. To find the greatest equation, place of the aphelion and excentricity of the orbit of *Saturn*. Between the opposition in 1686 and 1687 Saturn had moved through 12°. 38'. 20", and its mean motion in that interval being 12°. 39'. 34", Saturn was then very near its mean distance. Now Saturn was in opposition in

1686, March 16, 10*h*. 28' in - -	5ˢ. 26°. 47'. 6"
1701, September 16, 2*h*. in - -	11 . 23 . 21 . 16
Interval 15*y*. 186*d*. 15*h*. 32' - -	5 . 26 . 34 . 10
Mean motion in this interval - -	6 . 9 . 36 . 0
	13 . 1 . 50
Greatest equation - - - -	6 . 30 . 55

To find the place of the aphelion, and the time of coming to it. Saturn was in opposition in

1686, March 16, 10*h.* 28′ in - - - -	5ˢ. 26°. 47′. 6″
1693, June 9, 19*h.* 32′ in - - -	8 . 19 . 54 . 41
Interval 7*y.* 87*d.* 9*h.* 4′ - - -	2 . 23 . 7 . 35
Mean motion in this interval - -	2 . 28 . 29 . 27
	5 . 21 . 52
Greatest equation - - - -	6 . 30 . 35
Equation at *r* - - - - -	1 . 9 . 3

Hence, Saturn was not come to its aphelion in opposition 1693. Now the opposition happened in

1686, March 16, 10*h.* 28′ in - -	5ˢ. 26°. 47′. 6″
1694, June 21, 19*h.* 30′ in - - - -	9 . 1 . 6 . 40
Interval 8*y.* 99*d.* 10*h.* - - -	3 . 4 . 19 . 34
Mean motion in this interval - -	3 . 11 . 6 . 51
	6 . 47 . 17
Greatest equation - - -	6 . 30 . 55
Equation at *m* - - - -	16 . 22

Therefore Saturn had passed its aphelion in opposition 1694. Hence, 1°. 9′. 3″ + 16′. 22″ = 1°. 25′. 25″ : 1°. 9′. 3″ :: 11°. 12′ (the angle described between the oppositions in 1693 and 1694) : 9°. 3′. 20″, which added to 8ˢ. 19°. 54′. 41″ gives 8ˢ. 28°. 58′ for the place of the aphelion. And to find the time, we have 1°. 25′. 25″ : 1°. 9′. 3″ :: 376*d.* 23*h.* 58′ (the time between the oppositions in 1693 and 1694) : 305*d.* 16*h.* which added to 1693, June 9, 19*h.* 32′ gives 1694, April 11, 11*h.* 32′ the time when Saturn was in its aphelion. Dr. Halley, in his Tables, makes the greatest equation 6°. 32′. 4″. Cassini makes it 6°. 31′. 40″. M. de Lambre makes it 6°. 26′. 42″ in 1750, and supposes that it is diminished 1″,1 in a year, according to the determination of M. de la Place. From the mean of six excentricities, determined (231) from the greatest equation, Cassini found the excentricity to be ,56515, the mean distance of the earth from the sun being unity.

253. The same method may be applied to find the greatest equation, place of the aphelion and excentricity of *Jupiter's* orbit, although we cannot so readily meet with observations made in the proper places, because we have fewer oppositions of Jupiter in one revolution than of Saturn. The following however are proper for our purpose (*Elem. d'Astron.* page 423.) In 1723, on June 25, at

5

4*h*. Jupiter was in opposition in ♑ 3°. 21′. 22″, near its mean distance; on December 22, 1728, at 3*h*. 9′ the true place of Jupiter in opposition was ♋ 1°. 8′. 2″. The difference of these places is 5ˢ. 27°. 46′. 40″; and the mean motion being 5ˢ. 16°. 50′. 15″, the difference is 10°. 56′. 25″, the half of which is 5°. 28′. 12″,5 the greatest equation from these observations. On September 5, 1725, at 14*h*. 44′ Jupiter was in opposition in ♓ 13°. 18′; this compared with the opposition in 1723, gives 2ˢ. 9°. 56′. 38″ for the true motion of Jupiter in the interval; and the mean motion being 2ˢ. 6°. 47′. 24″, the difference is 3°. 9′. 14″, which subtracted from 5°. 28′. 12″ gives 2°. 18′. 58″ the equation at *r*. On October 13, 1726, at 6*h*. Jupiter was in ♈ 20°. 4′. 10″ in opposition; this compared with the opposition in 1723, gives 3ˢ. 16°. 52′. 48″ for the true motion in the interval; and the mean being 3ˢ. 10°. 15′. 39″, the difference is 6°. 37′. 9″, from which subtract 5°. 28′. 12″ and the remainder is 1°. 8′. 57″ the equation at *m*. Hence, 2°. 18′. 58″ + 1°. 8′. 57″ = 3°. 27′. 55″ : 1°. 8′. 57″ :: 36°. 46′. 10″ (the angle described between the oppositions in 1725 and 1726) : 12°. 15′, which subtracted from ♈ 20°. 4′. 10″ gives ♈ 7°. 49′. 10″ the place of the perihelion. The time of opposition is also found by saying, 3°. 27′. 55″ : 1°. 8′. 57″ :: 372*d*. 15*h*. 16′ (the interval of the oppositions in 1725 and 1726) : 134*d*. 5*h*. 5′, which subtracted from the opposition in 1726 on October 13, at 6*h*. gives the time at which Jupiter was in its perihelion to be on June 1, 0*h*. 55′. Also, the excentricity is found to be 0,04774, the mean distance of Jupiter from the sun being unity. It must be here observed, that the accuracy of this method depends upon the proximity of *r* and *m* to the aphelion or perihelion. CASSINI, in his Tables, makes the greatest equation 5°. 31′. 17″. Dr. HALLEY makes it 5°. 31′. 36″. M. de LAMBRE finds it to be 5°. 30′. 37″,7 in 1750, and to increase 55″,36 in 100 years.

As in the ancient observations of *Mars* mentioned by PTOLEMY, there are only three which were made in opposition, and as they are not in proper places for the application of the last method, we shall give another Rule to determine the greatest equation, the place of the aphelion and the excentricity, from any three heliocentric places of a planet, and its mean motion. This is resolved in the following manner, first upon the supposition of the *simple elliptic hypothesis* (227), and then correcting it.

254. Let *S* be the sun, *B*, *C*, *D* three places of the planet observed in opposition, *F* the other focus, *A* the aphelion, *Q* the perihelion; with the center *F* and radius *FM* equal to the major axis describe a circle, and produce *FB*, *FC*, *FD* to the circumference, and join *SG*, *SH*, *SE*. Now the angles *BSC*, *CSD* are known from observation; also, upon supposition of the *simple elliptic hypothesis*, equal angles are described about *F* in equal times; therefore the angles *BFC*, *CFD* are known, by taking them to four right angles as the intervals of time between the first and second, second and third observations, to the periodic FIG. 54.

time. Now as $FG=FB+BS$, therefore $SB=BG$; for the same reason $SC=CH$ and $SD=DE$. Hence, $2FGS=FBS=BFA-BSA$; also, $2FHS=FCS=CFA-CSA$; therefore $2FGS+2FHS=BFC-BSC$; hence, $FGS+FHS$ is known; but $FGS=BFA-GSA$, and $FHS=CFA-HSA$; therefore $FGS+FHS=BFC-GSH$, whence GSH is known. For the same reason HSE is known. Hence, the angles GSH, HSE, GSE, and BFC, CFD, BFD are known. Produce ES to L, and join HL, HG, GL, and assume SH of any value in order to get the relative values of the other parts of the figure. Then in the triangle SHL, we know SH, the angle HSL (which is the supplement of HSE) and the angle HLS (which is half the angle HFE); hence we know SL; therefore in the triangle SLG, we know SL, the angle LSG (which is the supplement of GSE) and the angle SLG (the half of EFG); hence we know SG; therefore in the triangle GSH, we know GS, SH and the angle GSH; hence we know HG and the angle SHG; therefore in the isosceles triangle HFG, we know HG and the angle HFG; hence we know $FH=FC+CS$ the major axis, and the angle GHF, which taken from the angle SHG leaves the angle SHF which is therefore known; therefore in the triangle SHF, we know SH, HF and the angle SHF, from whence we know SF twice the excentricity, and the angle HSF, from which take the angle HSC (which $=SHF$) and we get the angle CSA, the distance of the aphelion A from the observation at C.

255. This method, being the *simple elliptic hypothesis*, supposes that the angles described about F are proportional to the times, which will be sufficiently accurate for orbits whose excentricity is small, as that of the earth and Venus; for the orbits of the other planets it may be thus corrected.

256. Having determined, from the three observed places m, n, r, of the planet, the place of the aphelion and the excentricity from the *simple elliptic hypothesis*, with the distances a, b, c, of the planet from the aphelion so found, calculate (232) the equation upon the true or KEPLER's hypothesis, and you will get the mean anomalies a', b', c' upon the true hypothesis. Then with these mean anomalies a', b', c', find the true anomalies a'', b'', c'', upon the simple elliptic hypothesis, and the difference between a and a'', b and b'', c and c'' shows the difference of the places upon the two hypotheses. To the place of the aphelion first found add the distances a'', b'', c'', and you get the places of the planet in the simple elliptic hypothesis answering to the true place upon KEPLER's hypothesis. Then with these three places compute, as at first, the place of the aphelion and excentricity upon the simple elliptic hypothesis, and you will have the distances A, B, C, from the aphelion upon the simple elliptic hypothesis, to these apply the differences of the two hypotheses before found, adding or subtracting them according as the simple elliptic hypothesis gave the place less or greater than KEPLER's hypothesis, and you will have the distances from the aphelion upon the true or KEPLER's hypothesis; subtract these from the cor-

responding places m, n, r of the planet observed, and you will have the place of the aphelion once corrected, and also the excentricity. In like manner the correction may be made as often as may be found necessary. *Elem. d'Astron.* page 184.

In 1694 on January 17, at 4*h*. 20′. Mr. FLAMSTEAD observed the place of *Mars* to be in ♋ 28°. 12′; in 1698 on March 26, at 17*h*. 55′ in ♎ 7°. 4′. 18″, and in 1702 on July 8, at 12*h*. 58′ in ♑ 16°. 10′. 23″. These observations reduced (268) to the orbit of Mars give the three places in ♋ 28°. 12′. 34″, ♎ 7°. 3′. 26″, and ♑ 16°. 11′. 9″. Hence, by KEPLER's hypothesis, the place of the aphelion is found to be in ♍ 0°. 39′. 2″ with the excentricity ,09292, the semi-axis major being unity; and the greatest equation 10°. 39′. 29″. *Elem. d'Astron.* page 474.

The same method may be applied to *Venus* from the conjunctions observed in the years 1715, 1716 and 1718; from which it appears, that the places of Venus seen from the sun upon the ecliptic were in 1715 on January 26, at 8*h*. 34′. mean time, in ♌ 6°. 22′. 58″; in 1716 on August 28, at 16*h*. 36′. 42″ in ♓ 5°. 49′. 2″; and in 1718 on April 8, at 10*h*. 15′. 11″. in ♎ 18°. 42′. 13″; which places reduced to the orbit of Venus will be ♌ 6°. 25′. 52″, ♓ 5°. 49′. 53″ and ♎ 18°. 39′. 24″. Hence, by the simple elliptic hypothesis, the true place of the aphelion in 1716 is found to be ♒ 6°. 50′; the greatest equation 49′. 8″; and the excentricity 0,00715. As the orbit of Venus differs but very little from a circle, there is no occasion for any correction. *Elem. d'Astron.* page 562. CASSINI, in his Tables, makes the greatest equation 49′. 6″. Dr. HALLEY makes it 48′. M. de la LANDE makes it 47′. 20″.

Upon the same principle we may deduce the place of the aphelion, excentricity and equation of the orbit of *Mercury*; but as the proper observations for this purpose happen at a considerable distance of time from each other, it will be proper to allow for the motion of the aphelion in the intervals, which CASSINI assumes (from what he was best able to collect from the observations before made) at 1′. 20″ in a year, by which means the motion is reduced to the orbit as immoveable. In 1661 on May 3, at 4*h*. 48′. 28″ mean time, the true place of Mercury was found to be in ♏ 13°. 33′. 27″ in respect to the ecliptic, and 13°. 33′. 10″ on its orbit. In 1690 on November 9, at 18*h*. 6′. it was in ♉ 18°. 20′. 46″ in respect to the ecliptic, and 18°. 22′. 28″ on its orbit. In 1697 on November 2, at 17*h*. 42′ it was in ♉ 11°. 33′. 50″ in respect to the ecliptic, and 11°. 32′. 30″ on its orbit. Now between the two first observations the motion of the aphelion was, by supposition, 39′. 20″; and between the first and last it was 48′. 40″; these subtracted from the second and third observations give the places in the orbit ♉ 17°. 43′. 8″ and ♉ 10°. 43′. 50″ in respect to the first observation, the orbit being supposed at rest. Hence, by subtracting ♏ 13°. 33′. 10″ from ♉ 17°. 43′. 8″, we have 6*s*. 4°. 9′. 58″ for the sum of the two true

4.

anomalies of Mercury between the first and second observations, the aphelion lying between the two observed places; and by subtracting ♉ 10°. 43′. 50″ from ♉ 17°. 43′. 8″, we have 6°. 59′. 18″ for the difference of the true anomalies between the second and third observations. Also, if we subtract 39′. 20″ from 6ˢ. 26°. 20′. 35″ the mean motion between the two first observations, and 48′. 40″ from 6ˢ. 21°. 51′. 7″ the mean motion between the first and third observations, we shall have 6ˢ. 25°. 41′. 15″ and 6ˢ. 21°. 2′. 27″ for the sum of the mean anomalies in these intervals; hence, 4°. 38′. 48″ is the mean anomaly corresponding to the two last observations, answering to 6°. 59′. 18″ of true anomaly. Hence, from the *simple elliptic* hypothesis, the aphelion of Mercury at the second observation is found to be in ♐ 10°. 51′. 50″, excentricity 0,21574, the mean distance being unity; and the greatest equation 24°. 55′. 4″. This corrected several times gives the true place of the aphelion on November 9, 1690 in ♐ 12°. 22′. 25″, the excentricity 0,20878 and the greatest equation 24°. 3′. Cassini, in his Tables, makes it 24°. 2′. 58″. Dr. Halley makes it 23°. 42′. 36″. M. de la Lande makes it 23°. 40′.

257. Besides these methods of determining the position and excentricity of the planetary orbits, we shall explain another method, which may be sometimes very successfully used, and is moreover strictly geometrical. By Art. 217, we may find the distance of a planet from the sun in any point of its orbit. The Problem therefore is, given in length and position three lines drawn from the focus of an ellipse, to determine the ellipse.

258. Let SB, SC, SD be the three lines; produce CB, CD, and take $SB : SC :: EB : EC$, and $SC : SD :: CF : DF$, then $SC-SB : SC :: BC : EC = \dfrac{SC \times BC}{SC-SB}$, and $SC-SD : SC :: DC : CF = \dfrac{SC \times DC}{SC-SD}$. Join FE, and draw DK, CI, BH perpendicular to it. Now by similar triangles, $IC : HB :: EC : EB ::$ (by con.) $SC : SB$; also, $IC : KD :: CF : DF :: SC : SD$. Hence, the proportion of IC, HB, KD is the same as SC, SB, SD, consequently EF is the directrix of the ellipse passing through B, C, D. Through S draw $ASQG$ perpendicular to FE; take $GA : AS :: CI : CS$, and $GQ : SQ :: CI : CS$; then $CI+CS : CS :: GS : SQ = \dfrac{CS \times GS}{CI+CS}$; also, $AS = \dfrac{CS \times GS}{CI-CS}$, and A, Q will be the vertices of the conic section.

259. *Calculation.* In the triangles SBC, SCD we know two sides and the included angles, they being the distances of the observed places upon the orbit; hence we can find BC, CD and the angles BCS, SCD, and consequently BCD. Hence (258) we know CE and CF, and the angle ECF being also known, the angle CEF can be found. Therefore in the right angled triangle CIE, CE and the angle E are given; hence, CI is known. Join SI; then in the triangle SIC we know CI, CS and the angle SCI ($=ECI-BCS$); hence we know SI

and the angles CIS, CSI, and therefore the angle SIG is known; hence, in the right angled triangle SIG, we know SI and the angle SIG, from whence SG is found. Hence (258) we know SA, SQ, half the difference of which is the excentricity, and their sum $= AQ$. Lastly, in the triangle BSO (O being the other focus) we know all the sides, to find the angle BSA, the distance of the aphelion from the observed place B.

In the year 1740 on July 17, August 26, September 6, M. de la CAILLE found three distances of Mercury (the mean distance being 10000) as follows; $SB = 10351{,}5$, $SC = 11325{,}5$, $SD = 9672{,}166$, the angle $BSC = 3^s.\ 27^\circ.\ 0'.\ 35''$ and $CSD = 44^\circ.\ 40'.\ 4''$. Hence, $BCS = 29^\circ.\ 55'.\ 5''$, $BC = 18941$, $SCD = 56^\circ.\ 49'$, $CD = 8124{,}5$, $BCF = 86^\circ.\ 44'.\ 5''$, $CE = 215004$, $CF = 55647$, $CEF = 14^\circ.\ 41'.\ 44''$, $CI = 54543$, $CSI = 124^\circ.\ 47'.\ 45''$, $CIS = 9^\circ.\ 49'.\ 4''$, $SI = 47281$, $SIG = 80^\circ.\ 10'.\ 56''$, $SG = 46589$, $SP = 8010{,}5$, $SA = 12209$, $SO = 4198{,}5$; hence, the excentricity $= 2099{,}75$, $BSA = 71^\circ.\ 37'.\ 23''$ or $2^s.\ 11^\circ.\ 37'.\ 23''$ which added to $6^s.\ 2^\circ.\ 13'.\ 51''$, the position of SB, gives $8^s.\ 13^\circ.\ 51'.\ 14''$ for the place of the aphelion. Hence, the greatest equation is $24^\circ.\ 3'.\ 5''$.

260. Or from the same data the place of the aphelion and excentricity may be thus found. Put the semi-axis major $= 1$, $SB = a$, $SD = b$, $SC = c$, the angle $BSD = v$, $BSC = u$, $BSQ = x$, $OS = e$, half the parameter $= r$. Then (see my Conic Sect. Ellipse, Propos. 16.) $a = \dfrac{r}{1 + e\ \cos.\ x}$, $b = \dfrac{r}{1 + e.\cos.\ \overline{v+x}}$, $c = \dfrac{r}{1 + e.\ \cos.\ \overline{u+x}}$; hence, $r = a + ae.\ \cos.\ x = b + be.\ \cos.\ \overline{v+x} = c + ce.\ \cos.\ \overline{u+x}$; therefore $\dfrac{b-a}{a.\cos.\ x - b.\cos.\ \overline{v+x}} = e = \dfrac{c-a}{a.\ \cos.\ x - c.\ \cos.\ \overline{u+x}}$; now for $\cos.\ \overline{v+x}$ and $\cos.\ \overline{u+x}$ substitute $\cos.\ v.\ \cos.\ x - \sin.\ v.\ \sin.\ x$ and $\cos.\ u.\ \cos.\ x - \sin.\ u.\ \sin.\ x$ (Trig. Art. 102) and we shall have

$$\frac{b-a}{a.\cos.x - b.\cos.v.\cos.x + b.\sin.v\sin.x} = \frac{c-a}{a.\cos.x - c.\cos.u.\cos.x + c.\sin.u\sin.x};$$

divide each denominator by $\cos.\ x$, and we have $\dfrac{b-a}{a - b.\ \cos.\ v + b.\ \sin.\ v.\ \tan.\ x} = \dfrac{c-a}{a - c.\ \cos.\ u + c.\ \sin.\ u\ \tan.\ x}$; hence, $\tan.x = \dfrac{b.\overline{c-a}.\ \cos.v - c.\overline{b-a}.\cos.u - a.\overline{c-b}}{b.\overline{c-a}.\sin.\ v - c.\overline{b-a}.\ \sin.\ u}$, which gives the place of the perihelion. Hence, we know $e = \dfrac{c-a}{a.\cos.x - c.\cos.\overline{u+x}}$ the excentricity; consequently $1 - e$ and $1 + e$ the perihelion and aphelion distances are known. This Theorem was first given by E. PROSPERIN, *Astron. Observatore reg.* in the *Nova Acta reg. soc. scien. Upsaliensis*, Vol. III. Mr. ROBINSON afterwards demonstrated it by another method in the *Edin. Trans.* 1788, not knowing that it had been published before. The *Species* of the

ellipse being thus determined, its major axis may be thus found. Compute the mean anomaly corresponding to the angle CSB, then say, as that mean anomaly : $360°$ :: the time of describing the angle CSB : the periodic time. The periodic time being known, the major axis is found (218) by KEPLER'S Rule.

261. Having explained the different methods of finding the place of the aphelion, excentricity and greatest equation; it will be proper to explain the methods of examining at any time these elements in order to apply such corrections as may be found necessary. M. de la LANDE proposes to examine the place of the aphelion by two observations, one near the aphelion and the other near the mean distance, supposing the equation of the center to be known. Calculate for each observation the equation of the center from the supposed place of the aphelion, and take the *difference* of the equations, if the two observations be on the *same* side of the aphelion, but the *sum* if on *different* sides; and the difference or sum of the equations will show how much the true motion differs from the mean, the mean being known from the known interval of the observations. Hence, if the difference *calculated* agree with the difference observed, the place of the aphelion was rightly assumed; but if the true motion by calculation, differ more from the mean motion than the true motion by observation does, the place of the aphelion was too near to or too far from the observation made near the mean distance. Assume therefore another place for the aphelion, as you may judge proper from the circumstances, and trying again, you will soon find the true place. For at the mean distance, the equation being a maximum it alters there but a very little for some time; therefore the whole difference arises principally from the equation at the observation near the aphelion; consequently the alteration of the place of the aphelion will destroy that difference.

262. M. de la LANDE proposes another method of examining the place of the aphelion of the orbits of *Venus* and *Mercury*, by means of their greatest elongations when at their mean distances. Let E be the earth, V the place of the greatest elongation and A the aphelion according to the Tables to be examined; a the true place of the aphelion. Now the planet being near its mean distance, its computed heliocentric longitude will not be sensibly altered by a small alteration of the aphelion, but its distance from the sun will be most altered; we may therefore suppose the observed place to be at v; hence the difference (d) of the observed and computed longitudes is the angle VEv. For any assumed alteration ASa (m) of the aphelion compute the variation Vv of the distance, and thence find the corresponding angle VEv (n), and we have $n : d :: m :$ the alteration of the aphelion from the place A in the Tables in order to make the observed and computed places agree. The aphelion is too backward by the angle ASa, when the perihelion is in inferior conjunction and the computed lon-

gitude is less than the observed, or when the aphelion is in inferior conjunction and the computed longitude is greater than the observed. In all other cases the aphelion is too forward.

On May 24, 1764, at 8*h*. 7′. 50″ true time, M. de la LANDE observed the greatest elongation of Mercury at v, at about 9^s. 8° of anomaly in going from superior to inferior conjunction, to be 22°. 51′. 12″, and its longitude to be 2^s. 26°. 50′. 35″. Now by Dr. HALLEY's Tables, its longitude at V computed at that time was 2^s. 26°. 51′. 49″, which was 1′. 14″ greater than that observed. But in the orbit of Mercury, an angle ASa of 1° answers to an angle VEv of very nearly 5′; hence, to find the angle ASa corresponding to $VEv = 1'. 14''$, say, $5' : 1° :: 1'. 14'' : 14'. 48''$ the angle ASa; therefore the place of the aphelion in Dr. HALLEY's Tables was 14′. 48′ too backward, and the place thus corrected is found to agree with observation.

263. We have now fully explained the different methods of finding the place of the aphelion, excentricity and greatest equation; but as it may appear, by comparing the computations with observations that the elements may not be accurate, M. de la CAILLE has given the following method of correcting them, which will be best understood by an Example; we shall therefore give that published by himself in the *Histoire de l'Académie Royale des Sciences* for the year 1750, upon the elements of the theory of the sun. Let $AIOM$ be the earth's orbit, S the sun, M, I, O* three places of the earth observed on March 29, July 6, and October 3, in the year 1749; A the aphelion, supposed to be in 3^s. 8°. 38′. 51″ on January 1, 1749, and its annual motion 1′. 3″. The sun's mean longitude at the same time was supposed to be 9^s. 10°. 15′. 6″. By observation, M. de la CAILLE found the angle $ISM = 95°. 27'. 7''$, $ISO = 85°. 58'. 34''$, these being the differences of the three true anomalies; and the corresponding mean anomalies were 97°. 34′. 26″, and 87°. 42′. 26″. Now we first make two suppositions for the excentricity, and assume two true anomalies for the point M, and from thence calculate the angle ISM and compare it with the observation. FIG. 57.

* Two of the observations ought to be near the mean distance, and one near the apsides, or two near the aphelion and one near the mean distance, as such observations will add to the accuracy of the conclusion. Two observations near the apsides will best determine the place of the aphelion, and two near the mean distance will give most accurately the equation. The observations may be made at any intervals of time, provided we know the motion of the aphelion, so as to be able to reduce the observations to what they would have been if the orbit had been fixed. The longitudes should also be reduced (268) to the orbit of the planet. The time of the planet's revolution is also supposed to be known, in order to find (239) its mean motion.

	First Hypothesis	*Second Hypothesis*
Excentricity supposed - -	0,01681	0,01685
First assumed true anomaly of *M*	89°. 50′. 0″,0	89°. 50′. 0″,0
Hence, the true anomaly of *I* -	5 . 37 . 7	5 . 37 . 7
Mean anomaly of *M* by calculation	91 . 45 . 34, 6	91 . 45 . 50, 9
Mean anomaly of *I* by calculation	5 . 48 . 34, 5	5 . 48 . 36, 1
Sum of the two mean anomalies ex hyp.	97 . 34 . 9, 1	97 . 34 . 27
Sum of the two mean anomalies from obs.	97 . 34 . 26	97 . 34 . 26
Difference, or error of the hypothesis	− 16, 9	+ 1
Second assumed true anomaly of *M*	89 . 40 . 0, 0	89 . 40 . 0, 0
Hence, the true anomaly of *I* -	5 . 47 . 7	5 . 47 . 7
Mean anomaly of *M* by calculation	91 . 35 . 34, 7	91 . 35 . 50, 9
Mean anomaly of *I* by calculation	5 . 58 . 54, 9	5 . 58 . 56, 6
Sum of the two mean anomalies ex hyp.	97 . 34 . 29, 6	97 . 34 . 47, 5
Sum of the two mean anomalies from obs.	97 . 34 . 26	97 . 34 . 26
Difference, or error of the hypothesis	+ 3, 6	− 21, 5

Hence we have the following proportion for each hypothesis; *As the sum of the errors (or difference when they have the same sign) : the least error :: the difference of the two true anomalies, suppose in M, : the quantity to be applied to the assumed anomaly in M, answering to the least error ;* this quantity is to be added or subtracted according as the sign of the error was − or +. With the assumed anomaly in *M* thus corrected, and the same excentricity, we proceed as before;

	First Hypothesis	*Second Hypothesis*
Corrected anomaly of *M* - -	89°. 41′. 46″,0	89°. 50′. 26″,7
Hence, the anomaly of *I* - - -	5 . 45 . 22,	5 . 36 . 43,
Mean anomaly of *M* by calculation - -	91 . 37 . 19,7	91 . 46 . 14,9
Mean anomaly of *I* by calculation - -	5 . 57 . 6,3	5 . 48 . 11,2
Sum of the two mean anomalies ex hyp.	97 . 34 . 26,	97 . 34 . 26,1
Sum of the two mean anomalies from obs.	97 . 34 . 26,	97 . 34 . 26
Difference, or error of the hypothesis -	0 . 0 . 0	+ 0,1

We have therefore two suppositions of the excentricity which answer to the two observations in *M* and *I*. We must therefore next see how these hypotheses will agree with the observations in *I* and *O*. Assuming therefore the anomalies in *I* as above, we proceed thus :

	First Hypothesis	*Second Hypothesis*
The mean anomaly of I was found - -	5°. 57′. 6″,3	5°. 48′. 11″,2
Mean anomaly answering to the angle ISO	87 . 42 . 26	87 . 42 . 26
Hence, the mean anomaly of O is - -	93 . 39 . 32,3	93 . 30 . 37,2
True anomaly of I - - - - -	5 . 45 . 22,0	5 . 36 . 43,0
The angle ISO by observation - -	85 . 58 . 34	85 . 58 . 34
Hence, the true anomaly of O - -	91 . 43 . 56	91 . 35 . 17
Hence, the mean anomaly of O from obs.	93 . 39 . 24,5	93 . 31 . 2,6
Difference from that which we want to find	− 7,8	+ 25,4

Hence, we have the following proportion; *As the sum of the errors (or difference when they have the same sign) : the least error* (which here belongs to the first hypothesis) :: *the difference between the two supposed excentricities : the quantity to be applied to the first excentricity;* hence, 33″,2 : 7″,8 :: 0,00004 : 0,0000094; now one excentricity giving a result −, and the other +, the true excentricity must be between them; hence, 0,01681 + 0,0000094 = 0,0168194 the excentricity.

Again, *As the sum of the same errors : the least error :: the difference of the two true anomalies, suppose in M, : the quantity to be applied to the true anomaly in M answering to the least difference;* hence, 33″,2 : 7″,8 :: 8′. 39″ :: 2′. 2″, which added to 89°. 41′. 46″ gives 89°. 43′. 48″ the true anomaly of M. But the observed place of M was 8°. 55′. 21″ of longitude; hence, the place of the aphelion on March 29, was 3ˢ. 8°. 39′. 9″. And if we allow 15″ for the motion of the apogee in respect to the equinoctial points from January 1, we shall have the true place of the sun's apogee on January 1, 1749, to be 3ˢ. 8°. 38′. 54″. From the mean of several observations, M. de la CAILLE found the apogee at that time to be 3ˢ. 8°. 39′. 40″, and the excentricity 0,0168293.

264. All the epochs in our Astronomical Tables are reckoned from noon on December 31, in the common years, and from January 1, in the bissextiles. Hence, to find the epoch of the mean longitude, from the place of the aphelion and the true longitude of the planet at the time, you have the distance of the planet from the aphelion, or the true anomaly, from which find the mean anomaly and add it to the place of the aphelion, and you have the mean longitude at that time. Then take the interval from that time to that of the epoch, and find the mean motion corresponding, and add it to the mean longitude, and you have the mean longitude at the epoch. If you know the time when the planet passes the aphelion, you have then only to add to the place of the aphelion the mean motion from that time to the time of the epoch, because at the aphelion the true and mean longitudes are the same.

Ex. On June 29, 1744, at 15*h*. 57′. 46″ the sun was found in its aphelion in 3ˢ. 8°. 31′. 55″. From that time to the last day of December at noon is

184 days 8*h*. 2′. 14″, in which time the mean motion is 6ˢ. 1°. 40′. 21″, which added to 3ˢ. 8°. 31′. 55″ gives 9ˢ. 10°. 13′. 16″ for the mean longitude on December 31, 1744, at noon, as deduced from this one observation. From the mean of several observations, M. de la Caille makes the mean longitude at the beginning of 1749, to be 9ˢ. 10°. 15′. 17″,5.

On February 15, 1743, at 19*h*. 17′. 40″ true time, the mean anomaly of *Mars*, according to M. de la Lande, was 11ˢ. 25°. 6′. 42″, and the place of the aphelion 5ˢ. 1°. 20′. 39″, the sum of which is 4ˢ. 26°. 27′. 21″ the mean longitude of Mars at that time; and the mean motion of Mars from that time to January 1, 1744, (that being leap year) was 15ˢ. 17°. 16′. 53″; hence we find the mean longitude for January 1, 1744, to be 10ˢ. 13°. 45′. 14″. In like manner we find the epochs of the mean longitudes of all the planets.

MEAN LONGITUDE FOR THE MERIDIAN OF PARIS, FOR THE BEGINNING OF 1750.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Sun	9ˢ. 10°. 0′. 35″	9ˢ. 10°. 0′. 13″	9ˢ. 10°. 0′. 35″,5
Mercury	8. 13. 19. 5	8. 13. 7. 45	8. 13. 11. 15
Venus	1. 16. 19. 21	1. 16. 19. 23	1. 16. 20. 48
Mars	0. 21. 58. 43	0: 21. 58. 30	0. 21. 58. 47
Jupiter	0. 4. 0. 59	0. 4. 5. 17	0. 3. 42. 29
Saturn	7. 20. 41. 56	7. 20. 26. 24	7. 21. 20. 22

PLACE OF THE APHELIA FOR THE BEGINNING OF 1750.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	8ˢ. 13°. 41′. 18″	8ˢ. 13°. 27′. 12″	8ˢ. 13°. 33′. 58″
Venus	10. 7. 38. 0	10. 7. 18. 31	10. 7. 46. 42
Earth	3. 8. 27. 23	3. 8. 28. 43	3. 8. 37. 16
Mars	5. 1. 36. 9	5. 1. 31. 38	5. 1. 28. 14
Jupiter	6. 10. 14. 33	6. 10. 33. 46	6. 10. 21. 4
Saturn	8. 29. 13. 31	8. 29. 39. 58	8. 28. 9. 7

EXCENTRICITY OF THE ORBITS,

The mean distance of the Earth from the Sun being 100000.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	8092,5	7970	7955,4
Venus	517	504,985	498
Earth	1690	1691,9	1681,395
Mars	14155	14170	14183,7
Jupiter	25060	25078,6	25013,3
Saturn	54320	54381,4	53640,42
Georgian	* * *	* * *	90804

GREATEST EQUATIONS.

* * *	1740,	1719,	1750,
Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	24°. 2′. 58″	23°. 42′. 36″	23°. 40′. 0″
Venus	0. 49. 6	0. 48. 0	0. 47. 20
Earth	1. 55. 51	1. 56. 20	1. 55. 36,5
Mars	10. 39. 19	10. 40. 2	10. 40. 40
Jupiter	5. 31. 17	5. 31. 36	5. 30. 38,3
Saturn	6. 31. 40	6. 32. 4	6. 26. 42
Georgian	* * *	* * *	5. 27. 16

The place of the aphelion of the *Georgian* Planet in 1788, was 11ˢ. 16°. 19′. 30″, and mean longitude 3ˢ. 14°. 49′. 14″, according to M. de la Lande.

265. The greatest equations, and consequently the excentricities of the orbits, are subject to a variation, arising from the mutual attractions of the planets. M. de la GRANGE, in the *Berlin Acts* for 1782, has calculated the variation of the greatest equations for each, from the attraction of the others, and has found it for 100 years to be as in the following Table.

* * *	Mercury	Venus	Earth	Mars	Jupiter	Saturn
By Mercury	* * *	− 9″,02	− 0″,80	+ 0″,22	* * *	* * *
— Venus	+ 3″,04	* * *	+ 4, 18	+ 0, 22	* * *	* * *
— Earth	+ 0, 58	− 9, 02	* * *	+ 3, 66	* * *	* * *
— Mars	− 0, 22	− 0, 64	− 4, 94	* * *	− 0″,02	* * *
— Jupiter	− 1, 26	− 6, 16	− 16, 02	+ 31, 68	* * *	− 1′. 50″,6
— Saturn	+ 0, 02	− 0, 14	− 0, 08	+ 1, 30	+ 56, 28	* * *
Whole Change	+ 2, 16	− 24, 98	− 17, 66	+ 37, 08	+ 56, 26	− 1. 50,6

In this Table, the quantity of matter in Venus is supposed to be 1,31, that of the Earth being unity; but the density, and consequently the quantity of matter in Venus is subject to some uncertainty. If any other quantity of matter be assumed, the numbers in the horizontal line opposite to Venus will vary in the same ratio. The equation of the *Georgian* Planet is diminished 0″,01 by Jupiter, and 0″,1 by Saturn, according to M. de la GRANGE.

A new Method of correcting the Elements of the Orbit of a Planet.

266. LEMMA. If any quantity z be assumed, and the value of any function of it be computed; then if z be increased by any very small quantity v, the *variation* of the same function will be in proportion to v. This is a proposition well known to Mathematicians*.

267. Given three observed heliocentric longitudes of a planet, the times of observations, and its periodic time; also the place of the aphelion of its orbit, and its excentricity are supposed to be very nearly known; to correct these two elements. Let S be the sun, M, I, O the three given observed places of the planet, A the estimated place of the aphelion, and SC the supposed excen-

* The principle on which the truth of this depends, is this: let A be the result of the first computation. Then for z substitute $z+v$, and compute again the same quantity with this new value of z. Now as v is very small, we may reject all its powers above the first, consequently the second result will be $A \pm mv$, m being some known coefficient; because when $v=o$ the two results must be the same. Hence, the variation mv of the first result is in proportion to v.

tricity. As the intervals of time of the planet's motion from M to I and from I to O are known, and the periodic time is given, the mean anomalies between M and I, I and O will be known (222); call these p and q respectively; and as the points M, I, O, A are given, the angles ASM, ASI, ASO are known, the three true anomalies; compute therefore (232) the three corresponding mean anomalies, and from thence we shall know the mean anomalies between M and I, I and O; call these P and Q. Then if $P=p$, and $Q=q$, the computed agree with the observed places, and consequently the place of the aphelion and the excentricity were rightly assumed. But if P be not equal to p, let it be, for instance, less by m, and let Q be less than q by n. Now increase the place of the aphelion by a very small quantity x, and compute the mean anomalies between M and I, I and O again, and let the corresponding errors be m' and n'. Hence from increasing the place of the aphelion by x, the alteration of the mean anomalies between M and I, I and O will be $m \pm m'$ and $n \pm n'$ respectively, according as the errors are of a different or of the same kind. Increase the excentricity by a very small quantity y, and let the errors of the mean anomalies between M and I, I and O be m'' and n''; then will $m \pm m''$ and $n \pm n''$ be the corresponding alterations of the mean anomalies from the increase y of excentricity. Let x' and y' be the alterations necessary to be made to the first assumed place of the aphelion and the excentricity, in order to correct the errors m and n. Then (266) $x : x' :: m \pm m' : \frac{x' \times \overline{m \pm m'}}{x}$ the change of mean anomaly between M and I from the alteration x'; also $y : y' :: m \pm m'' : \frac{y' \times \overline{m \pm m''}}{y}$ the change which arises from the alteration y'. But we want to increase the mean anomaly between M and I which arises from the first assumed place of the aphelion and the excentricity, by the quantity m; hence we must assume $\frac{x' \times \overline{m \pm m'}}{x} + \frac{y' \times \overline{m \pm m''}}{y} = m$. For the same reason, $x : x' :: n \pm n' : \frac{x' \times \overline{n \pm n'}}{x}$ the change of mean anomaly between I and O from the alteration x'; also $y : y' :: n \pm n'' : \frac{y' \times \overline{n \pm n''}}{y}$ the change arising from the alteration y'. But we want to increase the mean anomaly between I and O from the first assumption, by the quantity n; hence, we must assume $\frac{x' \times \overline{n \pm n'}}{x} + \frac{y' \times \overline{n \pm n''}}{y} = n$. Put $\frac{m \pm m'}{x} = a$, $\frac{m \pm m''}{y} = b$, $\frac{n \pm n'}{x} = c$, $\frac{n \pm n''}{y} = d$, and we have $ax' + by' = m$, $cx' + dy' = n$; hence, $x' = \frac{dm - bn}{da - bc}$ and $y' = \frac{cm - an}{cb - ad}$, the corrections to be made to the first

assumed place of the aphelion and the excentricity in order to make the computed agree with the observed mean anomalies. Thus we correct at once the two elements. If P or Q be greater than p or q, then, as the assumed place of the aphelion and the excentricity give the mean anomalies between M and I, I and O too great by m or n, it is manifest that the alteration which we want to produce, by altering these two elements, is to diminish the computed mean anomalies by m or n; to effect which, we must assume the alterations equal to $-m$ or $-n$. Regard must also be had to the signs of $m \pm m'$, $m \pm m''$, $n \pm n'$, $n \pm n''$, by considering, whether the assumed variations x and y have produced an increase or decrease of the mean anomalies between M and I, I and O, and writing them positive or negative accordingly. The circumstance of any particular case will immediately point out these matters.

To find the Reduction of a Planet to the Ecliptic.

268. Let ♈C be the ecliptic, AB the orbit of a planet, N the ascending node, ♈C the order of the signs, P the place of the planet, and Pm perpendicular to ♈C; then Nm, reckoned from N according to the order of the signs, is called the *argument of latitude*, because the latitude Pm depends upon Nm; hence to get the argument of latitude, we must always subtract the place of the node from the place of the planet reduced to the ecliptic, adding 12 signs to the latter if it be the least. Take $NA = N$♈, and the longitude of a planet upon its orbit is computed from the point A; hence, the longitude on the orbit is $AP = AN + NP$; and the longitude on the ecliptic is ♈$m =$ ♈$N + Nm$ $= AN + Nm$; the difference of these longitudes is the difference between NP and Nm, which difference applied to the longitude of the planet upon the ecliptic, adding it to or subtracting it from, according as Nm is less or greater than NP, that is, as Nm is between 0° and 90° or 180° and 270°, or between 90° and 180° or 270° and 360°, gives the longitude upon its orbit. This difference is called the *Reduction*.

269. To find the reduction, put $c =$ the cosine of the angle PNm, $t =$ the tangent of Nm the argument of latitude; then the cotan. $PN = \frac{\text{rad.} \times c}{t}$; hence, 10, $+\log. c. - \log. t = \log.$ cotan. PN; and the difference between PN and Nm is the reduction required.

Ex. Let the inclination of the orbit of *Mercury* be 7°, and the argument of latitude 30°. 17'. 48"; then

7°. 0'. 0" - - -	cos. + 10, =	19,9967507
30. 17. 48 - - - - -	tan. =	9,7666171
30. 29. 1 - - - -	cot.	10,2301336
0. 11. 13 the Reduction.		

In the Tables of the planet's motions, a Table of reductions is given, which applied to *NP* gives *Nm*; or applied to the longitude of a planet on its orbit gives the longitude upon the ecliptic; but if applied with a contrary sign to the longitude on the ecliptic it gives the longitude on its orbit. In like manner a reduction may be applied to the sun's longitude to find its right ascension or the contrary.

CHAP. XIV.

ON THE MOTION OF THE APHELIA OF THE ORBITS OF THE PLANETS.

Art. 270. **HAVING** explained in the last Chapter the methods of finding the place of the aphelia of the orbits of the planets, we proceed next to determine their motion, arising from their mutual attraction, which is immediately done by comparing the places as settled by the ancient and modern observations; or by comparing the length of an anomalistic with that of a tropical or sidereal revolution.

271. To find the motion of the *Earth's* apogee. HIPPARCHUS, 140 years before J. C. determined its place to be $2^s.\ 5^\circ\frac{1}{2}$; and by the observations of WALTHERUS in 1496, the place was found to be $3^s.\ 3^\circ.\ 57'.\ 57''$; from these observations, the motion of the apogee is $1'.\ 2''\frac{3}{4}$ in a year in respect to the equinoctial points. M. de la CAILLE determined the place of the apogee for the beginning of the year 1749 to be $3^s.\ 8^\circ.\ 39'$; which compared with the observation of WALTHERUS gives $1'.\ 6''$ for the yearly motion. In the year 1588, TYCHO determined the place of the apogee to be $3^s.\ 5^\circ.\ 30'$; and KEPLER in the same year determined its place to be $3^s.\ 5^\circ.\ 32'$. These compared with the observation of CASSINI in the year 1738, who determined its place to be then in $3^s.\ 8^\circ.\ 19'.\ 8''$, give about $1'.\ 7''$ for the annual motion. M. de la CAILLE determined the length of the anomalistic year to be $26'.\ 35''$ longer than the tropical year, which makes the motion of the apogee to be $1'.\ 5'',5$ in a year. KEPLER made it $1'.\ 2''$; RICCIOLUS, $1'.\ 2''.\ 4'''.\ 4''''$ in a year. MAYER in his Tables makes it $1'.\ 6''$. Dr. HALLEY makes it $1'.\ 1''$; and CASSINI about $1'.\ 1'',25$. M. de la LANDE in his Tables makes it $1'.\ 2''$ as computed by M. de LAMBRE from Dr. MASKELYNE'S observations in 1788; and this determination is most to be depended upon, as made by so eminent an Astronomer, from observations which are acknowledged to be the best that have been ever made. These motions are in respect to the equinox. If we assume it to be $1'.\ 2''$, and the precession of the equinoxes to be $50''\frac{1}{4}$, we shall have the *real* motion of the apogee to $11''\frac{3}{4}$ in a year.

272. To determine the motion of the aphelion of *Saturn*. The place of the aphelion in 1694 was $8^s.\ 28^\circ.\ 58'$; but from three oppositions observed in the years 127, 133 and 136, its place for the year 132 was $7^s.\ 24^\circ.\ 14'.\ 29''$, which makes the annual motion $1'.\ 20''$. TYCHO found the place of the aphelion on December 19, 1590, to be $8^s.\ 25^\circ.\ 40'.\ 51''$, which compared with the observation in 132 gives $1'.\ 18'',5$ for the annual motion. The same observation of TYCHO compared with the place of the perihelion on December 12, 1708, in

8^s. 28°. 25'. 10", gives 1'. 23",5 for the annual motion. If the same observation of TYCHO be compared with the place of the aphelion in April 1694 in 8^s. 28°. 58' it gives 1'. 55" for the annual motion. CASSINI conjectured from all this, that the motion of the aphelion was quicker now than formerly. He also found the perihelion in 1708 not so forward by a degree as it ought, when compared with the place of the aphelion in 1694 at the annual movement of 1'. 20"; from whence he suspected that the orbit had a librating motion, and that there ought to be an equation employed between the two points. The irregularities of Saturn, however, as we have before observed, are so great, that we need not wonder at these differences. KEPLER makes it 1'. 16". CASSINI supposes it to be 1'. 18", and Dr. HALLEY 1'. 20". M. de la GRANGE, from calculating the disturbing force of each planet upon the other, has determined the annual motion of the aphelion to be 1'. 6",3. M. de la PLACE makes it 1'. 6",07, which M. de la LANDE has employed in his Tables.

273. To determine the motion of *Jupiter's* aphelion. According to the observations of PTOLEMY, the aphelion was in ♍ 14°. 38' in the year 136; but in 1720 it was in ♎ 9°. 47'; this gives 57". 11‴ for the annual motion. In the year 1590, the place of the aphelion, calculated from the observations of TYCHO, was found to be in ♎ 6°. 30'. 43"; this compared with the observation in 1720, gives 1'. 30" for the annual motion. If we compare the places in 136, and 1590, they give 54" for the annual motion. This induced CASSINI to think, that the motion of the aphelion is accelerated; or that it was subject to some irregularities; he states the motion at 57". 24‴. KEPLER makes it 47". Dr. HALLEY makes it 72". M. JEAURAT computed the place of the aphelion in 1590 to be in ♎ 7°. 49'. 19", and in 1762 in ♎ 10°. 36'. 41"; from which he found the annual motion to be 58",4. EULER, from the theory of attraction, found it to be 55". M. de la GRANGE, 57",2. M. WARGENTIN says, that an annual motion of 62" best agrees with observation. M. de la LANDE has employed 56",73 in his last Tables, according to the theoretical determination of M. de la PLACE.

274. To determine the motion of the aphelion of *Mars*. From three oppositions observed by PTOLEMY, the place of the aphelion in 135 was found to be 3^s. 29°. 24'; and by the observations made at Greenwich in 1691, 1696 and in 1700, the place was found to be in 5^s. 0°. 31'. 34" in 1696; hence the annual motion of the aphelion is 1'. 11". 47‴. 20⁗. KEPLER makes it 1'. 7". Dr. HALLEY makes it 1'. 12". From comparing the place in 1748 in 5^s. 1°. 26'. 10" with the place in 1592 in 4^s. 28°. 49'. 50", the motion is 1'. The mean of these determinations is 1'. 7",5. M. de la LANDE supposes it to be 1'. 7".

275. To determine the motion of the aphelion of *Venus*. CASSINI has found from computing the place of the aphelion from the ancient observations, a difference of 15°, from which uncertainty it is more difficult to determine its annual motion. However, the place, computed from the observations in 136, 138 and

140, (and which he thinks are the most to be depended upon) was found in 138 to be in ♐ 21°. 29′; this compared with the observations in 1715, 1716 and 1718 when it was found to be in ♒ 6°. 50′ in 1716, the annual motion is found to be 1′. 42″. 50‴. From comparing the place in 1596 in ♒ 1°. 54′ with the place in 1716 in ♒ 6°. 50′, the motion is 2′. 28″. Horrox fixed the place of the aphelion in 1639 in ♒ 5°; this compared with the place in 1716, gives 1′. 26″ for the motion. M. de la Lande employed the same method to settle the place of the aphelion of Venus as for Mercury, which we have explained in Art. 262. By comparing the place of the aphelion in his first Tables with the place in Kepler's Tables, the annual motion comes out 2′. 41″,5. Cassini makes it 1′. 26″, and Dr. Halley 56″,5. Kepler makes it 1′. 18″. Amidst so much uncertainty, M. de la Lande thinks it better to depend upon the theory, which, according to M. de la Grange, makes it 48″,5, and which M. de la Lande employs in his Tables. On account of the small excentricity, this uncertainty of the place of the aphelion is not of so much consequence, as an error of 1° in the place of the aphelion will never produce an error of 1′ in the heliocentric longitude.

276. To determine the motion of the aphelion of *Mercury*. From the observations of the passages of Mercury over the sun in 1661, 1690 and 1697, Cassini determined the place of the aphelion on November 9, 1690, to be in 8ˢ. 12°. 22′. 25″; and upon supposition that the motion of the aphelion was 1′. 20″ in a year, he found that it represented the passages very well in 1631, 1672, 1723 and 1736. But as these passages were nearly at the same point of the orbit, it does not sufficiently establish 1′. 20″ to be the true motion, as it might answer to the same points nearly, but not to other parts of the orbit. We ought not therefore to be surprised, says M. de la Lande, that a motion of 52″,5 by Dr. Halley answers equally well to the same observations. Kepler makes it 1′. 45″. M. de la Lande found, by the greatest equation, that on May 6, 1753, the place of the aphelion was 8ˢ. 13°. 55′. From comparing this place with the place computed from 8 observations of Ptolemy, (rejecting 6 others, 2 of which did not appear to be reconcileable with each other, and 4 were too near the aphelion) he found the motion to be 1′. 10″ in a year, which he constructed his first Tables upon ; observing however at the same time, that this motion does not agree perfectly with the observations in this century. He has since found that a motion of 56″,25 will best agree with observation ; and this he has assumed in his last Tables. M. de la Grange makes it 57″ by theory. The motions of the aphelia here determined are their motions in longitude; if therefore we subtract 50″,25 (the annual precession of the equinoxes) from each, we shall get their real motions.

MOTION OF THE APHELIA IN ONE HUNDRED YEARS.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	2°. 13′. 20″	1°. 27′. 37″	1°. 33′. 45″
Venus	2. 23. 20	1. 34. 13	1. 21. 0
Earth	1. 42. 55	1. 41. 7	1. 43. 35
Mars	1. 59. 38	1. 56. 40	1. 51. 40
Jupiter	1. 35. 42	2. 0. 0	1. 34. 33
Saturn	2. 9. 44	2. 13. 20	1. 50. 7

According to the calculation of M. de la Grange, the aphelion of the *Georgian* Planet is progressive 3″,17 in a year, from the action of Jupiter and Saturn; consequently its motion in longitude is 50″,25 + 3″,17 = 53″,42. He has also calculated the effect of each planet in disturbing the aphelia of the rest. The following Table contains the annual effect.

ANNUAL MOTION OF THE APHELIA.

. .	Mercury	Venus	Earth	Mars	Jupiter	Saturn
By Mercury	. . .	− 4″,30	− 0″,42	0″,02	0″,00	0″,00
— Venus	4″,14	. . .	+ 5, 20	0, 70	0, 01	0, 00
— Earth	0, 84	− 5, 06	. . .	1, 92	0, 01	0, 00
— Mars	0, 04	+ 1, 18	+ 1, 54	. . .	0, 00	0, 00
— Jupiter	1, 56	+ 6, 38	+ 6, 79	12, 31	. . .	15, 99
— Saturn	0, 08	+ 0, 08	+ 0, 19	0, 70	6, 56	. . .
Real motion	6, 66	− 1, 72	13, 30	15, 65	6, 58	15, 99
Precession	50, 25	50, 25	50, 25	50, 25	50, 25	50, 25
Mot. in long.	56, 91	8, 53	63, 55	65, 90	56, 83	66, 24

M. de la Grange here supposes, as before, the density of Venus to be 1,31, but M. de la Lande makes it only 0,95; for this density therefore, the second horizontal line must be diminished in the ratio of 1,31 to 0,95.

ON THE MOTION OF THE APHELIA, &C.

Kepler makes the earth's apogee to have coincided with the equinoctial point ♈, on July 24, in the year 3993 before J. C. which, according to some Authors, is about the time of the Creation. At the same time he makes the aphelion of Saturn to be ♌ 24°. 28′. 6″; of Jupiter ♋ 23°. 34′. 18″; of Mars ♉ 15°; of Venus ♎ 0°. 0′. 0″; of Mercury ♋ 0°. 0′. 0″; and the apogee of the Moon ♎ 0°. 0′. 0″.

CHAP. XV.

ON THE NODES AND INCLINATIONS OF THE ORBITS OF THE PLANETS TO THE ECLIPTIC.

Art. 277. FROM observing the course of the planets for one revolution, their orbits are found to be inclined to the ecliptic, for they appear only twice in a revolution to be in the ecliptic; and as it is frequently requisite to reduce their places in the ecliptic, ascertained from observation, to the corresponding places in their orbits, it is necessary to know the inclinations of their orbits to the ecliptic, and the points of the ecliptic where their orbits intersect it, called the *Nodes*. But previous to this, we must show the method of reducing the places of the planets seen from the earth to the places seen from the sun, and how to compute the heliocentric latitudes.

278. Let E be the place of the earth, P the planet, S the sun, ♈ the first point of aries; draw Pv perpendicular to the ecliptic, and produce ES to a. Compute*, at the time of observation, the longitude of the sun seen at a, and you have the longitude of the earth at E, or the angle ♈SE; compute also the longitude of the planet, or the angle ♈ Sv, and the difference of these two angles is the angle ESv of *commutation*. Observe the place of the planet in the ecliptic; and the place of the sun being known, we have the angle vES of elongation in respect to longitude; hence we know the angle SvE, which measures the difference of the places of the planet seen from the earth and the sun; therefore the place of the planet seen from the earth being known, the place seen from the sun will be known. Also, tan. PEv : rad. :: vP : Ev FIG. 59.

rad. : tan. PSv :: vS : vP

∴ tan. PEv : tan. PSv :: vS : Ev :: sin. SEv : sin. ESv; that is, *the sine of elongation in longitude : sin. of the difference of the longitudes of the earth and planet :: tan. of the geocentric latitude : tan. of the heliocentric latitude.* When the latitude is small, vS : Ev very nearly as PS : PE, which, in opposition, is very nearly as PS : $PS-SE$. Or we may compute (223) the values of PS and SE, which we can do with more accuracy than we can compute the angles SEv and ESv. The *curtate* distance Sv of the planet from the sun may be found, by saying, rad. : cos. PSv :: PS : Sv.

279. *First* method, to find the place of the node. The most simple method, when it can be applied, is to observe when the planet has no latitude, and

* The method of making these computations will be shown in the third Volume of this Work.

then reduce (278) the apparent place to the place seen from the sun, and it gives the place of the node.

280. *Second* method. The place of the node may be determined by finding two equal heliocentric latitudes on each side of the node, and the middle point between the longitudes found at the same times, is the place of the node.

281. *Third* method. Find the planet's heliocentric latitudes just before and after it has passed the node, and let a and b be the places in the orbit, m and n the places reduced to the ecliptic; then the triangles amN, bnN (which we may consider as rectilinear) being similar, we have $am+bn : mn :: am : mN$, that is, *the sum of the two latitudes : the difference of the longitudes :: either latitude : the distance of the node from the longitude corresponding to that latitude.* Or if we take the two latitudes seen from the earth, it will be very nearly as accurate when the observations are made in opposition. If the distance of the observations should exceed a degree, this Rule will not be sufficiently accurate, in which case we must make our computations for spherical triangles thus. Put $mn=a$, $bn=\beta$, $am=b, nN=x$; then (Trig. Art. 212) $\frac{\sin.\,\overline{a-x}}{\tan.\,b}=\cot.\,N=\frac{\sin.\,x}{\tan.\,\beta}$; but $\sin.\,\overline{a-x}=\sin.\,a\times\cos.\,x-\sin.\,x\times\cos.\,a$; hence, $\frac{\sin.\,a\times\cos.\,x-\sin.\,x\times\cos.\,a}{\tan.\,b}=\frac{\text{n.}\,x}{\tan.\,\beta}$, $\therefore \frac{\sin.\,a\times\tan.\,\beta}{\tan.\,b+\cos.\,a\times\tan.\,\beta}=\frac{\sin.\,x}{\cos.\,x}=\tan.\,x$. This Rule is given by Mr. Bugge, Professor of Astronomy in the University of Copenhagen. See the *Phil. Trans.* 1787.

282. *Fourth* method. Let P be the pole of the ecliptic EC, am, bn two heliocentric latitudes of the planet, and produce ma, bn to P; then the angle at P is the difference of longitudes; and in the triangle aPb, we know aP, bP and the angle aPb, to find the angle b; therefore in the right angled triangle Nbn, we know bn and the angle b, to find Nn; and as the longitude of n is known, the longitude of the node N will be known.

Ex. To the *third* method. Mr. Bugge observed the right ascension and declination of *Saturn*, and from thence deduced (124, 278) the following heliocentric longitudes and latitudes.

1784, Apparent Time	Heliocentric longitude	Heliocentric latitude
July 12, at 12^h. $3'$. $1''$	9^s. 20°. $37'$. $29''$	0°. $3'$. $13''$N.
20, — 11 . 29. 9	9 . 20 . 51 . 53	0 . 2 . 41
Aug. 1, — 10 . 38. 25	9 . 21 . 13 . 17	0 . 1 . 34
8, — 10 . 9. 0	9 . 21 . 26 . 2	0 . 0 . 56
21, — 9 . 14. 59	9 . 21 . 49 . 27	0 . 0 . 2
27, — 8 . 50. 19	9 . 22 . 0 . 12	0 . 0 . 27S.
31, — 8 . 33. 47	9 . 22 . 7 . 32	0 . 0 . 50
Sept. 5, — 8 . 13. 45	9 . 22 . 16 . 28	0 . 1 . 21
15, — 7 . 33. 45	9 . 22 . 34 . 32	0 . 1 . 59
Oct. 8, — 6 . 4. 23	9 . 23 . 16 . 15	0 . 3 . 35

In computing these heliocentric latitudes and longitudes, Mr. Bugge added the corrections for the perturbations, after the principles of M. Lambert, in the *Memoirs de Berlin*, 1783.

From the observations on August 21 and 27, by considering the triangles as plane, $x=44'',5$; from those on 21 and 31, $x=42'',5$; and from those on August 21, and September 5, $x=40''$; the mean of these is $x=42''$; Mr. Bugge makes $x=41''$, probably by taking the mean of a greater number, or computing from considering them as spherical triangles; hence, the heliocentric place of the descending node was 9^s. 21°. $50'$. $8'',5$. Now on August 21, at $9h$. $12'$. $26''$ true time, *Saturn's* heliocentric longitude was 9^s. 21°. $49'$. $27''$, and on 27, at $8h$. $49'$. $23''$ true time, it was 9^s. 22°. $0'$. $12''$; therefore in five days $23h$. $36'$. $57''$ Saturn moved $10'$. $45''$ in longitude; hence, $10'$. $45''$: $41''$:: $5d$. $23h$. $36'$. $57''$: $9h$. $7'$. $44''$ the time of describing $41''$ in longitude, which therefore added to August 21, $9h$. $12'$. $26''$, gives August 21, $18h$. $20'$. $10''$ the time when Saturn was in its node.

283. To determine the inclination of the orbit, we have bn the latitude of the planet, and nN its distance upon the ecliptic from the node; hence, sin. nN : tan. bn:: rad. : tan. of the angle N. But the observations which are near the node must not be used to determine the inclination, as a very small error in the latitude will make a considerable error in the angle. If we take the observation on July 20, it gives the angle 2°. $38'$. $15''$; if we take that on October 8, it gives the angle 2°. $22'$. $13''$; the mean of these is 2°. $30'$. $14''$ the inclination of the orbit to the ecliptic. To get the inclination accurately, we must, after having settled the place of the node, observe a latitude and longitude at a considerable distance from it. From the observations of Dr. Maskelyne, M. de Lambre found the place of the node on July 12, 1784, to be 3^s. 21°. $48'$. $15''$. FIG. 60.

On December 12, 1704, at $18h$. $50'$ at Paris, *Jupiter* was observed in opposition in 2^s. 21°. $26'$. $22''$ with $28'$. $10''$ south latitude; and on January 14, 1706,

at 16*h*. 2′ it was in opposition in 3ˢ. 24°. 40′. 40″ with 29′. 56″ north latitude seen from the earth. Now at the first and second observations, the distance of Jupiter from the sun was to the distance of the earth as 51144 to 9839, and 52566 to 9840; hence (278), 51144 : 41305 :: 28′. 10″ : 22′. 45″; and 52566 : 42726 :: 29′. 56″ : 24′. 20″ the latitudes seen from the sun at the respective oppositions; also, the difference of the two longitudes was 33°. 14′. 18″; hence (281), 22′. 45″ + 24′. 20″ : 22′. 45″ :: 33°. 14′. 18″ : 16°. 3′. 36″, which added to 2ˢ. 21°. 26′. 22″ gives 3ˢ. 7°. 29′. 58″ the place of the ascending node from these observations, according to M. Cassini. It is difficult to determine accurately the place of Jupiter's node on account of the small inclination of its orbit. M. de Lambre, from observations in 1775, 1776, 1777, 1782 and 1783, found the longitude of the node in 1783, to be 3ˢ. 8°. 14′.

On May 3, 1700, at 12*h*. 24′, Mr. Flamstead found the latitude of *Mars* to be 10′. 9″ north; and on May 10, at 11*h*. 48′ to be 10′. 13″ south. Now as the corresponding longitudes are not given we must proceed thus. The time between the two observations was 6*d*. 23*h*. 24′; hence, 10′. 9″ + 10′. 13″ : 10′. 9″ :: 6*d*. 23*h*. 24′ : 3*d*. 11*h*. 40′, which added to the time of the first observation gives May 7, 0*h*. 4′ for the time when the planet was in its node, at which time, by calculation, its place was in ♏ 17°. 23′. 13″. Now the place of the planet computed at the time of opposition was in ♏ 18°. 5′; consequently the difference 41′. 47″ shows how much the computed place at the time of passing the node wanted of the computed place at the time of opposition, or the difference of the two places at those times; but the observed place in opposition was in ♏ 18°. 6′, from which therefore subtract 41′. 47″ and we have ♏ 17°. 24′. 13″ for the true place of the descending node. In this manner we may always correct a computed place, if we have an observed place near to it. In the *Phil. Trans.* for 1790, Mr. Bugge makes the place of the ascending node to be 1ˢ. 17°. 54′. 24″ for December 7, 1783, which is 10′. 35″ greater than the place by M. Cassini, 23′. 27″ greater than by Dr. Halley, and 2″ less than by M. de la Lande in his last Tables.

On June 11, 1705, at 1*h*. 11′, the latitude of *Venus* was 5′. 35″ north; and on June 12, it was 7′. 35″ south at 1*h*. 5′. By calculation the true places of Venus seen from the sun at those times was ♐ 13°. 22′. 37″, and ♐ 14°. 57′. 32″, the motion of Venus was therefore 1°. 34′. 55″ in this interval; hence, 5′. 35″ + 7′. 35″ : 5′. 35″ :: 1°. 34′. 55″ : 40′. 15″, which added to the place at the first observation gives ♐ 14°. 2′. 52″ for the place of the node. Mr. Bugge in the *Phil. Trans.* 1790, determined the place of the descending node of Venus on August 25, 1786, to be 8ˢ. 14°. 44′. 38″, which is 3′. 53″ less than by M. Cassini, 1′. 59″ greater than by Dr. Halley, and 36″ less than by M. de la Lande in his last Tables.

In like manner, the place of the node of *Mercury* may be determined; but the best method of finding the place of the nodes of Venus and Mercury is from their transits over the sun's disc, as will be explained when we treat on that subject.

LONGITUDES OF THE NODES FOR 1750.

Planets	M. Cassini	Dr. Halley	M. de la Lande
Mercury	$1^s. 15°. 25'. 20''$	$1^s. 15°. 21'. 58''$	$1^s. 15°. 20'. 43''$
Venus	2. 14. 27. 45	2. 14. 23. 42	2. 14. 26. 18
Mars	1. 17. 45. 45	1. 17. 56. 21	1. 17. 38. 38
Jupiter	3. 7. 49. 57	3. 8. 15. 49	3. 7. 55. 32
Saturn	3. 22. 51. 4	3. 21. 20. 5	3. 21. 32. 22

M. de la Place found the place of the node of the *Georgian* Planet in 1788 to be $2^s. 12°. 47'$.

To find the Inclination of the Orbits of the Planets to the Ecliptic.

284. *First* method. The most simple method is to observe the latitude of the planet when it is 90° from its node, and then reduce (278) the latitude seen from the earth to that seen from the sun, and you have the inclination.

285. *Second* method. Observe the latitude and longitude of the planet at FIG.
any other time when it is at some distance from the node, and reduce them 61.
(278) to the latitude and longitude seen from the sun; then the place of the node being known, the distance of the planet in longitude from the node will be known; and in the triangle bnN, we know bn, nN, therefore sin. nN : tan. bn :: rad : sin. of the angle bNn; the further the planet is from the node, the smaller will be the error in the angle, any given error being made in the latitude.

286. *Third* method. Let P be the place of a planet in its orbit, Nn the FIG.
line of the nodes, E the earth in that line; draw Pv perpendicular to the eclip- 62.
tic, and Pr, vr perpendicular to Nn; then (13) the angle Prv is the inclination of the orbit. Now $rv : vP ::$ rad. : tan. Prv

$$vP : vE :: \text{tan. } PEv : \text{rad.}$$

$\therefore rv : vE ::$ tan. PEv : tan. Prv; but $rv : vE ::$ sin. vEr : rad. hence, *sin.* vEr : *rad.* :: *tan.* PEv : *tan.* Prv, that is, *the sine of the difference of the longitudes of the sun and planet seen from the earth : rad. :: tan. of the geocentric latitude : tan. of the inclination.*

Ex. On January 11, 1747, at $18h. 6'. 33''$, M. de la CAILLE observed the longitude of *Saturn* to be $6^s. 26^\circ. 12'. 52''$, and the sun was then in $9^s. 21^\circ. 47'$ in the node of Saturn, or at least within about $12'$ of it; also, the observed latitude was $2^\circ. 29'. 18''$ north; hence by the third method, sin. $85^\circ. 34'. 8''$: rad. :: tan. $2^\circ. 29'. 18''$: tan. $2^\circ. 29'. 45''$ the inclination. CASSINI, from the mean of 7 determinations, makes it $2^\circ. 30'. 33''$. M. de la LANDE from Dr. MASKELYNE's observations in 1775, 1776, 1777, makes it $2^\circ. 30'$; in his Tables he makes it $2^\circ. 29'. 50''$ for 1780. M. de LAMBRE found it $2^\circ. 29'. 55''$ for 1750.

On March 28, 1661, *Jupiter* was, according to HEVELIUS, in ♎ $8^\circ. 58'$ in opposition to the sun, distant only about $1^\circ. 30'$ from its greatest distance from its node, and with $1^\circ. 38'. 25''$ apparent south latitude. Now the distance of Jupiter from the earth was to its distance from the sun as 44537 to 54535; hence, by the first method, 54535 : 44537 :: sin. $1^\circ. 38'. 25''$: sin. $1^\circ. 20'. 23''$ the heliocentric latitude, or the inclination of the orbit; for the distance of $1^\circ. 30'$ from the greatest distance of the node will not cause an error of more than $2''$ in the inclination. From the opposition of Jupiter on April 6, 1768, M. de la LANDE found the inclination to be $1^\circ. 19'. 4''$, Jupiter being then at its greatest latitude; he makes it $1^\circ. 18'. 56''$ for 1780 in his Tables. M. de LAMBRE makes it $1^\circ. 19'. 2''$ for 1750.

On March 27, 1694, at $7h. 4'. 40''$ at Greenwich, Mr. FLAMSTEAD determined the right ascension of *Mars* to be $115^\circ. 48'. 55''$, and its declination $24^\circ. 10'. 50''$ north; hence (124), the geocentric longitude was ♋ $23^\circ. 26'. 12''$, and lat. $2^\circ. 46'. 38''$. Let S be the sun, E the earth, P Mars, v the place reduced to the ecliptic. Now the true place of Mars (by calculation) seen from the sun was ♌ $28^\circ. 44'. 14''$, and the place of the sun was ♈ $7^\circ. 34'. 25''$; hence, subtracting the place of the sun from the place of Mars seen from the earth, we have the angle vES between the sun and Mars $105^\circ. 51'. 47''$; and the place of the earth being ♎ $7^\circ. 34'. 25''$, take from it the place of Mars, and we have the angle $ESv = 38^\circ. 50'. 11''$; hence, (278) sin. $105^\circ. 51'. 47''$: sin. $38^\circ. 50'. 11''$ :: tan. $PEv = 2^\circ. 46'. 38''$: tan. $PSv = 1^\circ. 48'. 36''$. Now the place of the node was in ♉ $17^\circ. 15'$, which subtracted from ♌ $28^\circ. 44'. 14''$ gives $101^\circ. 29'. 14''$ for the distance vN of Mars from its node; hence, sin. $vN = 101^\circ. 29'. 14''$: tan. $Pv = 1^\circ. 48'. 36''$ :: rad. : tan. $PNv = 1^\circ. 50'. 50''$ the inclination of the orbit. Mr. BUGGE makes the inclination to be $1^\circ. 50'. 56'',56$, for March, 1788. M. de la LANDE makes it $1^\circ. 51'$ for 1780.

The inclination of the orbit of *Venus* V, may be very accurately determined, when Venus is about 90° from its node N, and in its inferior conjunction; because at that time it being about three times nearer to the earth than to the sun S, any error in taking the apparent latitude will not cause an error of above one third part thereof in the inclination. Let E be the earth, and draw Vr

perpendicular to the ecliptic. On September 2, 1700, the latitude of Venus, in inferior conjunction, was observed at Paris to be 8°. 40′. 15″ S. and its longitude seen from the sun was 11^s. 10°. 20′. 20″, consequently it was 86°. 22′ from its node. Now at that time, SV was to SE as 72769 to 100750; hence, 72769 : 100750 :: sin. $SEV = 8°. 40'. 15''$: sin. $EVS = 167°. 57'. 7''$; therefore the angle ESV, or Vr, is 3°. 22′. 38″, and as $rN = 86°. 22'$, sin. 86°. 22′ : tan. 3°. 22′. 38″ :: rad. : tan. $VNr = 3°. 23'. 5''$. By a like observation on August 28, 1716, the inclination was found to be 3°. 23′. 10″. Mr. BUGGE makes it 3°. 23′. 38″,6 in 1784. M. de la LANDE, from two observations in 1780 and 1782, makes it 3°. 23′. 35″ for 1780.

On July 16, 1731, at 10h. 32′. 47″ in the morning, M. CASSINI determined the place of *Mercury* seen from the earth to be ♋ 3°. 2′. 35″, with 2°. 2′. 20″ south latitude. Let S be the sun, E the earth, v the place of Mercury at M reduced to the ecliptic, N the node. By calculation, the true place of Mercury seen from the sun was ♓ 25°. 54′. 9″, and the place of the node N was ♉ 15°. 10′, consequently $vN = 49°. 15'. 51''$. Now the sun was in ♋ 23°. 13′. 12″, from which take the apparent place of Mercury ♋ 3°. 2′. 35″, and we have the angle $SEv = 20°. 10'. 37''$. Subtract the place of the earth ♑ 23°. 13′. 12″ from the true place of Mercury ♓ 25°. 54′. 9″, and we have the angle $ESv = 62°. 40'. 57''$; hence, the sine $SEv = 20°. 10'. 37''$: sine $ESv = 62°. 40'. 57''$ :: tan. $vEM = 2°. 2'. 20''$: tan. MSv, or Mv, $= 5°. 15'. 30''$; and sine $Nv = 49°. 15'. 51''$: tan. $Mv = 5°. 15'. 30''$ :: rad. : tan. $vNM = 6°. 51'. 58''$ the inclination. He fixes it at 7°. M. le GENTIL observed Mercury in the meridian on October 5, 1750, and found its apparent longitude 217°. 18′. 19″, with 2°. 50′. 23″ south latitude; also, the place of the sun was 6^s. 12°. 8′. 52″,5, and the angle EvS of commutation 78°. 31′. 23″,5; hence, the heliocentric latitude was 6°. 31′. 23″, and thence the inclination 7°. 1′. Dr. HALLEY makes it 6°. 59′. 20″. M. de la LANDE employs 7° in his Tables. FIG. 64.

INCLINATION OF THE ORBITS.

Planets	KEPLER	Dr. HALLEY	M. CASSINI	M. de la LANDE
Mercury	6°. 54′. 0″	6°. 59′. 20″	7°. 0′. 0″	7°. 0′. 0″
Venus	3. 22. 0	3. 23. 20	3. 23. 20	3. 23. 35
Mars	1. 50. 30	1. 51. 0	1. 50. 54	1. 51. 0
Jupiter	1. 19. 20	1. 19. 10	1. 19. 30	1. 18. 56
Saturn	2. 32. 0	2. 30. 10	2. 30. 36	2. 29. 50

This determination of M. de la LANDE is for the year 1780. He makes the inclination of the orbit of the *Georgian* Planet to be 46′. 20″.

287. But the inclination of the orbits are subject to a variation, arising from their mutual attractions, as we shall afterwards explain. This variation is too small to be determined with sufficient accuracy from observations; but by theory, M. de la GRANGE has found it to be as follows; for Saturn − 23″,11; for Jupiter − 27″,19; for Mars + 3″,45; for Venus + 4″,47; for Mercury + 20″,43; this is the variation in 100 years.

On the Motion of the Nodes.

288. The motion of the nodes is found, by comparing their places at two different times; or it may be determined by theory, as we shall afterwards explain.

PTOLEMY mentions, that in the year 136 *Saturn* was at its greatest north latitude at the beginning of Libra, and consequently the node must have been in the beginning of Capricorn; now in the year 1700 it was in ♑ 21°. 13′. 30″; hence it had advanced 21°. 13′. 30″ in 1564 years, or at the rate of 48″. 51‴ in a year, and 1°. 21′. 26″ in 100 years. But as a variation of several degrees in the place of the node would have but a very small effect on the latitude when near its greatest, the observation of PTOLEMY cannot be depended upon for this purpose. On March 1, 228 before J. C. Saturn was observed, by the Chaldeans, to be about 5′ above the star in the south shoulder of Virgo, marked γ by BAYER; from this M. CASSINI found the place of the node to be 2^s. 21°, which compared with the place in 1720, gives 56″. 26‴ for the yearly motion. BULLIALDUS mentions an occultation of Saturn by the moon in the year 503, from whence he found the place of the node to be 3^s. 12°. 36′. 21″; in the year 1769, M. de la LANDE found the place to be 3^s. 21°. 40′. 47″; this gives 25″. 48‴ for the yearly motion of the nodes. TYCHO-BRAHE observed Saturn very near its node on December 29, 1592, from whence M. CASSINI found the place of the node to be 3^s. 20°. 21′. 5″; this observation compared with the place of the node in 1700, determined to be 3^s. 21°. 13′. 30″, gives 29″. 24‴ for the annual motion. From four observations of M. CASSINI (which M. de la LANDE thinks are most to be depended upon) reduced to the year 1700, the place of the node appears then to have been in 3^s. 21°. 11′. 20″; and comparing this with the place in 1769, the annual motion is 25″,6. M. de LAMBRE makes it 33″,35. M. de la GRANGE makes it 29″, from the theory of attraction. M. de la LANDE makes it 31″,7 in his Tables.

M. CASSINI found the place of the node of *Jupiter* in 1705, to be in 3^s. 7°. 37′. 50″. According to PTOLEMY, the place of the node in his time was in the

beginning of Cancer; this gives 17″ for the annual motion. By an observation on September 26, 508, in which Jupiter was in conjunction with *Regulus*, M. CASSINI computed the motion to be 24″. 37‴ from the same observation. M. le GENTIL calculated the places of the node from the observations of GASSENDI, Dr. HALLEY and himself, to be, in 1633, in 3ˢ. 6°. 4′. 50″; in 1716, in 3ˢ. 7°. 37′. 30″; and in 1753, in 3ˢ. 8°. 21′. 25″. The two last observations give 66″ for the annual motion; the first and last give also 66″; but these motions are too great, as they will not agree with other observations. From the mean of several observations made at Paris between 1692 and 1730, it comes out 34″. M. de LAMBRE makes it 35″,7, which M. de la LANDE has assumed in his Tables. M. de la GRANGE makes it 31″ by theory.

The place of the node of *Mars* on October 28, 1595, was found, from the observations of TYCHO, to be in ♉ 16°. 24′. 33″; and on November 13, 1721, M. CASSINI found it to be in ♉ 17°. 29′. 49″; these give 31″. 4‴ for the annual motion of the nodes. By comparing the same observation of TYCHO with those made at Paris and Greenwich in the year 1700, the former gives 38″. 15‴, and the latter 34″. 16‴. In the year 139, PTOLEMY says the greatest north latitude of Mars was at the end of Cancer, which gives the place of the node at the end of Aries; this compared with the place in 1721 gives 39″. 50‴. M. CASSINI thinks this latter is not much to be depended upon, and therefore takes the mean of the others, which gives 34″. 32‴ for the annual motion. Mr. BUGGE makes it 28″,2. M. de LAMBRE makes it 28″, which M. de la LANDE employs in his Tables. M. de la GRANGE makes it 25″,4 by theory.

The place of the node of *Venus* in its transit over the sun in 1769, was found by M. de la LANDE to be 2ˢ. 14°. 36′. 20″, with a probable error of not more than 30″. Dr. HORNSBY calculated the place of the node in its transit in 1639, from the observations of HORROX, and found it to be 2ˢ. 13°. 27′. 50″, which gives 31″,7 for its annual motion. TIMOCHARES, on October 11, 271 years before J. C. observed η in the south wing of Virgo to be eclipsed by Venus; from this observation, M. CASSINI found the place of the node to be 1ˢ. 24°. 2′; this compared with the place in 1698 in 2ˢ. 14°. 1′. 45″, gives 36″,5. The observations in 1639 and 1698 make it 34″; and as this agrees very nearly with the results from the observations in 1705, 1710 and 1731, M. CASSINI fixed the motion at 34″. M. de la CAILLE, on December 21, 1746, found the place of the node to be 2ˢ. 14°. 23′. 10″; this compared with the place of the node observed by M. de la HIRE on October 31, 1692, gives 38″ for the annual motion. Mr. BUGGE makes it 30″,37. M. de la LANDE makes it 31″, which he uses in his Tables. M. de la GRANGE makes it 30″,55 by theory.

The place of the node of *Merçury* on November 7, 1631, was found, from the observation of GASSENDI, to be in ♉ 13°. 30′. 47″; and on November 11, 1736, it was found to be in ♉ 15°. 14′. 5″; this gives the annual motion 59″. 2‴.

According to the observations of HEVELIUS, the true place of the node on May 3, 1661, was in ♉ 14°. 19′; this compared with the observation in 1736, gives the annual motion 43″. 42‴; the mean of these is 51″. 22‴. This is M. CASSINI's determination. M. le GENTIL, by comparing the place of the node in 1753, in ♉ 15°. 24′. 14″ with the place in 1677 in ♉ 14°. 21′. 3″, found the motion to be 50″, 21. M. de la LANDE, by comparing the places of the node of Mercury found from its transits over the sun, makes it 43″, and these observations are most to be depended upon. He employs this in his Tables. M. de la GRANGE finds it to be 41″, 3 by theory.

289. This motion of the nodes is in respect to the equinox; if therefore we subtract from each 50″,25 the precession of the equinoxes, it will give the motion in respect to the fixed stars, or the real motion. The motion in the following Table is in respect to the equinoxes.

MOTION OF THE NODES IN ONE HUNDRED YEARS.

Planets	M. CASSINI	Dr. HALLEY	M. de la LANDE
Mercury	1°. 24′. 40″	1°. 23′. 20″	1°. 12′. 10″
Venus	0. 56. 40	0. 51. 40	0. 51. 40
Mars	0. 56. 40	1. 3. 20	0. 46. 40
Jupiter	0. 40. 9	1. 23. 20	0. 59. 30
Saturn	1. 35. 11	0. 30. 0	0. 55. 30

The *Georgian* Planet has not been discovered long enough to determine the motion of its nodes from observation. M. de la GRANGE has found the annual motion to be 12″, 5 by theory.

Thus we determine all the elements necessary for computing the place of a planet in its orbit at any time; but to facilitate the operation, which would be extremely tedious if we had only the elements thus given, Astronomers have constructed Tables of their motions, by which their places at any time may be very readily computed. The construction and use of these Tables, we shall explain in the Introduction to the Tables in the third Volume.

CHAP. XVI.

ON THE GEORGIAN PLANET.

Art. 290. ON March 13, 1781, between ten and eleven o'clock in the evening, as Dr. HERSCHEL was examining the small stars near the feet of *Gemini*, he observed one considerably larger than the rest, but it not being quite so brilliant, he suspected that it might be a comet; in consequence of which he observed it with different magnifying powers, from 227 with which he discovered it, to 2010; and found that its apparent magnitude increased in proportion, contrary to what takes place in the fixed stars. He therefore measured its distance from some of the neighbouring fixed stars, and comparing its distance from them for several nights, he found that it moved at the rate of about $2\frac{1}{4}''$ in an hour. On this, DR. HERSCHEL wrote immediately to the Royal Society, that other Astronomers might join in observing it; upon which it was found and observed by Dr. MASKELYNE, who almost immediately declared, that he suspected it to be a Planet; and on April 1, he wrote an account of this discovery to the Astronomers at Paris, so that it was soon observed by all the Astronomers in Europe. Mr. LEXELL was then in England, and applied himself to compute the orbit, upon supposition that it was a comet; he therefore, according to the usual manner in such a case, supposed the orbit to be a parabola, and assumed several perihelion distances 6, 8, 10, 12, 14, 16, and 18 times the earth's distance from the sun; and found that any perihelion distance between 14 and 18 would answer very well to the observations. BOSCOVICH printed a memoir on the subject, in which he showed that there were four different parabolas in which the body might move, and yet the computed places would agree with the observations which had then been made. Other Astronomers however found that a circular orbit, whose radius was about 18 times the distance of the sun from the earth, would agree better with the observations; and this confirmed Dr. MASKELYNE's opinion that it was a planet. Upon supposition therefore of a circular orbit, M. de la LANDE proceeded to investigate its magnitude from the following observations. *Mem. de l'Acad. Roy. des Sci.* 1779.

Time of observation . .	April 25, 1781, at 9*h*. 47′.	July 31, 1781, at 15*h*. 33′.	Dec. 12, 1781, at 10*h*. 10′.
Right ascension observed	2ˢ. 25°. 15′. 27″.	3ˢ. 1°. 7′. 49″	3ˢ. 1°. 23′. 31″
North declination obs.	23. 35. 34	23. 40. 25	23. 42. 47
Longitude	2. 25. 39. 17	3. 1. 2. 7	3. 1. 16. 28
Latitude north	11. 36	12. 24	14. 54
Nutation in longitude	+10	+ 8	+ 7
Aberration in longitude	+19	+21	−18
Sun's longitude from the mean equinox	1. 5. 58. 53	4. 9. 7. 13	8. 21. 21. 50
Log. of the sun's distance	0,003196	0,006272	9,992993

291. From these observations, M. de la LANDE proceeded thus to find the circular orbit. He assumed the radius of the orbit, and then calculated the heliocentric places of the planet at the times of the first and last observation; consequently the angle described by the planet about the sun in that interval of 231 days 23′ was known; and hence the time of the whole revolution was known by proportion, upon supposition that the orbit was circular. Next, knowing the radius of the orbit compared with the mean distance of the earth from the sun, he calculated the periodic time by KEPLER's Rule (218); but as this time did not agree with that before found, he varied his supposition of the distance, until he found they agreed, in which case the radius of the orbit was found to be 18,931 times the mean distance of the earth from the sun, and the duration of the revolution 82,37 years. This circular orbit therefore agreed to the first and last observations; and by computing from it the place at the second observation, he found that it differed only 5″ from the observed place, which difference might easily arise from the unavoidable errors in the observation. He then calculated 32 other observations made by Dr. MASKELYNE, MONNIER, MESSIER, MECHAIN, d'AGELET, LEVESQUE and himself, and found they all agreed very well, except in April 1781, and July, August, and September 1782, the last differing more than two minutes. He then proceeded, as before, to find what radius would answer to the observation on April 25, 1781, and on July 21, 1782, at 15*h*. at Paris, when the longitude observed was 3ˢ. 4°. 42′. 39″; this radius he found to be 18,893, and the periodic time 82,12 years. But by using this radius, he found the calculations to differ 1′. 27″ from

the place observed in opposition in December 1781. This indicated an irregularity in the motion of the planet; but the irregularity was too small, and the observations too near together, to afford proper data for the investigation of the orbit. M. de la LANDE proceeded to determine the place of the node and inclination of the orbit; but on account of the small motion in latitude, great accuracy could not at that time be expected. The geocentric latitudes observed on April 25, and December 12, 1781, were 11′. 36″ and 14′. 54″ north, which give the heliocentric latitudes 11′. 59″ and 14′. 8″; and the motion in longitude being 2°. 46′. 3″ between the observations, he found the place of the node to be 2^s. 12°. 54′, and inclination of the orbit 0°. 46′. Again, the observed geocentric latitudes on April 16, 1781, and March 26, 1782, were 11′. 48″ and 15′. 5″, and hence the heliocentric latitudes were found to be 12′. 7″ and 15′. 10″; and the motion in longitude between the observations being 4°. 7′. 44″, the place of the node was found to be 2^s. 12°. 2′, and the inclination 0°. 44′. He further observes, that the planet was stationary 11 days before Dr. HERSCHEL first observed it, and therefore if his observations had been made 11 days sooner, he would not have perceived any motion, and the discovery might have been lost. It is probable, however, that if this had happened, the discovery would have been made; for from the singularity of its appearance, which alone made Dr. HERSCHEL pay attention to it, he would undoubtedly have continued to observe it, till he had discovered its motion, which must very soon have been perceived.

It having been found that the motion did not agree to that of any one circle, the next enquiry was to determine the ellipse in which it moved, supposing that, like the other planets, it revolves in such a curve, having the sun in one of its foci.

292. The methods of finding the orbit of a planet as described in Chap. XIII. are by three heliocentric places and the times between, or by three distances from the sun and the angles between. The first method may be applied from three observed oppositions; and to apply the other we must have five; but as the latter method is direct, and also so very simple when compared with the former, we shall prefer that, as there are now observations sufficient for it; if we had wanted the elements of the orbit before there had been sufficient data for the latter, we must have used the former method. By this, Mr. ROBISON, Professor of Natural Philosophy in the University of Edinburgh, has investigated the elements of the orbit, in the *Edinb. Trans.* Vol. I. 1788; we shall therefore fully explain the principles and computations as given by him; the method is capable of great accuracy, so far as the observations are accurate, and may be easily understood by those who are well acquainted with only the elementary parts of Mathematics and Philosophy.

The observations upon which the investigation is founded, are as follows:

True Time at Edinburgh.		Longitude.	N. Lat.
Dec. 21, 1781, - - -	$17^h. 44'. 33''$ - - -	$3^s. 0°. 52'. 11''$ - - -	$15'. 7''$
—— 26, 1782, - - -	8. 56. 56 - - -	3. 5. 20. 29 - - -	18. 56
—— 31, 1783, - - -	0. 46. 24 - - -	3. 9. 50. 52 - - -	22. 10
Jan. 3, 1785, - - -	17. 28. 56 - - -	3. 14. 23. 2 - - -	25. 40
—— 8, 1786, - - -	10. 39. 31 - - -	3. 18. 57. 5 - - -	28. 52

293. We have here the times of five successive oppositions, as deduced from observations, and the corresponding heliocentric longitudes and latitudes. Hence the longitude of the node on January 1, 1786, was $2^s. 12°. 48'. 45''$, and inclination of the orbit $46'. 26''$. The place of the node and the inclination of the orbit being determined, the places of the planet reduced (268) to the orbit will be known, and thus we may find the arcs described in the orbit itself between the above oppositions.

294. Mr. Robison next took the opposition on December 31, 1783, for an epoch to which the other observations were to be reduced. The interval between this and the preceding opposition was *369d. 15h.* $49'. 28''$; from this opposition he counted back the same interval of time; and in like manner he counted forwards from the epoch two equal intervals; thus he got four equal intervals of time, to which times he found the places of the planet upon its orbit; and upon comparing their differences, he discovered that they had irregularities not consistent with the motion of a body in an ellipse; these therefore must have arisen from some inaccuracies in the observations; and as, upon account of the small intervals of the places, such errors would be the cause of great errors in the elements of the orbit, it was necessary to correct these inaccuracies, so as to give the differences such a law, as near as possible, that they ought to have.

295. The next consideration was, upon what principle this correction was to be made; and this was, by finding, as nearly as possible, about what part of the ellipse the planet was in at the time of the above observations, and then by observing in similar parts of the ellipses described by the other planets, what law the first and second differences of the angles described in equal times observe. The places of the planet in the ecliptic at five points of time being known, its place at any other point of time may be very accurately found by interpolation. Now on March 6, 1782, at *6h.* $14'. 56''$ mean time (at which time the planet was stationary), its apparent longitude upon the ecliptic was observed to be $2^s. 28°. 49'. 27''$; the heliocentric longitude was also found by interpolation; hence the distance of the planet from the sun came out 18,9053, the earth's distance from

the sun being unity. By interpolating the place of the planet for March, 7*d*. 6*h*. 14′. 56″, it was found to have moved 43″,4365 in 24 hours; but a planet revolving about the sun in a circle whose radius is 18,9053, will have its diurnal motion = 43″,1647. Now the angular velocity of a body in an ellipse is to the angular velocity in a circle at the same distance, in the subduplicate ratio of half the latus rectum to the distance; hence, the planet's distance from the sun was less than half the latus rectum. Also, by a like process for April 1781, it appears, that at that time the angular motion of the planet exceeded, by a very little, the angular motion of a body in a circle at the same distance; therefore its distance from its perihelion could be but a very little less than 90°. We find moreover, that the angular velocity of the planet about the sun was continually accelerated at the time of the above observations, and therefore the planet was approaching its perihelion. Now by examining the tables of the planet's motions in similar situations, it appears that, in equal intervals of time, the first differences decrease very slowly, and the second differences increase very slowly. Mr. Robison therefore gave to the first differences a very small diminution, and to the second differences a very small increase, and this correction was made without altering any of the longitudes more than 3″; for the first observation had its longitude diminished 1″, the second and third increased 2″,5, and the fourth and fifth diminished by 3″, and this must be allowed to be within the limits of probability. The times corresponding to the above mentioned equal intervals, and the corresponding corrected longitudes, cleared from the effects of aberration and nutation, and reduced to the orbit, and the epoch of 1783, are as follows:

True time at Greenwich.	Longitude.
Dec. 21, 1781, 17h. 20′. 17″	3s. 0°. 53′. 50″
—— 26, 1782, 9. 9. 45	3. 5. 21. 16, 5
—— 31, 1783, 0. 59. 13	3. 9. 50. 37, 5
Jan. 3, 1785, 16. 48. 41	3. 14. 21. 52
—— 8, 1786, 8. 38. 9	3. 18. 54. 58

These give the following intercepted arcs, with their first and second differences:

4°. 27′. 26″,5		
	1′. 54″,5	
4. 29. 21		1″
	1. 53, 5	
4. 31. 14, 5		2
	1. 51, 5	
4. 33. 6		

From these data the elliptic orbit of the planet is to be constructed.

296. Let ACP be the orbit, P the perihelion, S the focus, A, B, C, D, E the places of the planet at the five oppositions; and draw the chords and the radii. Now we may conceive the chords AC, CE to be bisected by the radii SB, SD in x and g. For supposing them to be bisected, the triangle $ASx = CSx$, and the triangle $BxC = BxA$, by Euclid B. I. P. 38. And the ellipse being nearly a circle, Sx is nearly perpendicular to CA, and therefore the chords BC, BA, and consequently the two segments, will be very nearly equal; and each being also extremely small compared with the triangles CSB, ASB, the sectors CSB, ASB will be very nearly equal, and hence the times from A to B, and from B to C, may be considered as equal, without any sensible error, and therefore B will be the place of the planet at the second observation. In like manner, D will be the place at the fourth observation.

297. Let the given angles $ASB = u$, $BSC = v$, $CSD = x$, $DSE = y$, $ASC = w$, $CSE = z$; then $AS : Ax :: \sin. AxS : \sin. u$, and Cx, or $Ax : CS :: \sin. v : \sin. CxS$ or AxS; hence, $AS : CS :: \sin. v : \sin. u$; in like manner, $ES : CS :: \sin. x : \sin. y$; thus we know the ratio of AS, CS, ES, and the angles between them, consequently the species and position of the ellipse may (257) be found. The error arising from the supposition of the chords being bisected, is here so extremely small, that it may safely be neglected; however, as Mr. Robison has shown how it may be corrected, we shall explain the method, as it may, upon other occasions, be necessary.

298. Bisect AE in F, AC in H, CE in G, and draw SHb, SFc, SGd, OFk, OGv, O being the center of the ellipse. Since the angles kOv, cSd are very small, the triangles cFk, dGv are nearly similar, and cF, dG being considered as versed sines, they will be very nearly as the squares of the chords; hence the area $cFk : dGv :: cF^2 : dG^2 :: AE^4 : CE^4$. Now by the property of the ellipse, the area $EFk = AkF$, also $EFS = AFS$; hence, $SFkE = AkFS$; add Fkc to both, and $ScE = AkFS + Fkc = ScA + 2Fkc$; therefore $ScE - ScA = 2Fkc$; but as $SCE = SCA$, therefore $ScE - ScA = 2SCc$, consequently $Fkc = SCc$. For the same reason, $dGv = SDd$; but as SD, SC are very nearly equal, $Cc : Dd ::$ area $SCc : SDd :: Fkc : dGv :: AE^4 : CE^4$. And as the arcs AC, CE are very small and nearly equal, therefore $\varphi F = Cc$, and $Gg = dD$ very nearly; also $AE : CE :: 2 : 1$ very nearly; hence, $\varphi F : Gg :: 16 : 1$. For the same reason, $\varphi F : Hx :: 16 : 1$ nearly.

299. Let $ABCDE$ be the true ellipse; take $Se : SC :: \sin. x : \sin. y$, and $Sa : SC :: \sin. v : \sin. u$, and Se, Sa are the values of the first and last radii, as determined in Art. 297. consequently Ee, Aa are the errors to be found.

Now $SC : Cg :: \sin. g : \sin. x$
And $Cg : Eg :: Cg : Eg$
Also $Eg : SE :: \sin. y : \sin. g$

$$\therefore \quad SC : SE :: Cg \times \sin. y : Eg \times \sin. x$$
$$\text{But} \quad Se : SC :: \sin. x : \sin. y$$
$$\therefore \quad Se : SE :: Cg : Eg$$
$$\therefore \quad Ee : SE :: Cg - Eg : Eg :: 2gG : Eg.$$

In like manner, $Aa : SA :: 2xH : Ax ::$ (because the arcs AC, CE are very nearly equal) $2gG : Eg$, and hence, as $SE = SA$ nearly, $Ee = Aa$ nearly.

$$\text{Now } SE : E\varphi :: \sin. \varphi : \sin. z$$
$$\text{And } E\varphi : A\varphi :: E\varphi : A\varphi$$
$$\text{Also } A\varphi : SA :: \sin. w : \sin. \varphi$$
$$\therefore \quad SE : SA :: E\varphi \times \sin. w : A\varphi \times \sin. z$$
$$\text{Assume } SA : So :: \sin. z : \sin. w$$
$$\therefore SE : So :: E\varphi : A\varphi$$
$$\therefore SE : Eo :: E\varphi : A\varphi - E\varphi, \text{ or } 2\varphi F.$$

But as EA is nearly $= 2AC$, $E\varphi = 2Eg$ nearly; also $2\varphi F = 32Gg$;

$$\text{Hence, } SE : Eo :: 2Eg : 32Gg :: Eg : 16Gg$$
$$\text{But } Ee : SE :: 2Gg : Eg$$
$$\therefore Ee : Eo :: 2Gg : 16Gg :: 1 : 8.$$

Make $Sa : S\varepsilon :: \sin. z : \sin. w$, and then $Sa : S\varepsilon :: SA : So$, therefore $Aa : o\varepsilon :: Sq : S\varepsilon$; and as $S\varepsilon = Sa$ nearly, therefore $Aa = o\varepsilon$ nearly; but $Aa = Ee$ nearly, consequently $Ee = o\varepsilon$ nearly; but $Eo = 8Ee$, hence, $e\varepsilon = 6Ee$, and therefore $Ee = \frac{e\varepsilon}{6}$ nearly.

In like manner, find a point α as ε was found, by taking $Se : S\alpha :: \sin. w : \sin. z$, and a point o' as o was found, by taking $SE : So' :: \sin. w : \sin. z$, and by the same reasoning it will appear, that $Aa = \frac{a\alpha}{6}$. Hence we have the following construction to obtain the three radii. Take CS of any value; assume $SC : Sa :: \sin. u : \sin. v$, $SC : Se :: \sin. y : \sin. x$, $Se : S\alpha :: \sin. w : \sin. z$, and $Sa : S\varepsilon :: \sin. z : \sin. w$. Then make $SA = Sa + \frac{a\alpha}{6}$, and $SE = Se - \frac{e\varepsilon}{6}$, and SA, SE will be the other two radii. Hence by Art. 257. the angle $ESP = 2^s. 4^\circ. 14'. 53''$; and the excentricity $= 0{,}9006$, the mean distance of the earth being unity. Also (232) the mean anomalies corresponding to the true anomalies OSA, OSE will be known. Therefore the difference of these two mean anomalies : 360° :: time from A to E : the time of a sidereal revolution; and the square of a sidereal

year : square of this sidereal revolution :: 1 : the cube of the planet's mean distance from the sun. Hence we deduce the following elements.

Mean distance		19,08247			
Excentricity		0,9006			
Periodic time		83,359 years			
Mean anomaly at E		4^s.	0°.	$32'$.	$51''$
Long. of aphelion	for epoch Dec. 31. 1783	11.	23.	9.	51
Long. of the node	for epoch Dec. 31. 1783	2.	12.	46.	14
Inclination of the orbit		0.		46.	25
Equation of the center		5.		26.	56,6

300. These elements, says Mr. Robison, are as accurate as the observations on which they are founded can give them; and agree at present (1788) very well with the observations, the differences being as often as much in defect as in excess; but as the observations were made so near together, it cannot be expected that this agreement will last for a long time. As they may be found to vary from observations, they may be corrected by Art. 267, without computing them over again. The star N°. 964, observed by Mayer in 1756, is not now to be found; and by computing the place of this planet for the time of his observation, Mr. Robison found the planet to be only $3'. 52''$ westward of the star, and $1''$ northward, from which he suspected that it might have been this planet which Mayer observed. It will appear however that this was not the case. It was also conjectured by some Astronomers, that the star N°. 34, *Tauri*, of the British Catalogue, was the new Planet; but Mr. Robison thinks this conjecture by no means to be admitted, as it cannot be made to agree with the elements. Mr. Robison has computed tables of this planet's motion, and observes, that the deviations from observations made near the vernal stations are in defect, whilst those near the autumnal stations are in excess. Hence it may be presumed, that the mean distance and periodic time are somewhat too small, and the aphelion too forward. This he did not perceive till after he had computed his tables, and, he observes, the task was too tedious to make the computations anew. He therefore publishes them, not in the persuasion that they are perfect, but because they are more consistent with observations than those of M. de la Place, and Oriani, the only ones which he had then seen.

The Elements given by M. de la PLACE, *are,*

Mean longitude 1784 - - - - -	3^s. 14°. 43'. 18"
Aphelion - - - - - - -	11. 17. 6. 44
Node - - - - - - - -	2. 12. 46. 47
Equation - - - - - - - -	5. 21. 3,2
Inclination - - - - - - - - -	46. 16
Secular motion of the aphelion - - - -	1. 28. 0
——————— node - - - - - -	26. 10
Mean distance - - - - - - - -	19,18352

301. M. de la LANDE, in the *Histoire de l'Academie Royal des Sciences,* 1787, has corrected these elements, after determining two distances from the sun, the angle, and time between. We shall explain the manner in which he has reduced the Problem to these data. To examine more accurately the motion of this Planet, he settled, from the best observations, the places of those fixed stars with which the Planet had been compared.

302. Let S be the sun, E and F the places of the earth when the Planet was in quadratures at H and K. Now in the quadrature before opposition, the geocentric longitude computed was found to be greater than that by observation, and in the quadrature after opposition, to be less. Draw SGH, SIK, and suppose G and I to be the computed places; then as the difference between the true and computed distances from the sun cannot sensibly vary between the two quadratures, we may suppose $GH=IK$, and consequently the angle HEG, $=KFI$; and as the difference between the true and computed angular velocities will not sensibly vary, we may suppose the true places to be at H and K, when the computed are at G and I. Hence, on the contrary, when the angles HEG, KFI are observed to be equal, the true places will be at H and K, and the computed ones at G and I. Now the distance SG compared with SE being given, and the angle SEG a right angle, if we assume the angle $HEG = 10''$, we shall find $GH=0,017$. At the quadratures at E on November 21, 1788, the error HEG was found to be 23", and the error KFI in the preceding quadrature May 8, was 20"; we will therefore take the mean 21",5 for each error; hence, $10'' : 0,017 :: 21'',5 : 0,03655$ the quantity by which you must augment the computed distance in order to get the true distance. M. de la LANDE makes it 0,04. Now from the position of E and S in respect to G, as the computed geocentric longitude of G was diminished 1",5, the corresponding computed heliocentric longitude will be diminished by about the same quantity; subtract therefore 1",5 from the computed heliocentric longitude, and you will have the true heliocentric longitude. Repeat the same for any other quadrature, and you will get the two distances from the sun, with the angle and time FIG. 66.

CHAP. XVII.

ON THE APPARENT MOTIONS AND PHASES OF THE PLANETS.

Art. 312. AS all the planets revolve about the sun as their center, it is manifest, that to a spectator at the sun they would appear to move in the direction in which they really do move, and shine with full faces. But to a spectator on the earth which is in motion, they will sometimes appear to move in a direction contrary to their real motion, and sometimes appear stationary; and as the same face is not always turned towards the earth as towards the sun, some part of the disc which is towards the earth will not be illuminated. These, with some other appearances and circumstances which are observed to take place among the planets, we shall next proceed to explain; and as these are matters in which great accuracy is never requisite, being of no great practical use, but rather subjects of curiosity, we shall consider the motion of all the planets as performed in circles about the sun in the center, and lying in the plane of the ecliptic.

313. To find the *position* of a planet when stationary. Let S be the sun, E the earth, P the cotemporary position of the planet, XY the sphere of the fixed stars to which we refer the motions of all the planets; let EF, PQ be two indefinitely small arcs described in the same time, and let EP, FQ produced, meet at L; then it is manifest, that whilst the earth was moving from E to F, the planet appeared stationary at L; and on account of the immense distance of the fixed stars, EPL, FQL, may be considered as parallel. Draw SE, SFw, SvP and SQ; then as EP and FQ are parallel, the angle $QFS-PES=PwS-PES=ESF$, and $SPw-SQF=SvF-SQF=PSQ$, that is, the cotemporary variations of the angles E and P are as $ESF : PSQ$, or (because the angular velocities are inversely as the periodic times, or inversely in the sesquiplicate ratio of the distances) as $SP^{\frac{3}{2}} : SE^{\frac{3}{2}}$, or as $a^{\frac{3}{2}} : 1^{\frac{3}{2}}$. But the sines of the angles E and P being in the constant ratio of $a : 1$, the cotemporary variations of these angles will (as is well known) be as their tangents. Hence, if x and y be the sines of the angles E and P, we have $x : y :: a : 1$, and $\frac{x}{\sqrt{1-x^2}} : \frac{y}{\sqrt{1-y^2}}$ $:: a^{\frac{3}{2}} : 1$, whence $x^2=\frac{a^3-a^2}{a^3-1}=\frac{a^2}{a^2+a+1}$, and $x=\frac{a}{\sqrt{a^2+a+1}}$ the sine of the planet's elongation from the sun, when stationary.

Ex. If P be the earth, and E Venus; and we take the mean distances of the earth and Venus to be 100000 and 72333, we find $x=0{,}48264$ the sine of

28°. 51'. 5", the elongation of Venus when stationary, upon the supposition of circular orbits.

For excentric orbits, the points will depend upon the position of the apsides and place of the bodies at the time. We may however get a very near approximation thus. Find the time when the planet would be stationary if the orbits were circular, and compute for several days, about that time, the geocentric place of the planet, so that you get two days, on one of which the planet was direct and on the other retrograde, in which interval it must have been stationary, and the point of time when this happened may be determined by interpolation. The arc of retrogradation must manifestly be different in different parts of the orbit. M. de la LANDE has given us the following circumstances respecting the stationary situations, and retrograde motions of the planets. The *first* stationary, means the stationary position after the planet has been direct; and the *second* stationary, after it has been retrograde. The titles above show the places of the planet and the earth in their orbits when the planet is *first* stationary; all other elongations at the time they are stationary, arcs and durations of retrogradation, must necessarily be contained within these limits. If the time of retrogradation be subtracted from the time of a *synodic* revolution, the remainder gives the time in which the motion of the planet has been *direct*.

MERCURY.

	☿ in perihelion ⊖ in aphelion	☿ in aphelion ⊖ in perihelion
Elongation at the first stationary	15°. 23'. 34"	18°. 39'. 23"
——— second stationary	20. 50. 55	14. 48. 39
Arc of retrogradation	15. 43. 58	9. 21. 56
Duration of retrogradation	21 *days* 12*h*.	23 *days* 12*h*.

VENUS.

	♀ in perihelion ⊖ in aphelion	♀ in aphelion ⊖ in perihelion
Elongation at the first stationary	29°. 6'. 42"	28°. 28'. 0"
——— second stationary	29. 40. 42	27. 41. 0
Arc of retrogradation	17. 12. 15	14. 35. 58
Duration of retrogradation	43 *days* 12*h*.	40 *days* 21*h*

MARS.

	♂ in perihelion ⊖ in aphelion	♂ in aphelion ⊖ in perihelion
Elongation at the first stationary	$4^{s}.\ 25^{\circ}.\ 3'.\ 9''$	$4^{s}.\ 10^{\circ}.\ 18'.\ 59''$
——— second station.	4. 26. 36. 51	4. 8. 44. 20
Arc of retrogradation	0. 10. 6. 11	0. 19. 34. 38
Duration of retrogradation	60 *days* 18 *hours*	80 *days* 15 *hours*

JUPITER.

	♃ in perihelion ⊖ in aphelion	♃ in aphelion ⊖ in perihelion
Elongation at the first stationary	$4^{s}.\ 0^{\circ}.\ 7'.\ 47''$	$3^{s}.\ 24^{\circ}.\ 2'.\ 35$
——— second station.	3. 26. 41. 49	3. 23. 35. 18
Arc of retrogradation	0. 9. 51. 30	0. 9. 59. 23
Duration of retrogradation	116 *days* 18 *hours*	122 *days* 12 *hours*

SATURN.

	♄ in perihelion ⊖ in aphelion	♄ in aphelion ⊖ in perihelion
Elongation at the first stationary	$3^{s}.\ 20^{\circ}.\ 19'.\ 38''$	$3^{s}.\ 17^{\circ}.\ 51'.\ 5''$
——— second station.	3. 20. 45. 50	3. 17. 24. 48
Arc of retrogradation	0. 6. 55. 44	0. 6. 40. 39
Duration of retrogradation	135 *days* 9 *hours*	138 *days* 18 *hours*

GEORGIAN.

	♅ in perihelion ⊖ in aphelion	♅ in aphelion ⊖ in perihelion
Elongation at the first stationary	$3^{s}.\ 12^{\circ}.\ 23'$	$3^{s}.\ 13^{\circ}.\ 33'$
——— second station.	3. 15. 5	3. 13. 47
Arc of retrogradation	0. 4. 13	0. 4. 3
Duration of retrogradation	151*d*. 12*h*.	149*d*. 18*h*.

314. To find the *time* when a planet is stationary, we must know the time of its opposition, or inferior conjunction. Let m and n be the daily motions of the earth and planet, and v the angle PSE when the planet is stationary; then $m-n$, or $n-m$, is the daily variation of the angle at the sun between the earth and planet, according as it is a superior or inferior planet; hence $m-n$, or $n-m$, : v :: 1 day : $\frac{v}{m-n}$ or $\frac{v}{n-m}$ the time from opposition or conjunction to the stationary points both before and after. Hence, the planet must be stationary twice every *synodic** revolution.

Ex. Let P be the earth, E Venus; then by the Example to Art. 313. the angle $SPE=28°.51'.5$, therefore $PSE=13°$; also $n-m=37'$; hence $37'$: $13°$ :: 1 day : 21 days the time between the inferior conjunction and the stationary positions.

315. If the elongation be observed when stationary, we may find the distance of the planet from the sun, compared with the earth's distance, supposed to be unity. For (313) $x^2=\frac{a^2}{a^2+a+1}$; hence, $a^2+\frac{x^2}{x^2-1}\times a=-\frac{x^2}{x^2-1}$ =(if t = the tangent of the angle whose sine is x) $a^2-t^2a=t^2$; consequently $a=\frac{1}{2}t^2+t\sqrt{1+\frac{t^2}{4}}$, upon the supposition of circular orbits.

316. A superior planet is retrograde in opposition, and an inferior planet in its inferior conjunction. For let E be the earth, P a superior planet in opposition; then as the velocities are in the inverse square roots of the radii of the orbits, the superior planet moves slowest; hence, if EF, PQ be two indefinitely small cotemporary arcs, PQ is less than EF, and on account of the immense distance of the sphere YZ of the fixed stars, FQ must cut EP in some point x between P and m, consequently the planet has appeared to move retrograde from m to n. If P be the earth, and E an inferior planet in inferior conjunction, it will have appeared to have moved retrograde from v to w. Hence, from this and the last Article, a superior planet appears to move retrograde from its stationary point before opposition to its stationary point after; and an inferior planet, from its stationary point before inferior conjunction to its stationary point after. FIG. 69.

317. If S be the sun, E the earth, V Venus or Mercury, and EV a tangent to the orbit of the planet, then will the angle SEV be the greatest elongation of the planet from the sun; which angle, if the orbits were circles having the sun FIG. 70.

* A *Synodic* revolution is the time between two conjunctions or oppositions of a planet.

in their center, would be found by saying, $ES : SV$:: rad. : sin. SEV. But the orbits are not circular, in consequence of which the angle EVS will not be a right angle, unless the greatest elongation happens when the planet is at one of its apsides. The angle SEV is also subject to an alteration from the variation of SE and SV. The *greatest* angle SEV happens, when the planet is in its *aphelion* and the earth in its *perigee;* and the *least* angle SEV, when the planet is in its *perihelion* and the earth in its *apogee.* M. de la LANDE has calculated these greatest elongations, and finds them 47°. 48′ and 44°. 57′ for *Venus*, and 28°. 20′ and 17°. 36′ for *Mercury*. If we take the mean of the greatest elongations of Venus, which is 46°. 22′,5, it gives the angle VSE=43°. 37′,5; and as the difference of the daily *mean* motions of Venus and the earth about the sun is 37′, we have 37′ : 43°. 37′,5 : 1 day . 70,7 days, the time that would elapse between the greatest elongations and the inferior conjunction, if the motions had been uniform, which will not vary much from the true time.

Dr. MASKELYNE gives the following rule for finding the time of the greatest elongation of an inferior planet. Take the difference of the sun's and that of the planet's longitude for every three days, about the time of the greatest elongation, and note on which day (the 25th in this example for Mercury) the elongation is the greatest (21°. 56′). Then as the elongation was greater on the 28th than on the 22nd, the 28th was nearer the greatest elongation than the 22nd. The greatest elongation, therefore, was after the 25th, and call the time (the decimal of a day) h; and the greatest elongation, 21°. 56′$+x$. Hence, on the 22nd, the distance of the time to the greatest elongation, was $3+h$; and the difference from the greatest elongation, was 21°. 56′$+x-$21°. 31′$=$25′$+x$.

June.	☉'s Long.	Mer. Long.	Elong.
16	2ˢ. 24°. 22′	2ˢ. 5°. 41′	18°. 41′
19	2. 27. 14	2. 6. 48	20. 26
22	3. 0. 6	2. 8. 35	21. 31
25	3. 2. 58	2. 11. 2	21. 56
28	3. 5. 49	2. 14. 6	21. 43

On the 28th, the distance of time from the greatest elongation, was 21°. 56′$+x$ $-$21°. 43′$=$13′$+x$. Therefore, on the 22d, 25th, 28th, the intervals from the times of the greatest elongation, and the excesses of the greatest elongation above the computed elongations, were $3+h$, h, $3-h$ and 25′$+x$, x, 13′$+x$

respectively; but as, for small quantities, the spaces vary very nearly as the squares of the times, $9+6h+h^2 : h^2 :: 25+x : x$, and $h^2 : 9-6h+h^2 :: x : 13'+x$; hence, $h=\frac{108}{228}=\frac{9}{19}$ of a day, the time after the 25th for the greatest elongation $=11\frac{7}{19}$ hours; and $x=13'\times\frac{h^2}{9-6h}=28''$, and the greatest elongation is $21°.\ 56'.\ 28''$.

318. To delineate the appearance of a planet at any time. Let S be the sun, E the earth, V Venus, for example, aVb the plane of illumination perpendicular to SV, cVd the plane of vision perpendicular to EV, and draw av perpendicular to cd; then ca is the breadth of the visible illuminated part, which is projected into cv, the versed sine of cVa, or SVZ, for SVc is the complement of each. Now the circle terminating the illuminated part of the planet, being seen obliquely, appears to be an ellipse; therefore if $cmdn$ represent the projected hemisphere of Venus next to the earth, mn, cd, two diameters perpendicular to each other, and we take cv=the versed sine of SVZ, and describe the ellipse mvn, then $mcnvm$ will represent the visible enlightened part, as it appears at the earth; and from the property of the ellipse, this area varies as cv. Hence, *the visible enlightened part* : *the whole disc* :: *the versed sine of* SVZ : *diameter*. FIG. 71.

Hence, *Mercury* and *Venus* will have the same phases from their inferior to their superior conjunction, as the moon has from the new to the full; and the same from the superior to the inferior conjunction, as the moon has from the full to the new. *Mars* will appear gibbous in quadratures, as the angle SVZ will then differ considerably from two right angles, and consequently the versed sine from the diameter. For *Jupiter*, *Saturn* and the *Georgian*, the angle SVZ never differs enough from two right angles to make them appear gibbous, so that they always appear to shine with a full face.

319. Let V be the moon; then as EV is very small compared with VS, ES, these lines will be very nearly parallel, and the angle SVZ very nearly equal to SEV; hence, *the visible enlightened part of the moon varies very nearly as the versed sine of its elongation.*

320. Dr. HALLEY proposed the following Problem: To find the position of *Venus* when brightest, supposing its orbit, and that of the earth, to be circular, having the sun in the center. Draw Sr perpendicular to EVZ, and put $a=SE$, $b=SV$, $x=EV$, $y=Vr$; then $b-y$ is the versed sine of the angle SVZ; and as the intensity of light varies inversely as the square of its distance, the quantity of light received at the earth varies as $\frac{b-y}{x^2}=\frac{b}{x^2}-\frac{y}{x^2}$; but by Euclid, B. II. P. 12. $a^2=b^2+x^2+2xy$; hence, $y=\frac{a^2-b^2-x^2}{2x}$; substitute this for y, and

we get the quantity of light to be as $\frac{b}{x^2} - \frac{a^2 - b^2 - x^2}{2x^3} = \frac{2bx - a^2 + b^2 + x^2}{2x^3}$ = a maximum; put the fluxion equal to nothing, and $x = \sqrt{3a^2 + b^2} - 2b$. Now if $a = 1$, $b = ,72333$ as in Dr. HALLEY's Tables, then $x = ,43036$; hence, the angle $ESV = 22°. 21'$; but the angle ESV at the time of the planet's greatest elongation is $43°. 40'$; hence, Venus is brightest between its inferior conjunction and its greatest elongation; also, the angle $SEV = 39°. 44'$ the elongation of Venus from the sun at the same time. The angle $SVZ = VSE + VES = 62°. 5'$, the versed sine of which is 0,53, radius being unity; hence (318), the visible enlightened part : whole disc :: 0,53 : 2; Venus therefore appears a little more than one fourth illuminated, and answers to the appearance of the moon when five days old. The diameter of Venus is about 39″, and therefore the enlightened part is about 10″,25. At this time, Venus is bright enough to cast a shadow at night. This situation happens about 36 days before and after its inferior conjunction; for the daily variation of the angle ESV is the *difference* of the daily motions of the earth and Venus about the sun, which (taking their mean motions) is 37′; an angle ESV therefore of $22°. 21'$ corresponds to about 36 days. It passes the meridian about 2*h*. 31′ before or after the sun, according as we take the situation after or before the inferior conjunction. If instead of supposing Venus and the earth at their mean distances, we suppose Venus in its perihelion and the earth in its apogee, the elongation of Venus when brightest would be 39°. 6′; and if Venus were in its aphelion and the earth in its perigee, it would be 40°. 20′. *Memoirs de Berlin*, 1750.

321. If we apply this to Mercury, $b = ,3171$, and $x = 1,00058$; hence, the angle $ESV = 78°. 55'\frac{3}{4}$; but the same angle at the time of the planet's greatest elongation is $67°. 13'\frac{1}{4}$. Hence, Mercury is brightest between its greatest elongation and superior conjunction. Also, the angle $SEV = 22°. 18'\frac{1}{2}$ the elongation of Mercury at that time.

322. When Venus is brightest, and at the same time is at its greatest north latitude, it can then be seen with the naked eye at any time of the day; for when its north latitude is the greatest, it rises highest above the horizon, and therefore is more easily seen. This happens (325) once in about eight years, Venus and the earth returning nearly to the same parts of their orbits after that interval of time.

323. *Venus* is a *morning* star from inferior to superior conjunction, and an *evening* star from superior to inferior conjunction. For let S be the sun, E the earth, $ACBD$ the orbit of Venus, *arm*, *csn*, two tangents to the earth, representing the horizon at each place. Then the earth revolving about its axis according to the order *abc*, when a spectator is at *a*, the part *rCm* of the orbit of Venus is above the horizon, but the sun is not yet risen; therefore Venus, in going from *r* through *C* to *m*, appears in the morning before sun rise. When

the spectator is carried by the earth's rotation to c, the sun is then set, but the part nDs of Venus' orbit is still above the horizon; therefore Venus, in going from n through D to s, appears in the evening after sun set.

324. If two planets revolve in circular orbits, to find the time from conjunction to conjunction. Let $P=$ the periodic time of the earth, $p=$ that of the planet, suppose an inferior, $t=$ the time required. Then $P : 1 \text{ day} :: 360^\circ : \frac{360^\circ}{P}$ the angle described by the earth in 1 day; for the same reason, $\frac{360^\circ}{p}$ is the angle described by the planet in 1 day; hence, $\frac{360^\circ}{p}-\frac{360^\circ}{P}$ is the daily angular velocity of the planet *from* the earth. Now if they set out from conjunction, they will return into conjunction again after the planet has gained 360°; hence, $\frac{360^\circ}{p}-\frac{360^\circ}{P} : 360^\circ :: 1 \text{ day} : t=\frac{Pp}{P-p}$. For a superior planet, $t=\frac{pP}{p-P}$. This will also give the time between two oppositions, or between any two similar situations.

325. To find the time when a planet and the earth return to the same point of the Heavens. Find, from a Table of their mean motions, a number of years agreeing to a complete number of revolutions of the planet. Now *Mercury* in 13 years, (of which three are bissextiles) and three days, make 54 revolutions and 2°. 55′ over; and the earth has made 13 revolutions and 2°. 49′ over. In this time therefore the earth and Mercury return to the same situation in the heavens, very nearly. It will be 13 years and two days, if there be four bissextiles. *Venus*, after a space of eight years, is found within 1°. 32′ of the same place, and the earth within 4′. *Mars*, in 15 years wanting 18 days, has changed its place 11ˢ. 11°. 26′, and the earth 11ˢ. 11°. 38′; if there have been four bissextiles, it will be 15 years wanting 19 days. But in 79 years and 4 days, supposing there are 20 bissextiles, Mars returns to the same situation within 3°. 39′, and the earth within 3°. 48′. *Jupiter* in 83 years returns to the same point within 12′, and the earth within 6′. The period of 12 years 5 days approaches very near, for Jupiter has in that time made 4°. 47′ above one revolution, and the earth 5°. 1′ above 12 revolutions. *Saturn* in 59 years and two days returns to the same situation within 1°. 45′, and the earth within 1°. 41′. M. de la LANDE, who has given these returns of the planets and earth to the same point of the Heavens, has also added the following GRAND CONJUNCTIONS.

On May 22, 1702, *Jupiter* and *Saturn* were within 1°. 4′ of each other. *Miscel. Berolin.* p. 217.

On February 11, 1524, *Venus*, *Mars*, *Jupiter* and *Saturn* were very near

each other, and *Mercury* not above 16° from them, according to the Ephemeris of STOFFLER.

On November 11, 1544, *Mercury*, *Venus*, *Jupiter* and *Saturn* were within the space of 10°.

On March 17, 1725, *Mercury*, *Venus*, *Mars* and *Jupiter* appeared within the same telescope. SOUCIET, *Obs. Mathem.* T. 1. p. 103.

On December 23, 1769, *Venus*, *Mars* and *Jupiter* were within 1° of each other.

CHAP. XVIII.

ON THE MOON'S MOTION FROM OBSERVATION, AND ITS PHÆNOMENA.

Art. 326. THE moon being the nearest, and most remarkable body in our system next to the sun, and also useful for the division of time, it is no wonder that the ancient Astronomers were attentive to discover its motions; and it is a very fortunate circumstance, that their observations have come down to us, as from thence its mean motion can be more accurately settled, than it could have been by modern observations only; and it moreover gave occasion to Dr. HALLEY, from the observations of some ancient eclipses, to discover an acceleration in its mean motion. The proper motion of the moon in its orbit about the earth is from west to east; and from comparing its place with the fixed stars in one revolution, it is found to describe an orbit inclined to the ecliptic; its motion also appears not to be uniform; and the position of the orbit, and the line of its apsides are observed to be subject to a continual change. These circumstances, as they are established by observation, we come now to explain; the physical causes thereof will afterwards become the subject of our consideration.

To determine the Place of the Moon's Nodes.

327. *First Method.* Let AE be the ecliptic, A the first point of Aries, OL the moon's orbit, N the node, m the place of the moon in its orbit when it passes the meridian on the day before it comes to the ecliptic, n the place when it passes the day after, and draw mv, nw perpendicular to EA. Find (124) its latitudes mv, nw on these two days, and its longitudes Av, Aw; then $mv + nw : mv :: vw :: vN$, which added to Av gives the longitude of the node. To find the time when the moon is in the node, we have $vw : vN ::$ the interval of time between the passages of the moon over the meridian : the interval from the time of the first passage over the meridian till it comes to the node; this interval therefore added to the time of that passage, gives the time of the passage through the node. FIG. 73.

328. *Second Method.* In a *central* eclipse of the moon, the moon's place at the middle of the eclipse is directly opposite to the sun, and the moon must also then be in the node; calculate therefore the true place of the sun, or which is more exact, find its place by observation, and the opposite point will be the true place of the moon, and consequently the place of its node.

Ex. M. CASSINI, in his Astronomy, pag. 281, informs us, that on April 6, 1707, a central eclipse was observed at Paris, the middle of which was determined to be at $13h.\ 48'$ apparent time. Now the true place of the sun calculated for that time was $0^s.\ 26^\circ.\ 19'.\ 17''$; hence, the place of the moon's node was $6^s.\ 26^\circ.\ 19'.\ 17''$. The moon passed from north to south latitude, and therefore this was the descending node.

329. *Third Method.* To find the place of the node by a partial eclipse. Find, by observation, the magnitude AB of the eclipse at the middle, and subtract it from the semidiameter AD of the earth's shadow, and we have DB, to which add BC the semidiameter of the moon, and we have CD. Now at the time of a lunar eclipse, we may suppose the angle $CND=5^\circ.\ 17'$, from which it will never differ but a very little. Hence, in the right angled triangle DCN, right angled at C, we have DC and the angle DNC, to find DN, and as the point D is opposite to the true place of the sun, which is known by computation, the place N of the node will be known.

Ex. On March 26, 1717, the middle of an eclipse was observed at Paris at $15h.\ 16'$, and the digits eclipsed were $7\frac{17}{60}$ towards the north. Now the semidiameter of the moon was $15'.\ 46''$, and that of the shadow $42'.\ 43''$; hence, 12 dig. : $7\frac{17}{60}$ dig. :: $31'.\ 32''$ the diameter of the moon : $19'.\ 8''=AB$; therefore $BD=23'.\ 35''$, to which add $BC=15'.\ 46''$, and we have $CD=39'.\ 21''$, which is south, because the shadow upon the moon is towards the north. Hence, in the right angled triangle DCN, we have $CD=39'.\ 21''$, and the angle $N=5^\circ.\ 17'$, consequently $DN=7^\circ.\ 8'.\ 26''$, which is the distance of the center of the earth's shadow from the ascending node, because the shadow of the earth is on the north side of the moon and the latitude is decreasing. Now the true place of the sun at that time was $0^s.\ 6^\circ.\ 20'.\ 43''$, and therefore the true place of the center D of the earth's shadow was $6^s.\ 6^\circ.\ 20'.\ 43''$, to which add $DN=7^\circ.\ 8'.\ 26''$ and we get the true place of the ascending node of the moon to be in $6^s.\ 13^\circ.\ 29'.\ 9''$. M. de la LANDE makes the epoch of the ascending node for 1780, to be $2^s.\ 0^\circ.\ 3'.\ 2''$.

On the Mean Motion of the Nodes.

330. To determine the mean motion of the nodes, find (327) the place of the nodes at different times, and it will give their motion in the interval. We must first compare the places at a small interval, to get nearly their mean motion, and then at a greater interval to get it more accurately. Now on April 16, 1707, at $13h.\ 48'$ at Paris, the ascending node was in $0^s.\ 26^\circ.\ 19'$; and on March 26, 1717, at $15h.\ 16'$, the place of the same node was in $6^s.\ 13^\circ.\ 29'$; also by an eclipse observed at the same place on September 9, 1718, at $8h.\ 4'$,

the place of the ascending node was in 5^s. 16°. 40′. From the two last observations it appears that the node is retrograde. Now the interval of these two observations was *531d. 0h.* 16′. 48″, during which time the nodes moved retrograde through 26°. 49′, which gives the diurnal motion 3′. 2″. If we compare the first and last observations, they give the daily motion 3′. 10″.

331. But to determine the mean motion of the nodes with greater accuracy, we must compare together more distant observations. PTOLEMY, in his *Almagest*, mentions three lunar eclipses, that were observed at Babylon by the Chaldeans. The first was total on March 19, 720 years before J. C. the beginning was at *7h.* 30′ in the evening, and the middle was at *9h.* 30′. The second was on March 8, 719 years before J. C. the middle of which happened at midnight, and the greatest quantity eclipsed was 3 digits towards the south. The third happened on September 1, 719 years before J. C. the middle of which was at *8h.* 30′ in the evening, and the moon was eclipsed a very little more than one half towards the north. Now it being uncertain whether the first eclipse was central, M. CASSINI takes the second; and the difference of the meridians of Babylon and Paris being *2h.* 42′, it gives the middle of the eclipse at Paris *9h.* 18′ in the evening. And, by computation (329), we find the center of the earth's shadow to be 8°. 24′. 50″ from the node. The middle of the third eclipse happened at Paris at *5h.* 48′, and M. CASSINI takes the digits eclipsed to be $6\frac{1}{4}$, and computes (329) the distance of the center of the shadow from the node to be 8°. 15′. 28″. But we cannot tell from either of these observations, whether the latitude of the moon was ascending or descending, and therefore we do not know at which node the eclipses happened. To determine this, take the total eclipse on March 19, the middle of which was at *9h.* 30′ at Babylon, or *6h.* 48′ at Paris, at which time the sun's place, by computation, was 11^s. 21°. 27′, therefore the moon's place, was 5^s. 21°. 27′. Between this time and the eclipse on September 1, there was very nearly 18 months, in which time the nodes had moved retrograde about 29°, which subtracted from the place of the moon in the observation on March 19, which we suppose to be nearly the same as that of its node, as the eclipse was total, gives the place of the node on September 1, in 4^s. 23°, and the opposite node in 10^s. 23°. Now the true place of the sun at the middle of this eclipse was 5^s. 1°. 7′, and consequently that of the moon 11^s. 1°. 7′. Hence, the place of the moon in this eclipse was about 8° *before* the place of the node, and the moon being eclipsed on the north side, this must have been the descending node. Hence, if we subtract 8°. 15′, the distance of the node from the center of the shadow on September 1, 719, from 11^s. 1°. 7′ the place of the center of the shadow, we shall have 10^s. 22°. 52′ for the place of the descending node on September 1, 719 years before J. C. consequently the true place of the ascending node was 4^s. 22°. 52′. Now the place of the ascending node on September 9, 1718, at *8h.* 4′ of the evening,

was $5^s. 16°. 40'$; and as the motion of the nodes is retrograde, the node in this latter case wants $23°. 48'$ of being up to the place of the node in the former case; consequently in this interval of time, which is 2437 years (of which 608 were bissextiles) $19d. 2h. 16'$, the nodes made a certain number of revolutions and $336°. 12'$ over. Now Art. 330. gives $3'. 10''$ for the mean diurnal motion of the nodes, and consequently in the above time, the nodes must have made 131 complete revolutions; if therefore we divide $2437y. 19d. 2h. 16'$ by 131 revolutions $336°. 12'$, it gives $6798d. 7h.$ for the time of a mean revolution of the nodes; hence, if we divide $6798d. 7h.$ by $365d.$ it gives $19°. 19'. 45''$ for the mean motion of the nodes in a common year of 365 days; and if we divide $19°. 19'. 45''$ by 365, it gives $3'. 10'. 38'''$ for the mean daily motion of the nodes. This differs only $38'''$ from the motion determined from the observations in 1707 and 1718. The motion of the nodes is not uniform; certain equations therefore are necessary to be applied to the mean place in order to get the true place at any time. MAYER in his Tables makes the mean annual motion $19°. 19'. 43'',1$.

If we examine the motion of the nodes from the eclipses on March 8, and September 1, 719 years before J. C. it gives $3'. 10''. 20'''$ for their mean daily motion. We have no reason therefore to think, that the mean motion of the nodes is subject to any change.

On the Inclination of the Orbit of the Moon to the Ecliptic.

332. To determine the inclination of the orbit, observe the moon's right ascension and declination when it is 90° from its nodes, and thence compute its latitude (124), which will be the inclination at that time. Repeat the observation for every distance of the sun from the earth, and for every position of the sun and moon in respect to the moon's nodes, and you will get the inclination at those times. From these observations it appears, that the inclination of the orbit to the ecliptic is variable, and that the *least* inclination is about 5°, which is found to happen when the nodes are in quadratures; and the *greatest* is about $5°. 18'$, which is observed to happen when the nodes are in syzygies. The inclination is found also to depend upon the sun's distance from the earth.

On the Mean Motion of the Moon.

333. The mean motion of the moon is found from observing its place at two different times, and you get the mean motion in that interval, supposing the

moon to have had the same situation in respect to its apsides at each observation; and if not, if there be a very great interval of the times, it will be sufficiently exact. To determine this, we must compare together the moon's places, first at a small interval of time from each other, in order to get very nearly the mean time of a revolution; and then at a greater interval, in order to get it more accurately. The moon's place may be determined directly from observation, or deduced from an eclipse.

334. M. Cassini, in his Astronomy, pag. 294, observes, that on September 9, 1718, the moon was eclipsed, the middle of which happened at *8h.* 4′, when the sun's true place was 5^s. 16°. 40′. This he compared with another eclipse, the middle of which was observed at *8h.* 32′ on August 29, 1719, when the sun's place was 5^s. 5°. 47′. In this interval of *354d.* 28′ the moon made 12 revolutions and 349°. 7′ over; divide therefore *354d.* 28′ by 12 revolutions 349°. 7′, and it gives *27d.* *7h.* 6′ for the time of one revolution. This is sufficiently accurate to compare eclipses at a greater interval.

335. On March 26, 1717, the middle of a lunar eclipse was observed at *15h.* 16′ at Paris, when the sun's place was 0^s. 6°. 21′. And on March 15, 1699, an eclipse was observed, the middle of which was at *7h.* 23′ at which time the sun's place was 11^s. 25°. 30′. In this interval of 18 years (of which 4 were bissextiles) *11d.* *7h.* 53′, the moon, besides a certain number of revolutions, was advanced 10°. 51′. This interval of *6585d.* *7h.* 53′ divided by *27d.* *7h.* 6′ gives 241 revolutions and about $\frac{1}{4}$, which shows that the number of complete revolutions must have been 241. Hence, if we divide *6585d.* *7h.* 53′ by 241 revolutions 10°. 51′, it gives *27d.* *7h.* 43′. 6″ for the time of one revolution. This will be sufficiently accurate to give the time for the most distant eclipses.

336. The moon was observed at Paris to be eclipsed on September 20, 1717, the middle of which was at *6h.* 2′. Now Ptolemy mentions that a total eclipse of the moon was observed at Babylon on March 19, 720 years before J. C. the middle of which happened at *9h.* 30′, at that place, which gives *6h.* 48′ at Paris. The interval of these times was 2437 years (of which 609 were bissextiles) 174 days wanting 46′; divide this by *27d.* *7h.* 43′. 6″ and it gives 32585 revolutions and a little above $\frac{1}{2}$. Now the difference of the two places of the sun, and consequently of the moon, at the times of observation, was 6^s. 6°. 12′. Therefore in the interval of *2437y.* *174d.* wanting 46′ the moon had made 32585 revolutions 6^s. 6°. 12′, which gives *27d.* *7h.* 43′. 5″ for the mean time of a revolution. This determination is very exact, as the moon was at each time very nearly at the same distance from its apside. Hence, the mean *diurnal* motion is 13°. 10′. 35″, and the mean *hourly* motion 32′. 56″. 27‴$\frac{1}{2}$. M. de la Lande makes the mean *diurnal* motion 13°. 10′. 35″,02784394. This is the mean time of a revolution in respect to the equinoxes. The place of the moon at the middle of the eclipse has here been taken the same as that of the sun, which is not accurate, except for a cen-

tral eclipse; it is sufficiently accurate, however, for this long interval. From the unequal angular motion of the moon about the earth, the hourly motion of the moon is subject to change from 29'. 55" to 38'. 22"; the excentricity of the orbit produces a variation of 3'. 36"; the evection produces one of 42", and the variation produces one of 40". The corrections for all the inequalities of the moon's motion will be found in the Tables of the moon.

337. Hence, to find the mean motion for any other time, say, the interval between the eclipses 2437*y*. 174*d*. wanting 46' : any other time :: 32585 revolutions 6^s. 6°. 12' : the mean motion in that time. This is more exact than taking the mean diurnal motion 13°. 10'. 35" and multiplying it by the time, as small errors are thus multiplied and become considerable. M. de LAMBRE makes the secular motion to be 10^s. 7°. 53'. 12", which M. de la LANDE uses in his Tables. MAYER in his Tables makes it 10^s. 7°. 53'. 35". In this motion of 100 years, 25 are supposed to be bissextiles.

338. As the precession of the equinoxes is 50",25 in a year, or about 4" in a month, the mean revolution of the moon in respect to the fixed stars must be greater than that in respect to the equinox by the time the moon is describing 4" with its mean motion, which is about 7". Hence, the time of a sidereal revolution of the moon is 27*d*. 7*h*. 43'. 12".

339. M. de la LANDE has determined the revolution in respect to the equinoxes to be 27*d*. 7*h*. 43'. 4",6795, which does not differ $\frac{1}{2}$" from the above; and hence he makes the sidereal revolution 27*d*. 7*h*. 43'. 11",5259. Hence, the mean synodic revolution (324) is 29*d*. 12*h*. 44'. 2",8283. If we take unity to represent the mean motion of the moon in respect to the fixed stars, then will 0,004021853526 represent the motion of the node, found by comparing their mean motions; hence, as the nodes move retrograde, the sidereal revolution of the moon, 27*d*. 7*h*. 43'. 11",5259, : its revolution in respect to its nodes :: 1,004021853526 : 1, the moon approaching the node with the sum of the velocities; hence, the revolution of the moon in respect to the nodes is 27*d*. 5*h*. 5'. 35",603. This is the determination of the mean revolutions to the beginning of this century.

To determine the Place of the Moon's Apogee, and the Equation of its Orbit.

340. Compare the observed place of the moon at any time with the place observed at any time afterwards; take the mean motion corresponding to the interval of time, and add it to the moon's place at the first observation, and the difference between that sum and the moon's place at the second observation shows the effect of the equation of the orbit between these two situations of the moon. Repeat this for a great many intervals, and mark those where the difference between the sum before mentioned and the moon's true place is

greatest both in excess and defect. If the greatest excess and defect be equal, it is a proof that at the time of the first observation, the moon was in its apogee or perigee, and that its true and mean places were the same. In this case each of these differences is the greatest equation of the moon's orbit. If the greatest excess and defect be not equal, half the sum will measure the greatest equation; and if from the greatest equation we subtract the least of the differences, we shall have the equation of the moon at the time of the first observation. M. CASSINI uses the place of the moon as determined from its eclipses, selecting those which were proper for this purpose; and although the apogee has moved in the interval, yet, as the true and mean place of the moon always coincide at the apogee, it will not affect the conclusion. *Elem. d'Astron.* pag. 297.

341. Hence, to find the place of the apogee, let $AMPV$ be the orbit of the moon, A the apogee, P the perigee, C the center of the orbit, T the earth in the focus, F the other focus, M the place of the moon at the time of the first observation; produce TM to R, take $MR=MF$, and join RF. From the greatest equation find (231) the ratio of AC to CT; this being koown, we have, $TF : TR :: \sin. TRF : \sin. TFR$, or AFR; now $FRT=\frac{1}{2}FMT$ the equation of the moon at the first observation, upon the *simple elliptic* hypothesis (227); hence, we know AFR, from which subtract FRT, and we get ATM the moon's distance from its apogee.

342. Let the first eclipse, with which the others are to be compared, be a total one, the middle of which happened at Paris on December 10, 1685, at $10h.\ 38'.\ 10''$ mean time. The true place of the sun at that time, by calculation, was $8^s.\ 19^\circ.\ 40'$, and consequently the moon's place was $2^s.\ 19^\circ.\ 40'$. Let the next eclipse be the total one on May 16, 1696, the middle of which was $12h.\ 7'.\ 56''$ mean time at Paris, and the moon's place was $7^s.\ 26^\circ.\ 53'.\ 35''$. Now in this interval of 10 years (of which 3 were bissextiles) $157d.\ 1h.\ 29'.\ 46''$, the mean motion of the moon, omitting the complete revolutions, was $5^s.\ 12^\circ.\ 53'.\ 10''$; this added to $2^s.\ 19^\circ.\ 40'$, the place at the first eclipse, gives $8^s.\ 2^\circ.\ 33'.\ 10''$ for the mean place at the second eclipse, the difference between which and the true place $7^s.\ 26^\circ.\ 53'.\ 35''$ is $5^\circ.\ 39'.\ 35''$. The next eclipse compared with the first was that on March 15, 1699, the middle of which was at $7h.\ 14'$ mean time at Paris, at which time the moon's true place was $5^s.\ 25^\circ.\ 28'.\ 41''$. Now in this interval of 13 years (of which 3 were bissextiles) $94d.\ 20h.\ 35'.\ 50''$, the mean motion of the moon, omitting the revolutions, was $3^s.\ 1^\circ.\ 24'.\ 47''$; this added to $2^s.\ 19^\circ.\ 40'$, the place at the first eclipse, gives $5^s.\ 21^\circ.\ 4'.\ 47''$ for the mean place at this third eclipse, the difference between which and $5^s.\ 25^\circ.\ 28'.\ 41''$ the true place is $4^\circ.\ 23'.\ 54''$. In the former case, the true place was less than the mean place by $5^s.\ 39^\circ.\ 35'$, and in the latter case, the mean place is the least by $4^\circ.\ 23'.\ 54''$. These are the greatest differences of

all the eclipses between 1685, and 1720. Now the sum of these differences is $10°. 3'. 29''$, and the half sum is $5°. 1'. 44'',5$ the greatest equation of the moon's orbit deduced from these observations. And if from $5°. 1'. 44'',5$ we take $4°. 23'. 54''$, the least difference, we have $37'. 50'',5$ for the equation of the moon at the time of the first eclipse; and this taken from $2^s. 19°. 40'$, the true place of the moon at that time, gives $2^s. 19°. 2'. 10''$ for the mean place of the moon on December 10, 1685, at $10h. 38'. 10''$ mean time at Paris. This therefore may be considered as an *epoch* of the mean place of the moon. This is the method used by M. Cassini. But the best method is, to observe accurately the place of the moon for a whole revolution as often as it can be done, and by comparing the true and mean motions, the greatest difference will be double the equation. If two observations be found, where the difference of the true and mean motions is nothing, the moon must then have been in its apogee and perigee. Mayer makes the mean excentricity 0,05503568, and corresponding greatest equation $6°. 18'. 31'',6$. It is $6°. 18'. 32''$ in his last Tables published by Mr. Mason, under the direction of Dr. Maskelyne.

343. To determine the place of the apogee, from M. Cassini's observations, we have the greatest equation $= 5°. 1'. 44'',5$, therefore (231), $57°. 17'. 48'',8 : 2°. 30'. 52'',25 :: AC = 100000 : CT = 4388$ for the moon's excentricity. Also, $TF = 8776 : TR = 200000 ::$ sin. $TRF = 18'. 55'',25 :$ sin. TFR, or AFR, $= 7°. 12'. 20''$, from which take $TRF = 18'. 55'',25$, and we have $ATM = 6°. 53'. 25''$ the distance of the moon from its apogee; add this to $2^s. 19°. 40'$, the true place of the moon, and it gives $2^s. 26°. 33'. 25''$ for the place of the apogee on December 10, 1685, at $10h. 38'. 10''$ mean time at Paris. This therefore may be considered as an *epoch* of the place of the apogee.

344. If we compare the same eclipse in 1685 with two others, one of which happened on July 7, 1675, and the other on April 14, 1642, we shall get the equation of the orbit $5°. 2'. 14''$, differing only $37''$ from the other determination. Also, the place of the apogee at the eclipse in 1685, comes out $2^s. 25°. 57'. 58''$, which is $35'. 27''$ less advanced than by the former case. If the moon's place be determined by observation at any time when it is not eclipsed, the same method may be applied.

345. It has been here supposed, that at the time of the eclipses the moon was at its mean distance, and of the great number of observations which were compared in order to get the greatest difference of the true and mean places, it is supposed that those which gave the greatest differences were so circumstanced. Also, no other inequalities have been supposed, but those which arose from the excentricity of the moon's orbit. The way therefore to get at the greatest accuracy is to make a great number of such comparisons, and take the mean.

346. The place of the moon's apogee may also be thus found. Take a great many measures of the moon's diameter, when at, or very near, the full, with a micrometer, and reduce them to the measure at the same altitude, and note the times of observation; seek out amongst them, those which are the greatest or the least, and you have the time when the moon was in its perigee or apogee. Or if you find two diameters equal to each other, very near together, the moon must have been in its apogee or perigee in the middle point of time. Now at the apogee, the difference between the true and mean motion of the moon for every degree is about 5′; and at the perigee, about 5′. 30″. Hence, the places of the moon being known at the time when the two diameters were found equal, and the mean motion between the times being known, the mean motion from one of the times to the middle point of time between will be known; therefore, as the difference of the true and mean motions is known, the true motion is known from one of the times to the half interval of time, and consequently the true place of the moon at the half interval, or place of the apogee or perigee, will be known. But on account of the great difficulty of measuring accurately the diameter of the moon, this method cannot be depended upon to a great degree of accuracy. It was from observing the diameter of the moon, that Horrox found the motion of the apogee was sometimes in antecedentia, and sometimes in consequentia; and that the excentricity of the orbit must be variable, in order to account for the *second* equation (349) observed by Ptolemy. By this method, M. Cassini found, from the eclipse on December 10, 1685, at 10*h*. 38′. 10″ apparent time at Paris, the place of the apogee to be 2^s. 25°. 41′. 30″. From the mean of a great number of observations, he determined, at the above time, the place of the apogee to be 2^s. 24°. 32′, and the greatest equation 4°. 58′. 44″. But the excentricity, and consequently the greatest equation, is subject to a variation; and the excentricity here determined is about the least. According to Mayer, the mean excentricity is 0,05503568, and the corresponding greatest equation 6°. 18′. 31″,6.

To determine the mean Motion of the Apogee.

347. Find its place at different times, and compare the difference of the places with the interval of the time between. To do this, we must first compare observations at a small distance from each other, lest we should be deceived in a whole revolution; and then we can compare those at a greater distance. Now we may either compute (343) the place of the apogee at several times, or we may find it from knowing the place once, according to the following method, given by M. Cassini in his Astronomy, page 307. The place of the apside has been determined for Dec. 10, 1685; and to find from thence its

place at any other time, observe the true place of the moon at that time, and find the mean motion corresponding to that interval, and add it to, or subtract it from, the place of the apogee on December 10, 1685, according as the time was after or before that, and you have the mean place of the moon at that time; the difference between which and the true place observed, is the equation of the orbit at that time; if the mean place be forwarder than the true, the moon is in the first six signs; if backwarder, in the last six. But the same equation may answer to two different mean anomalies; this therefore leaves an uncertainty in respect to the place of the apogee. Now from the mean place of the moon subtract each mean anomaly, and it gives the place of the apogee corresponding to each; consequently you get the motion of the apogee corresponding to each place thus found; and to determine which is the true motion, repeat the operation for some other time compared with the place of the apogee on December 10, 1685, and you will get the motion corresponding to two places again. Then compare these two motions with the other two, and those two which agree, must be the true motion.

348. By thus comparing the place of the apogee on December 21, 1684, at 10*h*. 55′. 58″ apparent time, with the place determined on Dec. 10, 1685, M. CASSINI found the time of a revolution of the apsides to be either 8 years and nearly 9 months, or about 3 years. And by comparing the place of the apogee on Nov. 29, 1686, at 11*h*. 7′. 18″ apparent time, with the place on December 10, 1685, he found that the motion of the apsides, deduced from thence, came out, one between eight and nine years, but that the other motion did not agree with either of the former. The time of a revolution therefore must be about 8 years 9 months. The time being thus nearly determined, he computed the motion from more distant observations, and from a mean of the whole, he found the time of a revolution of the apsides to be 8 common years, 311*d*. 8*h*. and hence the mean annual motion is 1^s. 10°. 39′. 52″, and daily motion 6′. 41″. 1‴. MAYER in his Tables makes the annual motion 1^s. 10°. 39′. 50″. This is the mean motion in respect to the equinoxes. M. de la LANDE makes the daily motion in respect to the equinoxes, 6′. 41″,069815. Hence he deduces the daily motion in respect to the fixed stars to be 6′. 40″,932238. If we take unity to represent the mean motion of the moon in respect to the fixed stars, then will the motion of its apogee be represented by 0,00845226445, found by comparing their mean motions; hence, as the motion of the apogee is direct, the sidereal revolution of the moon, 27*d*. 7*h*. 43′. 11″,4947, : its revolution in respect to its apogee :: 1 – 0,00845226445 : 1, the moon approaching the apogee with the difference of the velocities; hence, the revolution of the moon in respect to its apogee is 27*d*. 13*h*. 18′. 33″,95. The motion of the apogee is not uniform, as is implied in this method of determining its mean motion, and therefore it will be subject to a small error, unless the equation should be the

same at both observations; this error may be corrected, by reducing the true to the mean place at each observation. HORROX from observing the diameters of the moon, found the apogee subject to an annual equation of 12°,5. Having thus explained the methods of determining the moon's mean motions, situation of its apogee, and the equation of its orbit, or first inequality, we proceed to show how that, and some of the other principal inequalities were discovered.

349. The motion of the moon having been examined for one month, it was immediately discovered, that it was subject to an irregularity, which sometimes amounted to 5° or 6°, but that this irregularity disappeared about every 14 days. And by continuing the observations for different months, it also appeared, that the points where the inequalities were the greatest, were not fixed, but that they moved forwards in the Heavens about 3° in a month, so that the motion of the moon in respect to its apogee was about $\frac{1}{120}$ less than its absolute motion; thus it appeared that the apogee had a progressive motion. PTOLEMY determined this *first* inequality, or equation of the orbit, from three lunar eclipses observed in the years 719 and 720, before J. C. at Babylon by the Chaldeans; from which he found it amounted to 5°. 1' when at its greatest. But he soon discovered that this inequality would not account for all the irregularities of the moon. The distance of the moon from the sun observed both by HIPPARCHUS and himself, sometimes agreed with this inequality, and sometimes it did not. He found that when the apsides of the moon's orbit were in quadratures, this *first* inequality would give the moon's place very well; but that when the apsides were in syzygies, he discovered that there was a further inequality of about 2°$\frac{2}{3}$, which made the whole inequality about 7°$\frac{2}{3}$. This *second* inequality is called the *Evection*, and arises from a change of excentricity of the moon's orbit. The inequality of the moon was therefore found, by PTOLEMY, to vary from about 5° to 7°$\frac{2}{3}$, and hence the mean quantity was 6°. 20'. MAYER makes it 6°. 18'. 31'',6. It is very extraordinary, that PTOLEMY should have determined this to so great a degree of accuracy. This mean quantity is the greatest equation of the orbit for the mean excentricity, and is called the *first* equation. The *Evection*, or variation of the equation of the orbit from the mean equation, is at its maximum 1°. 20'. 28'',9 according to MAYER. Hence, when the apsides are in syzygies, at which time the excentricity is found to be the greatest, the greatest equation is 7°. 39'. 0'',5; and when the apsides are in quadratures, at which time the excentricity is found to be the least, it is only 4°. 58'. 2'',7. D'ARZACHEL, an Arab, who observed in Spain about the year 1080, from comparing the observations of PTOLEMY and those of D'ALBATEGNIUS with his own, discovered that the apsides were sometimes progressive and sometimes regressive; and that the excentricity was subject to a change. KEPLER believed this to be the case. HORROX discovered the same from his own observations; he found that when the

distance of the sun from the apogee of the moon was about 45°. and 225°, the apogee was more advanced by 25° than when the distance was about 135° and 315°, in such a manner that the mean motion was not uniform, but subject from thence to an equation of about 12°,5. He first made the moon revolve in an ellipse about the earth in its focus, and although some difficulties arose from this supposition, yet, he says, he durst not give up the hypothesis.

350. TYCHO explained these irregularities thus. Let the earth be at T the center of the circle $sqxg$, whose radius is 100000; Tr, the semidiameter of the circle $Tdet$, = 21741 the circle of excentricity, in whose circumference the center of excentricity is supposed to move in consequentia Tde, with a motion equal to double the distance of the moon from the sun; so the radius of the circle $acbo = 5800$; and om, the radius of $mwzv$, = 2900. Let $sq = 90°$; and let the moon move from its syzygies and apogee at s to quadratures at q, and conceive in the same time the center of excentricity to move from T through d to e, with twice the angular velocity of the moon from the sun. Then, considering r as the mean place of the center, when the moon comes to q, the equation is the angle $eqr = 1°.\ 15'$, which is to be subtracted in the first quadrature at q, and added in the third quadrature at g; this will produce an inequality of 2°. 30′, and account for the *Evection.* But instead of supposing the moon to revolve in the circumference $sqxg$, let the center of the circle $oacb$ revolve in consequentia, and the moon revolve in antecedentia in the circumference $obca$, and be at o when the moon is in its apogee, and to descend through b and arrive at c when the moon comes to its perigee; this will produce an inequality of 3°. 19′, which is part of the equation of the center. Lastly, let us suppose the moon to revolve in the circumference $zvmw$ in consequentia, whilst the center o moves. When the center is at o let the moon be at z, and when the center has moved to b or a, let the moon be at m; this will produce an equation of 1°. 40′; which added to the last gives 4°. 59′. In this manner TYCHO represented the irregularities of the moon discovered by PTOLEMY, who explained the *Evection*, by making the center of excentricity describe a circle $Tdet$, and the *equation of the center*, by one circle $obca$. HORROX explained the *second* inequality thus. Let E be the earth, C the mean place of the center of the orbit, $EBCA$ the corresponding line of the apsides, EC the mean excentricity of the orbit; and if we suppose the center of the orbit, instead of being at C, to describe the circle ADB, and take the angle ACD double the distance of the apogee from the sun, then AED will represent the equation of the apogee, and ED the excentricity. Sir I. NEWTON followed the same hypothesis.

351. But TYCHO being able to determine more accurately, from his observations, the true place of the moon, found that the place, computed from the above principles, would not agree with observations out of syzygies and quadra-

tures, particularly in the octants, where the difference was the greatest, and where he found it from 37′ to 40′. Thus TYCHO discovered a *third* irregularity, which he called the *Variation.* To explain this, he substituted another circle *ni*, and gave the center of the circle *obca* a libratory motion in the diameter *ni* perpendicular to *Ts*, corresponding to a motion in the circumference, which is double the distance of the moon from the sun. Thus, with the center *s* and radius equal to the variation, describe the circle *iyn;* take hy = double the distance of the moon from the sun, draw *yp* perpendicular to *in*, and where it cuts *in* in *s* will be the place of the moon corrected for the *Variation.* For the different methods by which the inequalities of the moon's motion have been represented, see RICCIOLI *Almagestum Novum.* Sir I. NEWTON makes this inequality vary from 33′. 14″ to 37′. 11″, it depending upon the sun's distance from the earth. HORROX makes it 36′. 27″ in his Tables. MAYER makes it 36′. 59″,8.

352. TYCHO also discovered another, called the *annual* equation, because it depends upon the distance of the earth from the sun, the variation of which causes a variation of the effect of the sun's action upon the earth and moon. CASSINI makes this equation 9′. 44″. Sir I. NEWTON makes it 11′. 50″. TYCHO observed, that the mean motion of the moon, in order to be uniform, required an equation of days, different from that which the motion of the sun gave; but this did not agree with the equation which we now employ. KEPLER also employed an equation for this purpose, which, he said, arose either from the motion of the moon, or the equation of time. HORROX, after employing the three equations already mentioned, corrected the apparent time at which he would calculate the true place of the moon by the equation of time, additive in the first six signs, which at the mean distance went as far as 13′. 24″, which is the same as if he had added 7′. 21″ to the mean longitude; at the same time, he neglected one part of the true equation, which would have been 7′. 42″ subtractive, in such a manner that it would have added 4′. 14″ to the place of the moon; these two would have made the whole 11′. 35″, which agrees with the annual equation. FLAMSTEAD observed, that this equation of time was not the equation belonging to the solar system; nevertheless he granted that this equation ought to be employed, which he says is peculiar to the moon, it being affected by the earth. Afterwards Dr. HALLEY observed that the moon moved fastest when the sun was in its apogee; and he fixed this equation at about 13′. MAYER makes it 11′. 8″,8.

353. It is very easy to conceive how this annual equation might be discovered by observation. By computing the moon's place for a great many times in the year, allowing for the equation of the orbit, the evection and variation, and comparing it with the observed place, it would appear that they agreed very well about the beginning of January and July, but that they differed considerably at the beginning of April and October. This would point out an equation.

But besides these four principal equations, the only ones deduced solely from observation, there are a great many others which are smaller, which are found by theory and corrected by observations. The theory of the moon must therefore be consulted by those, who would wish to have an intimate knowledge of the subject. We shall afterwards give so much of it, as is consistent with the plan of this Work.

354. TYCHO found that the motion of the nodes and variation of the inclination of the orbit, were subject to an irregularity, and might be represented by the motion of the pole of the orbit in a circle $ECFG$, whose radius $GD = 9'.\ 30''$, half the difference of the greatest and least inclinations, the center D being $5°.\ 8'$ from the pole A of the ecliptic, that being the mean inclination of the orbit, according to TYCHO, or mean distance of the poles of the ecliptic and moon's orbit. By more accurate observations, $GD = 8'.\ 48''$, and the mean inclination $5°.\ 8'.\ 49''$. Let the pole of the lunar orbit move in the circumference GEC, and be at G in syzygies and C in quadratures, and at F and K in octants, its motion being twice the true distance of the sun from the moon. Then when the pole is at any point H, HA is the inclination, and the angle HAD the equation of the node, the angle ADH being double the distance of the moon from the sun. At F this equation is the greatest, and $= 1°.\ 46'$, found from the triangle DFA. Hence, MAYER gave a method of finding the equation of latitude, of which the following is the investigation, given by M. de la LANDE in his Astronomy.

355. Let L be the moon 90° from the true pole E of its orbit, D being the mean pole, draw LEM, and DM perpendicular to it; then as the angle DLM is very small, we may suppose $LD = LM$, and consequently $EM = LD - LE$. Now as DA is very small compared with DL, LE and LD will be very nearly circles of latitude, and therefore their difference EM, will be the equation of latitude, being the difference of the distances of the moon from the true and mean pole. Draw DB perpendicular to AD, and it must pass through the nodes, therefore LDB is the moon's distance from the node, or the argument of latitude, and which is equal to ADM, MDB being the complement of each; also, ADE is twice the distance of the moon from the sun. Now $EM = ED \times$ sin. $EDM = ED \times$ sin. $\overline{ADE - ADM}$; that is, the *equation of the moon's latitude is* $8'.\ 48''$ *multiplied by the sine of double the distance of the moon from the sun—the argument of latitude.* TYCHO, and after him FLAMSTEAD, HALLEY, NEWTON, &c. considered the equations of the nodes and inclination separately.

Elements of the Theory of the Moon according to Observation.

Secular motion for 100 years, of which 25 are bissextiles.	KEPLER and HORROX - - -	10s. 7°. 48'. 51"
	NEWTON, FLAMSTEAD, and HALLEY -	10. 7. 50. 25
	CASSINI - - - - - -	10. 7. 49. 52
	MAYER, (second Tables) - -	10. 7. 53. 35
	M. de LAMBRE - - - - -	10. 7. 53. 12
Secular motion of the Apogee.	KEPLER - - - - - -	3s. 19°. 14'. 16"
	HORROX - - - - - -	3. 19. 4. 16
	CASSINI - - - - - -	3. 19. 14. 16
	FLAMSTEAD, HALLEY, and MAYER -	3. 19. 11. 15
Secular motion of the Node.	KEPLER and HORROX - -	4s. 14°. 11'. 7"
	FLAMSTEAD and HALLEY - -	4. 14. 11. 15
	CASSINI - - - - - -	4. 14. 11. 5
	MAYER - - - - - -	4. 14. 11. 15
Epoch of the mean longitude of the moon for 1750.	KEPLER - - - - - -	6s. 8°. 18'. 54"
	HORROX - - - - - -	6. 8. 17. 54
	FLAMSTEAD - - - - -	6. 8. 21. 24
	CASSINI - - - - - -	6. 8. 20. 0
	MAYER (second Tables) - -	6. 8. 22. 24
	MASON - - - - - -	6. 8. 22. 21
	M. de LAMBRE - - - - -	6. 8. 22. 20
Epoch of the longitude of the Apogee for 1750.	KEPLER - - - - - -	5s. 21°. 30'. 36"
	HORROX - - - - - -	5. 20. 30. 36
	FLAMSTEAD and HALLEY - -	5. 20. 58. 55
	CASSINI - - - - - -	5. 21. 1. 24
	MAYER - - - - - -	5. 20. 55. 54
	MASON - - - - - -	5. 20. 54. 56

Epoch of the Longitude of the Node for 1750.	KEPLER - - - - -	9ˢ. 10° 33′. 13″
	HORROX - - - - -	9. 10. 15. 13
	FLAMSTEAD - - - - -	9. 10. 14 59
	HALLEY - - - - -	9 10. 13. 58
	CASSINI - - - - -	9. 10. 18. 7
	MAYER - - - - -	9. 10. 19. 8
	MASON - - - - -	9. 10. 19. 59
Mean Equation of the Orbit.	FLAMSTEAD - - - - -	6°. 18′. 43″
	EULER - - - - - -	6. 18. 18
	D'ALEMBERT - - - - -	6. 18. 43
	CLAIRAUT - - - - - -	6. 18. 1
	MAYER - - - - - -	6 18. 32

Times of the Revolutions of the Moon, of its Apogee and Node, as determined by M. de la LANDE.

Tropical revolution - - - - -	27ᵈ. 7ʰ. 43′. 4″,6795
Sidereal revolution - - - - -	27. 7. 43. 11, 5259
Synodic revolution - - - - -	29. 12. 44. 2, 8283
Anomalistic revolution - - - - -	27. 13. 18. 33, 9499
Revolution in respect to the node - - -	27. 5. 5. 35, 603
Tropical revolution of the apogee - -	8ʸ. 311. 8. 34. 57, 6177
Sidereal revolution of the apogee - -	8. 312. 11. 11. 39, 4089
Tropical revolution of the node - -	18. 228. 4. 52. 52, 0296
Sidereal revolution of the node - -	18. 223. 7. 13. 17, 744
Diurnal motion of the moon in respect to the equinox - - - -	13°. 10′. 35″,02784394
Diurnal motion of the apogee - - - -	0. 6. 41, 069815195
Diurnal motion of the node - - - - -	0. 3. 10, 638603696

The years here taken are the common years of 365 days.

According to PTOLEMY, the mean annual motion of the moon is 4ˢ. 9°. 22′. 46″, and the diurnal motion 13°. 10′. 34″. 58‴; the mean annual motion of the nodes in antecedentia is 19°. 20′. 0″. 58‴, and the diurnal motion 3′. 10″. 41‴,25; the mean annual motion of the apogee is 40°. 39′. 35″,75, and the diurnal motion 6′. 41″. 2‴,25; and the time of a mean synodic revolution is 29*d.* 13*h.* 5′. 39″. If the reader will compare these, with our present Tables, he will be

surprised at their accuracy; and if he consider also that PTOLEMY discovered the two first irregularities, the lunar motions will be found to have been known to a very considerable degree of accuracy, near 2000 years ago.

On the Acceleration of the Moon's Motion.

356. Dr. HALLEY, by comparing the ancient eclipses observed at Babylon, with those observed by ALBATEGNIUS in the ninth century, and with those observed in his own time, discovered the mean motion of the moon to be accelerated; and says, that he could have found out the quantity of acceleration, if he had had the longitude of Bagdat, Alexandria and Aleppo; because in, or near these places, the observations were made; for it is necessary to know their longitudes, in order to reduce the times, to those under the meridian in which the modern observations are made. In the *Phil. Trans.* 1749, Mr. DUNTHORNE has examined some ancient eclipses observed under known meridians, and determined what the acceleration is. The eclipses which he has chosen for this purpose are these.

357. An eclipse of the sun was observed at Alexandria, by THEON, in the year 364, on June 16; the beginning was in the afternoon at 3*h*. 18′, and the end at 5*h*. 15′. In the year 977, an eclipse of the sun was observed, at Grand Cairo, on December 13; the beginning was at 8*h*. 25′, and the end at 10*h*. 45′, apparent time, in the morning, and the digits eclipsed were 8; also, the sun's altitude at the beginning was 15°. 43′, and at the end 33°½. In the year 978, at the same place, the sun was observed to be eclipsed on June 8; the beginning was at 2*h*. 31′, and the end at 4*h*. 50′, apparent time, in the afternoon. Mr. DUNTHORNE then computed the distance of the moon from the sun in longitude, at the beginning of each eclipse, from the above *data*; he also computed their distance at that time from the Tables of the sun's and moon's motion, and found that at the beginning of the first eclipse, the Tables gave the difference of the sun's and moon's places less than that deduced from the observation by 4′. 16″; in the second eclipse it was greater by 7′. 36″; and in the third, greater by 8′. 45″; the computed places at the two last eclipses being so much *before* the observed places, but at the first eclipse the computed place was so much *behind*. The agreement of the two last shows, that the Tables represent the mean motion of the moon's apogee very well for above 700 years, the moon having been very near its perigee at the time of one of those eclipses, and near its apogee at the time of the other. Now HIPPARCHUS mentions an eclipse observed at Babylon, which happened on December 22, in the year 313 before J. C. when a small part of the moon was eclipsed on the north east, half an hour before the end of the night, and the moon set eclipsed;

Mr. DUNTHORNE, from his Tables, makes the middle of it at 9*h.* 4′ apparent time, and the duration 1*h.* 37′; PTOLEMY makes it 1*h.* 30′ nearly. Hence, the beginning, according to Mr. DUNTHORNE's calculation, was about 8*h.* 15′ after midnight. But, according to PTOLEMY, the sun rose at 7*h.* 12′, and as the moon had then south latitude, and was not quite come to the sun's opposition, its apparent setting must have been a little sooner, that is, above an hour before the beginning of the eclipse, according to his Tables; whereas the moon was seen eclipsed some time before its setting, which proves that its true place was then forwarder than the Tables make it, by 40′ or 50′. In the year 201 before J. C. on September 22, an eclipse was observed at Alexandria, when the moon began to be eclipsed about half an hour before its rising, and ended about 3½*h.* in the morning. Now by the Tables, the middle of the eclipse was at 7*h.* 44′ apparent time, and the duration, 3*h.* 4′, which makes the beginning at 6*h.* 12′, or about 10′ after the moon rose, and consequently 40′ after the time by observation; which makes the moon's true place forwarder than by the Tables, by about 20′. In the year 721 before J. C. on March 19, an eclipse was observed at Babylon, the middle of which, by the Tables, was at 10*h.* 26′ apparent time, and the beginning was at 8*h.* 32′; but the beginning, by observation, was at 6*h.* 46′, or 1*h.* 46′ sooner than by the Tables; therefore the moon's true place preceded its place by the Tables, by a little more than 50′. Hence, as the same Tables represent the moon's place in the ancient eclipses *behind* its true place, and in the two eclipses observed in 977 and 978 *before* it, it follows that its mean motion in ancient times was slower, and in latter times quicker than the Tables give, and therefore it must have been accelerated. There must also have been a time when the Tables would give the true place. And although the ancient observations of the times of the eclipses were not very accurate, yet they were sufficiently so to prove, beyond all doubt, that the moon's motion is greater at this time, than it was at the times when the ancient eclipses were observed.

358. As we have no *data* to determine directly what this acceleration is, and at what point of time the moon's place from the Tables would agree with its true place, we must make a supposition for each, and then compute the errors of the Tables, and see how they agree with the above errors; and that supposition which, upon the whole, agrees best, must give the acceleration the most to be depended upon, and probably near the truth. Now whatever be the cause of this acceleration, it is very probable that it continues constant, or very nearly so, and therefore the quantity of acceleration will vary as the square of the time. Upon this principle, if we suppose the Tables to give the true place of the moon at the year 700, and the acceleration to be 10″ for the first 100 years from that time, it will give results agreeing better with the observations than any other supposition. These results may be thus computed. The quan-

tity of acceleration at the beginning of the successive centuries from 700 to 1700, will be, 10″, 40″, 90″, 160″, 250″, 360″, 490″, 640″, 810″, 1000″. Now as the whole acceleration in these 10 centuries is 1000″, the mean acceleration is 100″ in a year, and would be, in the above respective centuries, 100″, 200″, 300″, 400″, 500″, 600″, 700″, 800″, 900″, 1000″; subtract therefore the above actual accelerations from these mean ones which the Tables give, and we have 1′. 30″, 2′. 40″, 3′. 30″, 4′. 0″, 4′. 10″, 4′. 0″, 3′. 30″, 2′. 40″, 1′. 30″, 0′. 0″ for the error of the Tables at the beginning of every century from 700 to 1700, showing how much the Tables give the place too forward. If we go downwards from 700, the motion will be diminished at the same rate of 10″, 40″, 90″, &c. for every century; whereas our Tables give it increasing at the rate of 100″, 200″, 300″, &c. therefore the errors of the Tables will be the sums of these respective quantities, or 1′. 50″, 4′. 0″, 6′. 30″, 9′. 20″, 12′. 30″, 16′. 0″, 19′. 50″, 24′. 0″, 28′. 30″, 33′. 20″, 38′. 30″, 44′. 0″, 49′. 50″, 56′. 6″, showing how much the Tables give the place too backward at the beginning of each century from the year 700 to 700 before J. C. Hence, the following Table:

Years before Christ	Error of Tables.	Years after Christ	Error of Tables.	Years after Christ	Error of Tables
700	− 56′. 6″	0	− 19′. 50″	900	+ 2′. 40″
600	− 49. 50	100	− 16. 0	1000	+ 3. 30
500	− 44. 0	200	− 12. 30	1100	+ 4. 0
400	− 38. 30	300	− 9. 20	1200	+ 4. 10
300	− 33. 20	400	− 6. 30	1300	+ 4. 0
200	− 28. 30	500	− 4. 0	1400	+ 3. 30
100	− 24. 0	600	− 1. 50	1500	+ 2. 40
* *	* * *	700	0. 0	1600	+ 1. 30
* *	* * *	800	+ 1. 30	1700	0. 0

If we compare the errors of the Tables, in the eclipses above related, with the errors in this Table, they will be found to differ not more than might be expected from the uncertainty of the times of the eclipses, and the different errors which the Tables may be subject to at different times. These observations therefore make the secular variation 10″, and to vary as the square of the time. M. de la Lande, from the eclipses in 977 and 978, makes it 9″,886. In Mayer's Tables it is 9″, beginning from 1700.

359. M. de la Place, in the *Mem. de l'Acad. Roy. des Scien.* for 1786, has shown, that this acceleration of the moon's motion arises from the action of the

sun upon the moon, combined with the variation of the excentricity of the earth's orbit. The excentricity of the earth's orbit is, at present, diminishing, and this arises from the action of the planets upon the earth. The major axis of the earth's orbit is not altered by this, but the excentricity is. The mean force of the sun to dilate and contract the orbit of the moon depends on the square of the excentricity of the earth's orbit. By the diminution of the excentricity, the moon's mean motion is accelerated, and this is a circumstance which takes place at present. When the excentricity is come to its *minimum*, the acceleration of the mean motion will cease; after which the excentricity will increase and the moon's mean motion will be retarded. This therefore causes a secular equation of the moon's mean motion, the period of which is very long. If n be the number of centuries from 1700, M. de la PLACE has computed the secular equation to be $+ 11'',135\, n^2 + 0'',4398 n^3$; this however cannot be true whatever be the value of n, because the acceleration would then continually increase; but it may be extended back to the most ancient observations of the moon, and for 1000 or 1200 years to come, without any sensible error. M. de LAMBRE, from comparing the modern observations at about the distance of a century, found that the secular mean motion of the moon in the last Tables of MAYER was too great by $25''$; and that the place of the moon, calculated by these Tables, ought to be corrected by the quantity $- 25'' n + 2'',135 n^2 + 0'',04398\, n^3$. If the ancient observations of the moon be compared with the places calculated by MAYER's Tables with this correction, the errors will be comparatively very small, and no greater than what might arise from the inaccuracy of their observations. M. de la LANDE, in his Tables of the moon, has thus corrected MAYER's Tables. Hence, it appears, that the present acceleration of the moon is nothing more than an equation, the period of which is very long; it will be accelerated and retarded by the same quantity, and therefore if the mean motion be taken for the whole time of acceleration or retardation, it will be found never to vary.

The mean motion of the nodes and apogee of the moon's orbit is subject to a secular equation. The secular equation of the nodes is $- 2'',784\, n^2 - 0'',010995\, n^3$, which being negative shows that it is to be applied contrary to their mean motion. This secular equation is $\frac{1}{4}$ of the secular equation of the mean motion. The secular equation of the apogee is $\frac{7}{4}$ of the secular equation of the mean motion, and is therefore $- 19'',486 n^2 - 0'',07697 n^3$; where the negative sign shows that it is to be applied contrary to its mean motion. Hence, all the irregularities of the moon are but so many equations, which return again in their regular order; and the same is shown to be true of the irregularities of *Jupiter* and *Saturn*; also, as the major axes of their orbits remain undisturbed, it is manifest that the system can never be destroyed, all the irregularities being pe-

riodical, and confined to such small limits as to produce no inconvenience. These are circumstances which furnish great matter for our attention; the stability of the system shows the power and wisdom of the Framer.

On the Diameter of the Moon.

360. The diameter of the moon may be measured, at the time of its full, by a micrometer; or it may be measured by the time of its passing over the vertical wire of a transit telescope; but this must be when the moon passes within an hour or two at the time of the full, before the visible illumination is sensibly changed from a circle. To find the diameter by the time of its passage over the meridian, let d''=the horizontal diameter of the moon, c=sec. of its declination, and m=the length of a lunar day, or the time from the passage of the moon over the meridian on the day we calculate, to the passage over the meridian the next day. Then (108) cd'' is the moon's diameter in right ascension; hence, $360° : cd'' :: m$: the time (t) of passing the meridian; therefore, $d'' = 360° \times \frac{t}{cm}$. If we observe the time when the limb of the moon comes to the meridian, we can find the time when the center comes to it, by adding to, or subtracting from the time when the first or second limb comes to the meridian, half the time of the passage of the moon over the meridian. The time in which the semidiameter of the moon passes the meridian, may be found by two Tables, in the Tables of the moon's motion.

361. Albategnius made the diameter of the moon to vary from 29'. 30'' to 35'. 20'', and hence the mean 32'. 25''. Copernicus found it from 27'. 34'' to 35'. 38'', and therefore the mean 31'. 36''. Kepler made the mean diameter 31'. 22''. M. de la Hire made it from 29'. 30'' to 33'. 30''. M. Cassini made it from 29'. 30'' to 33'. 38''. M. de la Lande, from his own observations, found the mean diameter to be 31'. 26''; the extremes from 29'. 22'' when the moon is in apogee and conjunction, and 33'. 31'' when in perigee and opposition. The mean diameter here taken is the arithmetic mean between the greatest and least diameters; the diameter at the mean distance is 31'. 7''. Hence, according to the theory of Mayer, the horizontal diameter of the moon at any time is 31'. 7'' − 1'. 42'',3 cos. anom. + 5'',4 cos. 2 anom. + 13'',7 cos. 2 dist. ☾ from ☉ − 20'',2 cos. (2 dist. ☾ from ☉ − anom. ☾).

362. When the moon is at different altitudes above the horizon, it is at different distances from the spectator, and therefore there is a change of the apparent diameter. Let C be the center of the earth, A the place of a spectator on its surface, Z his zenith, M the moon; then sin. CAM or ZAM:

sin. $ZCM :: CM : AM = \frac{CM \times \text{sin. } ZCM}{\text{sin. } ZAM}$; but the apparent diameter is inversely as its distance; hence, the apparent diameter varies as $\frac{\text{sin. } ZAM}{\text{sin. } ZCM}$, CM being supposed constant. Now in the horizon, $\frac{\text{sin. } ZAM}{\text{sin. } ZCM}$ may be considered as equal to unity; hence, $1 : \frac{\text{sin. } ZAM}{\text{sin. } ZCM}$, or sin. ZCM : sin. ZAM, or cos. true alt. (a) cos. apparent alt. (A) :: the horizontal diameter : the diameter at the apparent altitude (A). Hence, the horizontal diameter : its increase :: cos. a : cos. $A -$ cos. $a = 2$ sin. $\overline{\frac{1}{2} a + \frac{1}{2} A} \times$ sin. $\overline{\frac{1}{2} a - \frac{1}{2} A}$; therefore the increase of the semidiameter = hor. semidiam. $\times \frac{\text{sin. } \overline{\frac{1}{2} a + \frac{1}{2} A} \times \text{sin. } \overline{\frac{1}{2} a - \frac{1}{2} A}}{\text{cos. } a}$; from this we may easily construct a Table of the increase of the semidiameter for any horizontal semidiameter; and then for any other horizontal semidiameter, the increase will vary in proportion.

363. Some Astronomers have thought, that in finding the time of the transit of the moon over the meridian, we ought to take the apparent diameter instead of that seen from the center of the earth. But this, as M. de la LANDE has observed, must not be; for although the apparent diameter is increased by the moon being nearer to the spectator, yet the angular velocity about the point where the spectator is situated is increased in the same ratio, the angular velocity about any point, and the apparent diameter, being inversely as the distance, and therefore the time of the transit is the same.

On the Phases of the Moon.

364. By Art. 319. the greatest breadth of the visible illuminated part of the moon's surface varies as the versed sine of the moon's elongation from the sun, very nearly; and the circle terminating the light and dark part being seen obliquely will appear an ellipse; hence the following delineation of the phases. Let E be the earth, S the sun, M the moon; describe the circle $abcd$, representing that hemisphere of the moon which is towards the earth projected upon a plane; ac, db two diameters perpendicular to each other; take $dv =$ the versed sine of elongation SEM, and describe the ellipse avc, and (318, 319) $adcva$ will represent the visible enlightened part; which will be horned between conjunction and quadratures, bisected at quadratures, and gibbous between quadratures and opposition, the versed sine being less than radius in the first case, equal to it in the second, and greater in the third. The visible enlightened part varying as dv, we have, *the visible enlightened part : whole :: versed sine of elongation : diameter.*

On the Libration of the Moon.

365. Many Astronomers have given maps of the face of the moon; but the most celebrated are those of HEVELIUS in his *Selenographia*, in which he has represented the appearance of the moon in its different states from the new to the full, and from the full to the new; these figures MAYER prefers. Figure 81 represents the face of the moon in its mean state of libration, as shown by the best telescopes. LANGRENUS and RICCIOLUS denoted the spots upon the surface by the names of philosophers, mathematicians, and other celebrated men, giving the names of the most celebrated characters to the largest spots; HEVELIUS marked them with the geographical names of places upon the earth. The former distinction is now generally followed, and is that which we have here given. The numbers in the figure represent, nearly, the order in which the spots are eclipsed, going from the east to the west.

1. Grimaldus
2. Galileus
3. Aristarchus
4. Keplerus
5. Gassendus
6. Schikardus
7. Harpalus
8. Heraclides
9. Lansbergius
10. Reinoldus
11. Copernicus
12. Helicon
13. Capuanus
14. Bullialdus
15. Eratosthenes
16. Timocharis
17. Plato
18. Archimedes
19. Insulasinus Medii
20. Pitatus
21. Tycho
22. Eudoxus
23. Aristoteles
24. Manilius
25. Menelaus
26. Hermes
27. Possidonius
28. Dionysius
29. Plinius
30. Theophilus
31. Fracastorius
32. Promontorium Acutum, Censorinus
33. Messala
34. Promontorium Somnii
35. Proclus
36. Cleomedes
37. Snellius
38. Petavius
39. Langrenus
40. Taruntius

A. Mare Humorum
B. Mare Nubium
C. Mare Imbrium
D. Mare Nectaris
E. Mare Tranquillitatis
F. Mare Serenitatis
G. Mare Fæcunditatis
H. Mare Crisium.

The spots upon the moon are caused by the mountains and vallies upon its surface; for certain parts are found to project shadows opposite to the sun; and when the sun becomes vertical to any of them, they are observed to have no shadow; these therefore are mountains; other parts are always dark on that side next the sun, and illuminated on the opposite side; these therefore are cavities. Hence, the appearance of the face of the moon continually varies, from its altering its situation in respect to the sun. The tops of the mountains, on the dark part of the moon, are frequently seen enlightened at a distance from the confines of the illuminated part. The dark parts have, by some, been thought to be seas; and by others, to be only a great number of caverns and pits, the dark sides of which, next to the sun, would cause those places to appear darker than others. The great irregularity of the line bounding the light and dark part, on every part of the surface, proves that there can be no very large tracts of water, as such a regular surface would necessarily produce a line, terminating the bright part, perfectly free from all irregularity. If there was much water upon its surface, and an atmosphere, as conjectured (377) by some Astronomers, the clouds and vapours might easily be discovered by the telescopes which we have now in use; but no such phænomena have ever been observed.

366. Very nearly the same face of the moon is always turned towards the earth, it being subject only to a small change within certain limits, those spots which lie near the edge appearing and disappearing by turns; this is called its *Libration*, and arises from four causes. 1. GALILEO, who first observed the spots of the moon after the invention of telescopes, discovered this circumstance; he perceived a small daily variation arising from the motion of the spectator about the center of the earth, which, from the rising to the setting of the moon, would cause a little of the western limb of the moon to disappear, and bring into view a little of the eastern limb; this is called the *diurnal* libration. 2. He observed likewise, that the north and south poles of the moon appeared and disappeared by turns; this arises from the axis of the moon not being perpendicular to the plane of its orbit, and is called a libration in *latitude*. 3. From the unequal angular motion of the moon about the earth, and the uniform motion of the moon about its axis, a little of the eastern and western parts must gradually appear and disappear by turns, the period of which is a month, and this is called a libration in *longitude*; the cause of this libration was first assigned by RICCIOLUS, but he afterwards gave it up, as he made many observations which this supposition would not satisfy. HEVELIUS however found that it would solve all the phænomena of this libration. 4. Another cause of libration arises from the attraction of the earth upon the moon, in consequence of its spheroidical figure.

367. If the angular velocity of the moon about its axis were equal to its

angular motion about the earth, the libration in *longitude* would not take place. For if E be the earth, $abcd$ the moon at v and w, and avc be perpendicular to $Ebvd$; then abc is that hemisphere of the moon at v next to the earth. When the moon comes to w, if it did not revolve about its axis, bwd would be parallel to bvd, and the same face would not be towards the earth. But if the moon, by revolving about its axis in the direction $abcd$, had brought b into the line Ew, the same face would have been towards the earth; and the moon would have revolved about its axis through the angle bwE, which is equal to the alternate angle wEc, the angle which the moon has described about the earth.

368. When the moon returns to the same point of its orbit, the same face is observed to be towards the earth, and therefore (367) the time of the revolution in its orbit is equal to the time about its axis. But in the intermediate points it varies, sometimes a little more to the east, and sometimes to the west, becomes visible; and this arises from its angular motion about the earth being not uniform, whilst the angular motion about its axis is so. Hence, the libration in *longitude* is nearly equal to the equation of the orbit, or about $7°\frac{1}{2}$ at its maximum, and would be accurately so, if the axis of the moon were perpendicular to its orbit. The same face will be towards the earth in apogee and perigee, for at those points there is no equation of the orbit. If E be the earth, M the moon, pq its axis, not perpendicular to the plane of the orbit ab; then at a the pole p will be visible to the earth, and at b the pole q will be visible; as the moon therefore revolves about the earth, the poles must appear and disappear by turns, causing the libration in *latitude*. This is exactly similar to the cause of the variety of our seasons, from the earth's axis not being perpendicular to the plane of its orbit. Hence, nearly one half of the moon is never visible at the earth. Also, the time of its rotation about its axis being a month, the length of the lunar days and nights will be about a fortnight each, they being subject but to a very small change, on account of the axis of the moon being nearly perpendicular to the ecliptic. Her libration in *latitude* is about 10°.

369. Hevelius (*Selenographia*, pag. 245.) observed, that when the moon was at its greatest north latitude, the libration in latitude was the greatest, the spots which are situated near to the northern limb being then nearest to it; and as the moon departed from thence, the spots receded from that limb, and when the moon came to its greatest south latitude, the spots situated near the southern limb were then nearest to it. This variation he found to be about 1′. 45″, the diameter of the moon being 30′. Hence it follows, that when the moon is at its greatest latitude, a plane drawn through the earth and moon perpendicular to the plane of the moon's orbit, passes through the axis of the moon; consequently the equator of the moon must intersect the ecliptic in a line parallel to the line of the nodes of the moon's orbit, and therefore, in

the Heavens, the nodes of the moon's orbit and of its equator coincide; and this will be further confirmed, when we treat on the situation of the moon's equator and axis.

370. It is a very extraordinary circumstance, that the time of the moon's revolution about its axis should be equal to that in its orbit. Sir I. NEWTON, from the altitude of the tides on the earth, has computed that the altitude of the tides on the moon's surface must be 93 feet, and therefore the diameter of the moon perpendicular to a line drawn from the earth to the moon, ought to be less than the diameter directed to the earth, by 186 feet; hence, says he, the same face must always be towards the earth, except a small oscillation; for if the longest diameter should get a little out of that direction, it would be brought into it again, by the attraction of the earth. The supposition of D. de MAIRAN is, that that hemisphere of the moon next the earth is more dense than the opposite one, and hence the same face would be kept towards the earth, upon the same principle as above. M. de la GRANGE, in the *Mem. de l'Acad. des Scien.* 1780, has examined this subject very fully, and shown, that from the attraction of the earth, that diameter of the moon's equator which is directed towards the earth, will be lengthened four times more than that which is perpendicular to it. If h be the semidiameter of the moon in parts of its mean distance from the earth, m the quantity of matter in the moon expressed in parts of that of the earth, he has shown that the increase of the semidiameters will be $\frac{5h^3}{m}$ and $\frac{5h^3}{4m}$, the radius being unity.

371. Sir I. NEWTON proposes the following method of representing the libration of the moon in *latitude* and *longitude*. Take a common globe, and elevate the pole to the zenith, so that the equator may coincide with the horizon, and let the ecliptic represent the moon's orbit. Conceive the center of this globe to represent the place of the earth, and the surface of the globe the sphere in which the moon revolves. Take two small spheres, having each a meridian, and suspend each by a string from one of its poles. Let one of these represent a fictitious moon carried uniformly round the earth, having its equator coinciding with the horizon of the globe, and revolving uniformly about its axis in the same time in which it revolves about the earth; then the same meridian of the moon will always pass through the earth, and the moon would not be subject to any libration. Let the other sphere, representing the true moon, be carried in the ecliptic with its proper angular motion about the earth, having its axis and meridian parallel to those of the other moon. Then as the true moon moves from the perigee to its apogee, preceding the fictitious moon, the meridian will appear towards the left of its disc, and the spots will appear to move towards the east, by as many degrees as there are between the longitudes of the true and fictitious moons, or by the equation of the orbit; when the true

moon moves from apogee to perigee, the meridian of the true moon will appear towards the right of the disc, and the spots will appear to move towards the west; thus representing the libration in longitude. When the true moon moves from its ascending node to its greatest north latitude, the north pole of the moon will disappear, and the south pole, with the spots about it, will come into view; and as the moon leaves this northern limit, they will begin to disappear, and when the moon has reached its greatest southern latitude, the northern pole, with the spots about it, will be brought into view, and appear furthest upon its disc; thus representing the libration in longitude. NICOLAI MERCATORIS, *Institut. Astron.* pag. 286.

372. When the moon is about three days from the new, the dark part is very visible, by the light reflected from the earth, which is moon light to the Lunarians, considering our earth as a moon to them; and in the most favourable state, some of the principal spots may be seen. But when the moon gets into quadratures, its great light then prevents the dark part from being visible. According to Dr. SMITH, the strength of moon light, at the full moon, is 90 thousand times less than the light of the sun; but from some experiments of M. BOUGUER, he concluded it to be 300 thousand times less. The light of the moon, condensed by the best mirrors, produces no sensible effect upon the thermometer. Our earth, in the course of a month, shows the same phases to the Lunarians, as the moon does to us; the earth is at the full at the time of the new moon, and at the new at the time of the full moon. The surface of the earth being about 13 times greater than that of the moon, it affords 13 times more light to the moon than the moon does to the earth. To a Lunarian, the earth appears nearly fixed in respect to his horizon.

On the Altitude of the Lunar Mountains.

373. The method used by HEVELIUS, and others since his time, to determine the height of a lunar mountain is this. Let SLM be a ray of light from the sun, passing by the moon at L, and touching the top of the mountain at M; then the space between L and M appears dark. With a micrometer, measure LM, and compare it with LC; then knowing LC, we know LM, and by Euc. B. 1. P. 47. $CM=\sqrt{CL^2+LM^2}$ is known; from which subtract Cp, and we get the height pM of the mountain. But, as Dr. HERSCHEL observes in the *Phil. Trans.* 1781, this method will only do when the moon is in quadratures; he has therefore given the following general method. Let E be the earth, draw EMn, and Lo, perpendicular to the moon's radius RC, and Lr parallel to on, also ME' perpendicular to SM. Now ML would measure its full length when seen from the earth in quadratures at E', but seen from E, it only measures

the length of Lr. As the plane passing through SM, EM, is perpendicular to a line joining the cusps, the circle RLV may be conceived to be a section of the moon perpendicular to that line. Now it is manifest, that the angle SLo or LCR, is very nearly equal to the elongation of the moon from the sun; and the triangles LrM, LCo being similar, $Lo : LC :: Lr : LM = \frac{LC \times Lr}{Lo} = Lr$ divided by the sine of elongation, radius being unity. Hence we find Mp as before.

Ex. On June 1780, at 7 o'clock, Dr. HERSCHEL found the angle under which LM, or Lr, appeared to be 40″,625, for a mountain in the south east quadrant; and the sun's distance from the moon was 125°. 8′, whose sine is ,8104; hence, 40″,625 divided by ,8104 gives 50″,13, the angle under which LM would appear, if seen directly. Now the semidiameter of the moon was 16′. 2″,6, and taking its length to be 1090 miles, we have, 16′. 2″,6 : 50″,13 :: 1090 : LM= 56,73 miles; hence, Mp = 1,47 miles.

374. Dr. HERSCHEL found the height of a great many more mountains, and thinks he has good reason to believe, that their altitudes are greatly overrated; and that, a few excepted, they generally do not exceed half a mile. He observes, that it should be examined whether the mountain stands upon level ground, which is necessary that the measurement may be exact. A low tract of ground between the mountain and the sun will give it higher, and elevated places between will make it lower, than its true height above the common surface of the moon.

375. The line Lr was measured thus. 1. Set the immoveable hair of the micrometer parallel to AB, then moving the other hair parallel to it from L to r, it gives the measure under which Lr appears. Or 2. Observe some spot near to L, to which the line rL is directed; or take a view of the shadow of some neighbouring mountains; either of these will indicate a line perpendicular to a line joining the cusps, sufficiently near to set the micrometer by. The last method Dr. HERSCHEL thinks the best. But if the micrometer be furnished with an hair perpendicular to the moveable wire, and that hair be made to coincide with Lr, it at once gives the position of the micrometer.

376. On April 19, 1787, Dr. HERSCHEL discovered three volcanos in the dark part of the moon; two of them seemed to be almost extinct, but the third showed an actual eruption of fire, or luminous matter, resembling a small piece of burning charcoal covered by a very thin coat of white ashes; it had a degree of brightness about as strong as that with which such a coal would be seen to glow in faint day-light. The adjacent parts of the volcanic mountain seemed faintly illuminated by the irruption. A similar irruption appeared on May 4, 1783. *Phil. Trans.* 1787. On March 7, 1794, a few minutes before 8 o'clock, in the evening, Mr. WILKINS of Norwich, an eminent Architect, observed, with

the naked eye, a very bright spot upon the dark part of the moon; it was there when he first looked at the moon; the whole time he saw it, it was a fixed, steady light, except the moment before it disappeared, when its brightness increased; he conjectures that he saw it about 5 minutes. The same phænomenon was observed by Mr. T. STRETTON, in St. John's Square, Clerkenwell, London. *Phil. Trans.* 1794. On April 13, 1793, and on February 5, 1794, M. PIAZZI, Astronomer Royal at Palermo, observed a bright spot upon the dark part of the moon, near Aristarchus. Several other Astronomers have observed the same phenomenon. See the *Memoirs de Berlin*, for 1788.

377. It has been a doubt amongst Astronomers, whether the moon has any atmosphere; some suspecting that at an occultation of a fixed star by the moon, the star did not vanish instantly, but lost its light gradually; whilst others could never observe any such appearance. M. SCHROETER of Lilianthan, in the dutchy of Bremen, has endeavoured to establish the existence of an atmosphere, from the following observations. 1. He observed the moon when two days and a half old, in the evening soon after sun set, before the dark part was visible; and continued to observe it till it became visible. The two cusps appeared tapering in a very sharp, faint, prolongation, each exhibiting its farthest extremity faintly illuminated by the solar rays, before any part of the dark hemisphere was visible. Soon after, the whole dark limb appeared illuminated. This prolongation of the cusps beyond the semicircle, he thinks must arise from the refraction of the sun's rays by the moon's atmosphere. He computes also the height of the atmosphere, which refracts light enough into its dark hemisphere to produce a twilight, more luminous than the light reflected from the earth when the moon is about 32° from the new, to be 1356 Paris feet; and that the greatest height capable of refracting the solar rays is 5376 feet. 2. At an occultation of Jupiter's satellites, the third disappeared, after having been about 1″ or 2″ of time indistinct; the fourth became indiscernible near the limb; this was not observed of the other two. *Phil. Trans.* 1792. If there be no atmosphere in the moon, the Heavens, to a Lunarian, must always appear dark like night, and the stars be constantly visible; for it is owing to the reflection and refraction of the sun's light by the atmosphere, that the Heavens, in every part, appear bright in the day.

On the Phænomenon of the Harvest Moon.

378. The full moon which happens at, or nearest to, the autumnal equinox, is called the *Harvest* moon; and at that time, there is a less difference be-

tween the times of its rising on two successive nights, than at any other full moon in the year; and what we here propose, is to account for this phenomenon.

379. Let P be the north pole of the equator QAU, HAO the horizon, EAC the ecliptic, A the first point of Aries; then, in *north* latitudes, A is the ascending node of the ecliptic upon the equator, AC being the order of the signs, and AQ that of the apparent diurnal motion of the heavenly bodies. When Aries rises in north latitudes, the ecliptic makes the least angle with the horizon; and as the moon's orbit makes but a small angle with the ecliptic, let us first suppose EAC to represent the moon's orbit. Let A be the place of the moon at its rising on one night; now, in mean solar time, the earth makes one revolution in $23h.\ 56'.\ 4''$, and brings the same point A of the equator to the horizon again; but in that time, let the moon have moved in its orbit from A to c, and draw the parallel of declination $tcns$; then it is manifest, that $3'.\ 56''$ before the *same* hour the next night, the moon, in its diurnal motion, has to describe cn before it rises. Now cn is manifestly the least possible, when the angle CAn is the least, Ac being given. Hence it rises more nearly at the same hour, when its orbit makes the least angle with the horizon. Now at the autumnal equinox, when the sun is in the first point of Libra, the moon, at that time of its full, will be at the first point of Aries, and therefore it rises with the least difference of times, on two successive nights; and it being at the time of its full, it is more taken notice of; for the same thing happens every month when the moon comes to Aries.

Hitherto we have supposed the ecliptic to represent the moon's orbit, but as the orbit is inclined to it at an angle of $5°.\ 9'$ at a mean, let xAz represent the moon's orbit when the ascending node is at A, and Ar the arc described in a day; then the moon's orbit making the least possible angle with the horizon in that position of the nodes, the arc rn, and consequently the difference of the times of rising, will be the least possible. As the moon's nodes make a revolution in about 19 years, the least possible difference can only happen once in that time. In the latitude of London the least difference is about $17'$.

380. The ecliptic makes the greatest angle with the horizon when the first point of Libra rises, consequently when the moon is in that part of its orbit, the difference of the times of its rising will be the greatest; and if the descending node of its orbit be there at the same time, it will make the difference the greatest possible; and this difference is about $1h.\ 17'$ in the latitude of London. This is the case with the vernal full moons. Those signs which make the least angle with the horizon when they rise, make the greatest angle when they set, and vice versâ; hence, when the difference of the times of rising is the least, the difference of the times of setting is the greatest, and the contrary.

381. By increasing the latitude, the angle rAn, and consequently rn, is diminished; and when the time of describing rn, by the diurnal motion, is $3'. 56''$, the moon will then rise at the same solar hour. Let us suppose the latitude to be increased until the angle rAn vanishes, then the moon's orbit becomes coincident with the horizon, every day, for a moment of time, and consequently the moon rises at the same sidereal hour, or $3'. 56''$ sooner, by solar time. Now take a globe, and elevate the north pole to this latitude, and marking the moon's orbit in this position upon it, turn the globe about, and it will appear, that at the instant after the above coincidence, one half of the moon's orbit, corresponding to Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini, will rise; hence, when the moon is going through that part of its orbit, or for 13 or 14 days, it rises at the same sidereal hour. Now taking the angle $xAE = 5°. 9'$, and the angle $EAQ = 23°. 28'$, the angle QAx, or QAH when the moon's orbit coincides with the horizon, is $28°. 37'$; hence, the latitude QZ is $61°. 23'$ where these circumstances take place. If the descending node be at A, then QAx, or $QAH = 18°. 19'$, and the latitude is $71°. 41'$. In any other situation of the orbit, the latitude will be between these limits. When the angle QAx is greater than the complement of latitude, the moon will rise every day sooner by sidereal time. As there is a complete revolution of the nodes in about 18 years 8 months, all the varieties of the rising and setting of the moon must happen within that time.

On the Horizontal Moon.

382. The phænomenon of the horizontal moon is this, that it appears larger in the horizon than in the meridian; whereas, from its being nearer to us in the latter case than in the former, it subtends a greater angle. Gassendus thought that, as the moon was less bright in the horizon, we looked at it there with a greater pupil of the eye, and therefore it appeared larger. But this is contrary to the principles of Optics, the image of an object upon the retina not depending upon the pupil. This opinion was supported by a French *Abbé*, who supposed that the opening of the pupil made the chrystalline humour flatter, and the eye longer, and thereby increased the image. But there is no connection between the muscles of the iris and the other parts of the eye, to produce these effects. Des Cartes thought that the moon appeared largest in the horizon, because, when comparing its distance with the intermediate objects, it appeared then furthest off; and as we judge its distance greatest in that situation, we of course think it larger, supposing that it subtends the same angle. This opinion was supported by Dr. Wallis in the *Phil. Trans.* N°. 187. Dr. Berkley accounts for it thus. Faintness suggests the idea of greater distance; the moon appearing most faint in the horizon, suggests the idea of greater distance, and,

supposing the visual angle the same, that must suggest the idea of a greater tangible object. He does not suppose the *visible* extension to be greater, but that the idea of a greater *tangible* extension is suggested, by the alteration of the appearance of the visible extension. He says, 1. That which suggests the idea of greater magnitude, must be something perceived; for what is not perceived can produce no effect. 2. It must be something which is variable, because the moon does not always appear of the same magnitude in the horizon. 3. It cannot lie in the intermediate objects, they remaining the same; also, when these objects are excluded from sight, it makes no alteration. 4. It cannot be the visible magnitude, because that is least in the horizon; the cause therefore must lie in the visible appearance, which proceeds from the greater paucity of rays coming to the eye, producing *faintness*. Mr. ROWNING supposes, that the moon appears furthest from us in the horizon, because the portion of the sky which we see, appears, not an entire *hemisphere*, but only a portion of one; and in consequence of this, we judge the moon to be furthest from us in the horizon, and therefore to be then largest. Dr. SMITH, in his *Optics*, gives the same reason. He makes the apparent distance in the horizon to be to that in the zenith as 10 to 3, and therefore the apparent diameters in that ratio. The methods by which he estimated the apparent distances, may be seen in Vol. I. pag. 65. The same circumstance also takes place in the sun, which appears much larger in the horizon than in the zenith. Also, if we take two stars near each other in the horizon, and two other stars near the zenith at the same angular distance from each other, the two former will appear at a much greater distance from each other, than the two latter. Upon this account, people are, in general, very much deceived in estimating the altitudes of the heavenly bodies above the horizon, judging them to be much greater than they are. Dr. SMITH found, that when a body was about 23° above the horizon, it appeared to be half way between the zenith and horizon, and therefore at that real altitude it would be estimated to be 45° high. Upon the same principle, the lower part of a rainbow appears broader than the upper part. And this may be considered as an argument that the phænomenon cannot depend entirely upon the greater degree of faintness in the object when in the horizon, because the lower part of the bow frequently appears brighter than the upper part, at the same time that it appears larger; also, this cause could have no effect upon the distance of the stars; and as the difference of the apparent distance of two stars, whose angular distance is the same, in the horizon and zenith, seems to be fully sufficient to account for the apparent variation of the moon's diameter in these situations, it may be doubtful, whether the faintness of the object enters into any part of the cause.

CHAP. XIX.

ON THE ROTATION OF THE SUN, MOON AND PLANETS.

Art. 383. THE time of rotation of the sun, moon and planets, and the position of their axes, are determined from the spots which are observed upon their surfaces. The position of the same spot, observed at three different times, will give the position of the axis; for three points of any small circle will determine its situation, and hence we know the axis of the sphere which is perpendicular to it. The time of rotation may be found, either from observing the arc of the small circle described by a spot in any time, or by observing the return of a spot to the same position in respect to the earth.

On the Rotation of the Sun.

384. It is doubtful by whom the spots on the sun were first discovered. SCHEINER, professor of Mathematics at Ingolstadt, observed them in May, 1611, and published an account of them in 1612, in a Work entitled, *Rosa ursina*. He supposed them not to be spots upon the body of the sun, but that they were bodies of irregular figures revolving about the sun, very near to it. GALILEO, in the Preface to a Work entitled, *Istoria, Dimostrazioni, intorno alle Macchie Solari, Roma* 1613, says, that being at Rome in April 1611, he then showed the spots of the sun to several persons, and that he had spoken of them, some months before, to his friends at Florence. He imagined them to adhere to the sun. KEPLER, in his Ephemeris, says, that they were observed by the son of DAVID FABRICIUS, who published an account of them in 1611. In the papers of HARRIOT, not yet printed, it is said, that spots upon the sun were observed on December 8, 1610. As telescopes were in use at that time, it is probable that each might make the discovery. Admitting these spots to adhere to the sun's body, the reasons for which we shall afterwards give, we proceed to show, how the position of the axis of the sun, and the time of its rotation, may be found.

385. To determine the position of a spot upon the sun's surface, find, by the method given in my *Practical Astronomy*, Art. 125, the difference between the right ascensions and declinations of the spot and sun's center; from which, find the latitude of the spot, and the difference between its longitude and that of the sun's center; this may be done thus. Let ♈ Q be the ecliptic, ♈ C the equator, AB the sun, S the center of its disc, v a spot on its surface; draw

St parallel to ♈C, and Sb, xva secondaries to ♈C, and vr perpendicular to ♈Q; then ab is the observed difference of the right ascensions of the spot and the sun's center, and vx the difference of their declinations. By Art. 13. $ab \times \cos. Sb = Sx$; hence, in the right angled triangle vSx, we know Sx and vx, to find vS, and the angle vSx; also in the right angled triangle ♈Sb, we know ♈b the sun's right ascension, and bS its declination, to find the angle ♈Sb, the difference between which and the right angle bSx gives ♈Sx, and as vSx is known, we get vSr; hence, in the right angled triangle vSr, we know vS and the angle vSr, to find vr the latitude of the spot, and rS the difference of longitudes of the spot and sun's center. Reduce its geocentric latitude and longitude to the heliocentric latitude and longitude. To do which, let $EACD$ be the projection of the sun's disc, ESC the ecliptic, S the center of the disc, M a spot on the surface; draw ML perpendicular to EC, and ML, LS, are the observed geocentric latitude of the spot, and difference of longitudes between that and the sun's center; hence we know SM, which is the projection of the arc of a great circle between the point S on the sun's surface to which the earth is vertical and the spot, into its sine. To find this arc, let E be the earth, Ea a tangent to the sun, and draw ab perpendicular to EeS; then the angle Sab being equal to SEa, the apparent semidiameter of the sun, the arc ae is the complement of the sun's semidiameter. Hence, if d be a spot upon the sun, and dc be perpendicular to Se: then, *as* ba *the observed semidiameter, the sine of the arc* ae :: *the observed angle under which* dc *appears* : *the sine of the arc* dc. Thus we find the arc corresponding to SM, or the angular distance of the spot upon the sun's surface from the middle of the sun's disc. Now the angle MSL in the projection, is equal to that upon the surface of the sun formed by the great circles; compute therefore this angle from the right angled plain triangle MLS. Let p be the pole of the ecliptic upon the surface of the sun. Then the angle pSL being a right angle, we know the angle pSM on the sun's surface, together with SM and Sp, Sp being $= 90°$; hence we find pM, and the angle MpS. Now as S is a point on the sun's disc, to which the earth is vertical, S seen from the sun's centre has the same longitude as the earth, and is therefore known; hence, if to that we add, or from it subtract, MpS, according as L is to the east or west of S, we get the longitude of M seen from the sun's center; and the difference of PM and Pv, or vM, is the heliocentric latitude of M.

386. To determine the pole P of the sun's equator $QnRN$, let ab be the path described by a spot, and M, N, O, three observed positions of that spot, the apparent motion of which is from east to west, the sun revolving about its axis according to the order of the signs; then (385) we know Mp, Np, Op, and the angles MpN, NpO; for as we know the angles which Mp, Np, Op make with pS, the angles between these circles will be known, which is the

difference of their longitudes. Join the points M, N, O, by three great circles, dotted in the Figure; then in each of the triangles MpN, MpO, NpO, we know two sides and the included angle at p, to find the arcs of the great circles MN, MO, NO, denoted by the dotted lines. Now to find the arcs of the small circle ab corresponding, take the sines of half the arcs of the great circles, and the double will be the chords. Let $aMNOb$ be the small circle, C its center, produce MC to V, and join OV; then knowing the chords MN, NO, MO, we know the angle ONM, the supplement of which is the angle OVM, the double of which is the angle OCM at the center, or the arc ONM of the small circle. Let OvM be an arc of the great circle passing through OM, whose radius OD is equal to the radius of the sphere; draw DCw, which must be perpendicular to OM; then the angle OCw shows the degrees contained in half the arc ONM of the small circle, and the angle ODw, half the degrees in the great circle OvM; and sin. OCw, or OCD, : sin. ODC :: OD : OC :: the radius of the sphere : radius of the small circle parallel to the solar equator, :: radius : cos. of the distance of the small circle ab from the solar equator; hence, the distance of this small circle from the pole P is known. Therefore in the triangle POM, we know all the sides, to find the angle PMO; and in the triangle pMO, we know all the sides, to find the angle pMO; hence we know the angle PMp, together with PM, pM; therefore we can find Pp, which measures the inclination of the sun's axis to the ecliptic.

387. Let N be the ascending node of the sun's equator; to find the situation of which from the sun's center, produce pP to t, then Pt passing through the poles of the ecliptic and equator must cut each 90° from the node N, therefore $Nt = 90$. Now to find the position of t, find, in the triangle OPp, the angle at p, which measures the arc te; find (385) also the angle OpS, or the arc eS; hence we know tS; but the longitude of S seen from the sun's center, is opposite to the sun's place in the ecliptic; find this therefore at the time of the observation at O, and we get the longitude of t, consequently we get the place N of the node. The best time to determine the place of the node and the inclination of the equator, is about the beginning of June and December, because at those times the earth being in the plane of the equator, the path of the spot is most inclined to the ecliptic, and its latitude changes the fastest.

388. To find the time of the sun's rotation, we have given the degrees of the arc MNO, and the time the spot is moving from M to O; hence, the arc MO : 360° :: the time of describing MO : the time of a revolution.

389. But there is a shorter and more elegant method of determining the place of the node and inclination of the axis, given by M. Cagnoli, in his Trigonometry, from the variation of a triangle when two of its sides remain con-

stant, and the third side varies by any finite quantity; this is the case with the triangles PpO, PpN, PpM, where Pp is constant, and $PO = PN = PM$. Now taking any two of these triangles, PpM, PpN, he proves that, sin. $\frac{1}{2} \times \overline{pN - pM}$: tan. $\frac{1}{2}$ MpN :: sin. $\frac{1}{2} \times \overline{Np + pM}$: cot. $\frac{1}{2} \times \overline{PMp + PNp}$, where all the terms are known, except the last, which therefore is known; in like manner, from the triangles PpN, PpO, we get $PNp + POp$; therefore if we put L', L'', L''' for the observed longitudes of the spot at M, N, O, and D', D'', D''' for PM, PN, PO; also, $a = \frac{1}{2} \times \overline{PMp + PNp}$, $b = \frac{1}{2} \times \overline{PMp + POp}$, $c = \frac{1}{2} \times \overline{PNp + POp}$; then

$$\text{tan. } a = \frac{\text{sin. } \frac{1}{2} \times \overline{D'' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L'' - L'}}{\text{sin. } \frac{1}{2} \times \overline{D'' + D'}}$$

$$\text{tan. } b = \frac{\text{sin. } \frac{1}{2} \times \overline{D''' - D'} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L'}}{\text{sin. } \frac{1}{2} \times \overline{D''' + D'}}$$

$$\text{tan. } c = \frac{\text{sin. } \frac{1}{2} \times \overline{D''' - D''} \times \text{cot. } \frac{1}{2} \times \overline{L''' - L''}}{\text{sin. } \frac{1}{2} \times \overline{D''' + D''}}$$

Also, tan. $\frac{1}{2}$ OpN : tan. $\frac{1}{2} \times \overline{PNp - POp}$ :: tan. $\overline{PpO + \frac{1}{2}\, OpN}$: tan. $\frac{1}{2} \times \overline{PNp + POp}$, where all the terms are known, except the third, of which one part OpN is $L''' - L''$; hence, tan. $\overline{PpO + \frac{1}{2} \times \overline{L''' - L''}} =$ tan. $\frac{1}{2} \times \overline{L''' - L''} \times$ tan. $c \times$ cot. $\overline{a - b}$, which put $=$ tan. x, and we have

$$PpO + \tfrac{1}{2} \times \overline{L''' - L''} = x$$

Add $\quad \frac{1}{2} \times \overline{L''' + L''} = \frac{1}{2} \times \overline{L''' + L''}$

$\therefore PpO + L''' = x + \frac{1}{2} \times \overline{L''' + L''}$ the longitude of the pole P of the sun, or of t, to which add 90°, and we get the longitude of the node N.

Now to find Pp, put $P =$ the longitude of the pole P, then $PpO = P - L''' = s$, and $POp = b + c - a = d$; consequently the tangent of half the difference of PM and Pp is $\dfrac{\text{tan. } \frac{1}{2}\, D''' \times \text{sin } \frac{1}{2}.\ \overline{s - d}}{\text{sin. } \frac{1}{2}.\ \overline{s + d}} =$ tan. y, and the tangent of half the sum is $\dfrac{\text{tan. } \frac{1}{2}\, D''' \times \text{cos. } \frac{1}{2}.\ \overline{s - d}}{\text{cos. } \frac{1}{2}.\ \overline{s + d}} =$ tan. z; hence, $z + y = PM$ (PM being greater than Pp) and $z \sim y = Pp$ the inclination of the solar equator to the ecliptic. If s be greater than 180°, take $360° - s$ for s; and the same for d. But if d be less than 90°, then $Pp = 180° - \overline{z + y}$, and $PM = 180° - \overline{z \sim y}$.

Ex. According to the observations of M. de la Lande, the three longitudes

of a spot seen from the center of the sun, and its distance from the pole to the ecliptic in 1775, were as follows;

	Longitude		Distance from the Pole
June 14,	$7^s.\ 8^\circ.\ 34'.\ 21'' = L'$	. . . . .	$90^\circ.\ 38'.\ 6'' = D'$
18,	$9.\ 5.\ 48.\ 51 = L''$	. . . . .	$97.\ 30.\ 8 = D''$
21,	$10.\ 19.\ 0.\ 14 = L'''$	. . .	$101.\ 35.\ 16 = D'''$

$$\text{Hence, tan. } a = \frac{\text{sin. } \tfrac{1}{2} \times \overline{D'' - D'} \times \text{cot. } \tfrac{1}{2} \times \overline{L'' - L'}}{\text{sin. } \tfrac{1}{2} \times \overline{D'' + D'}} = 6^\circ.\ 16'.\ 45''$$

$$\text{tan. } b = \frac{\text{sin. } \tfrac{1}{2} \times \overline{D''' - D'} \times \text{cot. } \tfrac{1}{2} \times \overline{L''' - L'}}{\text{sin. } \tfrac{1}{2} \times \overline{D''' + D'}} = 4^\circ.\ 34'.\ 10''$$

$$\text{tan. } c = \frac{\text{sin. } \tfrac{1}{2} \times \overline{D''' - D''} \times \text{cot. } \tfrac{1}{2} \times \overline{L''' - L''}}{\text{sin. } \tfrac{1}{2} \times \overline{D''' + D''}} = 5^\circ.\ 13'.\ 2''$$

Hence, $\text{tan. } \tfrac{1}{2} \times \overline{L''' - L''} \times \text{tan. } c \times \text{cot. } \overline{a - b} = \text{tan.}$ of $50^\circ.\ 26'.\ 50'' = x$; consequently $= PpO + L''' = x + \tfrac{1}{2} \times \overline{L''' + L''} = 11^s.\ 17^\circ.\ 51'.\ 20''$ the longitude P of the pole of the sun; hence, the longitude of the node N is $2^s.\ 17^\circ.\ 51'.\ 20''$.

Now $P - L''' = 28^\circ.\ 51'.\ 6''$, $b + c - a = 3^\circ.\ 30'.\ 27''$; hence,

$$\text{tan. } \tfrac{1}{2} \times \overline{PM \backsim Pp} = \text{tan. } y = \frac{\text{tan. } \tfrac{1}{2}\, D''' \times \text{sin. } \tfrac{1}{2} \times \overline{s - d}}{\text{sin. } \tfrac{1}{2} \times \overline{s + d}} = 43^\circ.\ 59'.\ 0''$$

$$\text{tan. } \tfrac{1}{2} \times \overline{PM + Pp} = \text{tan. } z = \frac{\text{tan. } \tfrac{1}{2}\, D''' \times \text{cos. } \tfrac{1}{2}\ \overline{s - d}}{\text{cos. } \tfrac{1}{2}.\ \overline{s + d}} = 51^\circ.\ 14'.\ 10''$$

Hence, (PM being greater than Pp,) we have $PM = z + y = 95^\circ.\ 13'.\ 10''$ the distance of the spot from the north pole of the sun; and $Pp = z - y = 7^\circ.\ 15'.\ 10''$ the inclination of the solar equator to the ecliptic.

390. M. Cassini, from his own observations, makes the inclination of the sun's axis $7\tfrac{1}{2}^\circ$, calling the inclination the distance from the perpendicular to the ecliptic; and the place of the node $2^s.\ 8^\circ$. Le P. Scheiner supposes the inclination to be 7°. M. de l'Isle found it $6^\circ.\ 35'$, from one set only of observations. The place of the node was determined by M. Cassini the Son, to be $2^s.\ 10^\circ$. M. de l'Isle found it $1^s.\ 26^\circ$. Le P. Scheiner, in 1626, fixed it at $2^s.\ 10^\circ$. From the difficulty of determining the exact position of the spots, the place of the node and inclination, more particularly the former, are subject to considerable errors, and accuracy can only be depended upon, from the mean of a great number of observations. It does not appear that the place of the node, and the inclination, are subject to any change.

391. M. de la Lande has given the following method of correcting the place of the node, and the inclination of the equator. He supposes the place of the node, and the inclination to be nearly known; and from three observed latitudes and longitudes of a spot, he computes its declination, which ought to

be the same in each case, if the above quantities be rightly assumed; if the declinations come out different, he changes the assumed place of the node and inclination, according to the errors, until the declination comes out the same for each observation, and then concludes the quantities to be rightly assumed, so far as the observations are true. For example; He assumes the place of the node n 8ˢ. 17°, and inclination 7°. 30′. Now in 1775, he found by observation on June 14, the latitude of a spot 0°. 38′ south, longitude 7ˢ. 8°. 34′; on June 18, the latitude 7°. 30′, and longitude 9ˢ. 5°. 49′; and on June 21, the latitude 11°. 35′; and longitude 10ˢ. 19°; hence (393) the corresponding declinations by calculation are 5°. 17′, 5°. 2′ and 4°. 57′. By making the inclination 7°. 20′, the first and last declinations become 5°. 11′ and 5°. 6′; therefore by diminishing the inclination 10′, the declinations of the spot at the first and last observations are brought nearer by 15′; hence, 15′ : 10′ :: 5′ (the difference of 5°. 11′ and 5°. 6′) : 3′, which subtracted from 7°. 20′ gives 7°. 17′ for the inclination, which will give the first and last declination 5°. 9′. With this inclination 7°. 17′, the second observed place gives 5°. 6′ for the declination, differing 3′ for the two other. His second hypothesis is to change the place of the node in order to make the declinations at the first and third observations agree; he therefore supposes the place of the node to be 8ˢ. 22°. And by going through the calculations as before, he finds, that an inclination of 7°. 10′ will give 5°. 33′ for the declination at the first and third observations, and 5°. 47′ at the second, differing 14′. Hence he arranges the two hypotheses thus.

Node	Inclination.	Decl. on June 14 and 21	Declination on June 18	Difference of Declnˢ.
8ˢ. 17°. 0′	7°. 17′	5°. 9′	5°. 6′	3′ less
8. 22. 0	7. 10	5. 33	5. 47	14 more
Diff. 5. 0	0. 7	0. 24	0. 41	17 diff.

Here a change of 5° of the node and 7′ in the inclination has made a difference of 17′ in the sum of the errors. Hence, to alter the place of the node and inclination to make both the differences 3′ and 14′ vanish, say, 17′ : 5° :: 3′ : 53′, which added to 8ˢ. 17° gives 8ˢ. 17°. 53′; also, 17′ : 7′ :: 3′ : 1′, subtract therefore 1′ from 7°. 17′ and it gives 7°. 16′ for the corresponding inclination. Lastly, to find the corresponding declinations, say, 17′ : 24′ :: 3′ : 4′, add this 4′ to 5°. 9′ and it gives 5°. 13′ for the declination on June 14 and 21; and 17′ : 41′ :: 3′ : 7′, add this 7′ to the declination 5°. 6′ on June 18, and it gives 5°.

13′ for the declination at that time. Hence, the place of the node 8^s. 17°. 53′, and inclination 7°. 16′, give 5°. 13′ for the declination of the spot at the three observations, and therefore we may conclude the place of the node and inclination to be truly ascertained, as near as the observations can give it. It will be always proper to go through with all the calculations again, after you have thus deduced the place of the node and inclination, and see whether they give the declinations the same at each observation; if not, another correction must be made in the same manner; but this will not be found necessary, unless you have considerably altered the place of the node and inclination; in which case, the approximations may not be sufficiently exact; and after all, the small errors which the observations must be subject to, renders it unnecessary to seek for a nearer agreement in the declinations than 3′ or 4′. This may be considered as a correction of the place of the node and inclination, as determined nearly by any other method.

392. When the earth is in the nodes of the sun's equator, it being then in its plane, the spots appear to describe straight lines; this happens about the beginning of June and December. As the earth recedes from the nodes, the path of a spot grows more and more elliptical, till the earth gets 90° from the nodes, which happens about the beginning of September and March, at which time the ellipse has its minor axis the greatest, and is then to the major axis, as the sine of the inclination of the solar equator to radius.

393. To find the right ascension nv of the spot at O from the descending node n, and the declination Ov, we have, in the right angled triangle neO, ne the difference of the longitudes of n and O, with eO the latitude of O, to find On, and the angle One; and as we know vne, we shall know vnO; hence, in the right angled triangle Ovn, we know nO and the angle Onv, to find vn the right ascension of O measured from the node n, and Ov its declination.

394. If the latitude, longitude and declination of a spot be known, we may find its right ascension thus. By spher. trig. rad. $\times$ cos. $nO =$ cos. $ne \times$ cos. Oe, and rad. $\times$ cos. $nO =$ cos. $nv \times$ cos. Ov; hence, cos. $ne \times$ cos. $Oe =$ cos. $nv \times$ cos. Ov, consequently the cos. of right ascen. $nv = \frac{\text{cos. } ne \times \text{cos. } Oe}{\text{cos. } Ov} = \frac{\text{cos. dist. from node} \times \text{cos. hel. lat.}}{\text{cos. hel. dec.}}$. If we therefore calculate the right ascension of the same spot at two different times, we get its motion in right ascension in the interval of these times; hence, that motion : 360° :: the interval of the times : the time of the rotation of the sun in respect to the nodes, or, as it does not appear that the node has any sensible motion, it gives the true time of rotation. Or the time may be determined by the return of a spot to the same declination or right ascension. Thus M. de la LANDE has found the time of rotation to be 25*d.* 10*h.* and the return of the spots to the same situation, to be

$27d.\ 7h.\ 37'.\ 28''$. M. CASSINI determined the time of rotation, from observing the time in which a spot returns to the same situation upon the disc, or to the circle of latitude passing through the earth. Let t be that interval of time, and let m be equal to the *true* motion of the earth in that time, and n equal to its *mean* motion; then $360° + m : 360° + n :: t$: the time of return if the motion had been uniform, and this, from a great number of observations, he determines to be $27d.\ 12h.\ 20'$; now the mean motion of the earth in that time is $27°.\ 7'.\ 8''$; hence, $360° + 27°.\ 7'.\ 8'' : 360° :: 27d.\ 12h.\ 20' : 25d.\ 14h.\ 8'$ the time of rotation. *Elem. d'Astron.* pag. 104. But this method is not capable of so much accuracy as the other.

395. There has been a great difference of opinions respecting the nature of the solar spots. SCHEINER supposed them to be solid bodies revolving about the sun, very near to it; but as they are as long visible as they are invisible, this cannot be the case. Moreover, we have a physical argument against this hypothesis, which is, that most of them do not revolve about the sun in a plane passing through its center, which they necessarily must, if they revolved, like the planets, about the sun. GALILEO confuted SCHEINER'S opinion, by observing that the spots were not permanent; that they varied their figure; that they increased and decreased, and sometimes disappeared. He compared them to smoke and clouds. HEVELIUS appears to have been of the same opinion; for in his *Cometographia*, page 360, speaking of the solar spots, he says, *hæc materia nunc ea ipsa est evaporatio et exhalatio (quia aliunde minime oriri potest) quæ ex ipso corpore solis, ut supra ostensum est, expiratur et exhalatur.* But the permanency of most of the spots is an argument against this hypothesis. M. de la HIRE supposed them to be solid, opaque bodies, which swim upon the liquid matter of the sun, and which are sometimes entirely immersed. M. de la LANDE supposes that the sun is an opaque body, covered with a liquid fire, and that the spots arise from the opaque parts, like rocks, which, by the alternate flux and reflux of the liquid igneous matter of the sun, are sometimes raised above the surface. The spots are frequently dark in the middle, with an umbra about them; and M. de la LANDE supposes that that part of the rock which stands above the surface forms the dark part in the center, and those parts which are but just covered by the igneous matter form the umbra. Dr. WILSON, Professor of Astronomy at Glasgow, opposes this hypothesis of M. de la LANDE, by this argument. Generally speaking, the umbra immediately contiguous to the dark central part, or nucleus, instead of being very dark, as it ought to be, from our seeing the immersed parts of the opaque rock through a thin stratum of the igneous matter, is, on the contrary, very nearly of the same splendour as the external surface, and the umbra grows darker the further it recedes from the nucleus; this, it must be acknowledged, is a strong argument against the hypothesis of M. de la LANDE. Dr. WILSON further observes,

that M. de la LANDE produces no optical arguments in support of the rock standing above the surface of the sun. The opinion of Dr. WILSON is, that the spots are excavations in the luminous matter of the sun, the bottom of which forms the umbra. They who wish to see the arguments by which this is supported, must consult the *Phil. Trans.* 1774 and 1783. Dr. HALLEY conjectured that the spots are formed in the atmosphere of the sun. Dr. HERSCHEL supposes the sun to be an opaque body, and that it has an atmosphere; and if some of the fluids which enter into its composition should be of a shining brilliancy, whilst others are merely transparent, any temporary cause which may remove the lucid fluid will permit us to see the body of the sun through the transparent ones. See the *Phil. Trans.* 1795. Dr. HERSCHEL on April 19, 1779, saw a spot which measured 1′. 8″,06 in diameter, which is equal in length to more than 31 thousand miles; this was visible to the naked eye. Besides the dark spots upon the sun, there are also parts of the sun, called *Faculæ*, *Luculi*, &c. which are brighter than the general surface; these always abound most in the neighbourhood of the spots themselves, or where spots recently have been. Most of the spots appear within the compass of a zone lying 30° on each side of the equator; but on July 5, 1780, M. de la LANDE observed a spot 40° from the equator. Spots which have disappeared have been observed to break out again. The spots appear so frequently, that Astronomers very seldom examine the sun with their telescopes, but they see some; SCHEINER saw 50 at once. The following phænomena of the spots are described by SCHEINER and HEVELIUS.

I. Every spot which hath a nucleus, hath also an umbra surrounding it.

II. The boundary between the nucleus and umbra is always well defined.

III. The increase of a spot is gradual, the breadth of the nucleus and umbra dilating at the same time.

IV. The decrease of a spot is gradual, the breadth of the nucleus and umbra contracting at the same time.

V. The exterior boundary of the umbra never consists of sharp angles, but is always curvilinear, however irregular the outline of the nucleus may be.

VI. The nucleus, when on the decrease, in many cases changes its figure, by the umbra encroaching irregularly upon it.

VII. It often happens, by these encroachments, that the nucleus is divided into two or more nuclei.

VIII. The nucleus vanishes sooner than the umbra.

IX. Small umbræ are frequently seen without nuclei.

X. An umbra of any considerable size is seldom seen without a nucleus.

XI. When a spot, consisting of a nucleus and umbra, is about to disappear, if it be not succeeded by a facula, or more fulgid appearance, the place it oc-

cupied, is, soon after, not distinguishable from any other part of the sun's surface.

On the Rotation of the Moon.

396. The latitude and longitude of some one spot, as seen from the moon's center, must be determined (385) as for the sun; but (referring to Fig. 89.) pS is not, as for the sun, equal to 90°, but it is the moon's distance from the pole of the ecliptic, for the pole of the ecliptic will not lie in the circumference of the moon's disc, as in the case of the sun, except when the moon is in the ecliptic; for as the moon leaves the ecliptic, it is manifest that the pole of the ecliptic will approach upon the disc, or recede behind the moon, by a quantity equal to the moon's latitude; at the time of observation therefore, pS will be known, by knowing the moon's latitude; also SM and the angle pSM are determined as for the sun; hence we can compute pM the distance of the spot from the north pole of the ecliptic, and the angle SpM the difference between the longitude of the spot and that of the earth seen from the moon, therefore the longitude of the earth being known, the longitude of the spot seen from the moon's center will be known. We thus find the latitude and longitude of a spot at three different times, seen from the center of the moon, in respect to the ecliptic, or to a circle drawn through the center of the moon parallel to the ecliptic; and with these three observations, we can determine the situation of the lunar equator, in the same manner as for the sun; but MAYER has given another method by approximation, by which he can employ more observations for one operation, and thereby increase the accuracy of the conclusion. Those spots near the center are the best for this purpose, because their change is most sensible; MAYER has therefore chosen that called Manilius, the observations upon which are contained in the following Table.

True Time at Nuremberg.	$SM=$	$pSM=$	Apparent long. of ☾	Appar. lat. of ☾	$pM=$	$SpM=$	Longitude of Manilius at ☾'s center
1748. D. H. M.	° ′	° ′	s ° ′	° ′	° ′	° ′	s ° ′
April 11. 11. 1	17. 20	58. 11	6. 0. 35	4. 16 s.	76. 50	15. 4	0. 15. 39
—— 13. 9. 30	15. 8	58. 35	6. 27. 24	5. 27 -	76. 52	13. 14	1. 10. 39
May 11. 10. 56	15. 29	60. 40	7. 6. 19	5. 51 -	76. 48	13. 50	1. 20. 9
—— 16. 16. 11	13. 26	28. 45	9. 22. 14	2. 31 -	75. 45	6. 38	3. 28. 52
—— 17. 15. 56	14. 23	20. 49	10. 6. 33	1. 17 -	75. 18	5. 14	4. 11. 37
June 5. 9. 58	18. 2	62. 16	6. 2. 53	4. 56 -	76. 59	16. 20	0. 19. 13
—— 13. 14. 0	14. 18	25. 48	10. 0. 24	1. 41 -	75. 29	6. 23	4. 6. 47
—— 14. 12. 50	15. 12	16. 47	10. 14. 43	0. 25 -	75. 3	4. 30	4. 19. 13
July 2. 9. 23	18. 2	61. 56	5. 28. 25	4. 54 -	76. 55	16. 17	0. 14. 42
—— 4. 6. 49	17. 36	64. 29	6. 23. 11	5. 48 -	76. 57	16. 16	1. 9. 27
—— 5. 8. 4	17. 23	64. 49	7. 7. 18	6. 8 -	76. 48	16. 7	1. 23. 25
—— 6. 8. 34	16. 20	62. 37	7. 21. 34	5. 57 -	76. 49	14. 52	2. 6. 26
—— 7. 9. 4	15. 43	58. 10	8. 6. 15	5. 30 -	76. 26	13. 42	2. 19. 57
—— 8. 10. 4	15. 8	52. 0	8. 21. 33	4. 44 -	76. 7	12. 14	3. 3. 47
—— 9. 11. 15	14. 38	44. 26	9. 7. 12	3. 38 -	76. 2	10. 30	3. 17. 42
—— 10. 12. 5	14. 34	34. 40	9. 22. 50	2. 19 -	75. 46	8. 29	4. 1. 19
—— 11. 13. 15	15. 23	23. 24	10. 8. 37	0. 51 -	75. 4	6. 16	4. 14. 53
—— 12. 13. 5	16. 0	16. 57	10. 23. 34	0. 30 N.	75. 13	4. 46	4. 28. 20
—— 15. 13. 35	19. 38	2. 14	0. 6. 37	3. 41 -	74. 4	0. 47	6. 7. 24
Aug. 3. 7. 5	16. 10	60. 27	7. 29. 58	5. 46 s.	76. 31	14. 25	2. 14. 23
—— 14. 11. 34	20. 23	4. 16	1. 11. 2	4. 25 N.	74. 5	1. 33	7. 12. 35
Nov. 1. 5. 44	19. 27	15. 33	11. 24. 42	3. 4 -	74. 21	5. 19	6. 0. 1
—— 2. 6. 29	20. 26	11. 50	0. 9. 1	3. 46 -	73. 51	4. 16	6. 13. 17
Dec. 27. 4. 47	20. 54	7. 19	0. 14. 44	4. 21 -	73. 36	2. 43	6. 17. 27
1749.	* *	* *	* * *	* *	* *	* *	* * *
Jan. 28. 3. 59	8. 56	9. 59	2. 16. 0	3. 0 -	74. 22	3. 21	8. 19. 21
Feb. 25. 11. 43	17. 30	14. 53	2. 27. 53	2. 0 -	75. 6	4. 35	9. 2. 28
March 4. 11. 42	14. 46	54. 26	5. 22. 9	4. 42 s.	76. 53	12. 17	0. 4. 26

397. Let QDV represent the face of the moon next to the earth; C the center of the moon's disc; QNX the lunar equator, P its pole; DNW the ecliptic referred to the moon's surface, or rather a circle passing through its center parallel to the ecliptic, and which extended to the heavens may be considered as coinciding with it, p its pole, which is not, as in the sun, in the outward circle QDV; M Manilius, through which draw the great circles pMB, PML; and let ♈ be the first point of Aries seen from the moon's center; then MB is the latitude of Manilius, which is a variable quantity, and known from observation, and therefore we know pM its complement; Pp is the distance of the two poles, or the inclination of the lunar equator to the ecliptic;

ML is the *declination** of Manilius; and ♈N is the longitude of the node N of the lunar equator. Now when p falls between P and M, Mp is the least; and when p is opposite to that situation, Mp is the greatest; and half the difference gives Pp the distance of the poles, or the inclination of the lunar equator to the ecliptic. But as Mp is the complement of latitude of M, it is manifest that the above mentioned half difference is half the difference of the greatest and least complements of latitudes of M. Now by inspection in the Table, the greatest observed value of pM is 76°. 59', and the least value is 73°. 36', half the difference of which is 1°. 41',5, which is nearly the value of Pp, and would be accurately so, if we could be sure that the above values of pM were the greatest and least possible. Also, (369) the node N of the lunar equator coincides, or nearly so, with the node of the lunar orbit. Put $a=Pp$, $b=LM$, $g=$ ♈B, $h=pM$, $t=$ the distance of the node N of the lunar equator from the node of its orbit, $k=$ the longitude of the ascending node of the orbit; then $k+t=$ ♈N the longitude of the node of the lunar equator; hence, $g-k-t=NB$, or the angle NpB, and therefore $MpP=90°-g+k+t$, because the great circle passing through the poles of any two great circles must be 90° from their intersection. Now in the triangle MPp, (Trig. Art. 243.) cos. $PM=$ cos. $Pp\times$ cos. $pM+$ sin. $Pp\times$ sin. $pM\times$ cos. PpM, that is, cos. $\overline{90°-b}=$ cos. $a\times$ cos. $h+$ sin. $a\times$ sin. $h\times$ cos. $\overline{90°-g+k+t}$, or sin. $b=$ cos. $a\times$ cos. $h+$ sin. $a\times$ sin. $h\times$ sin. $\overline{g-k-t}$. Now by plain trig. sin. $\overline{g-k-t}=$ sin. $\overline{g-k}\times$ cos. $t-$ sin. $t\times$ cos. $\overline{g-k}$; but as t is very small, we may assume the cos. $t=1$; and as a is also very small, cos. $a=1$; hence, by substitution and transposition, sin. $b-$ cos. $h=$ sin. $a\times$ sin. $h\times$ sin. $\overline{g-k}-$ sin. $a\times$ sin. $h\times$ sin. $t\times$ cos. $\overline{g-k}$. But as Pp is very small, $b=90°-h+x$, where x must be very small, it never being more than Pp; hence, sin. $b=$ cos. $\overline{h-x}=$ cos. $h\times$ cos. $x+$ sin. $h\times$ sin. $x=$ (as cos. $x=1$ very nearly, and sin. $x=x$) cos. $h+x\times$ sin. h, therefore sin. $b-$ cos. $h=x\times$ sin. $h=b-\overline{90°-h}\times$ sin. h. Substitute this quantity for sin. $b-$ cos. h in the above equation, divide by sin. h, and for sin. a substitute a, and we have $b-\overline{90°-h}=a\times$ sin. $\overline{g-k}-a\times$ sin. $t\times$ cos. $\overline{g-k}$. Now the quantities g, h, k are known from observation, to find a, b, t; to do which, we must form three equations from three different values of g, h and k, from whence we can find a, b, t.

For this purpose, MAYER has taken the observations on July 2, 10, and 15, in the Tables; hence,

* Writers upon this subject call this the *Lunar Latitude*, but this makes a confusion of terms; I have chosen to call it *Declination*, it being the distance of the spot from the lunar equator.

Times of observation	July 2, at $9h.\ 23'$	July 10, at $12h.\ 5'$	July 15, at $13h.\ 35'$
$g=$	$0^s.\ 14^\circ.\ 42'$	$4^s.\ 1^\circ.\ 19'$	$6^s.\ 7^\circ.\ 24'$
$h=$	0. 76. 55	0. 75. 46	0. 74. 4
$k=$	10. 9. 14	10. 8. 48	10. 8. 32
sin. $\overline{g-k}$	+0,9097	+0,1302	−0,8560
cos. $\overline{g-k}$	+0,4152	−0,9915	−0,5170

These values substituted into the above equations give,

$$b-13^\circ.\ 5' = +0{,}9097a - 0{,}4152a \times \sin.\ t.$$
$$b-14^\circ.\ 14' = +0{,}1302a + 0{,}9915a \times \sin.\ t.$$
$$b-15^\circ.\ 56' = -0{,}8560a + 0{,}5170a \times \sin.\ t.$$

Subtract the first from the second, and the second from the third, and

$$-171' = -1{,}7657a + 0{,}9322a \times \sin.\ t.$$
$$-102' = -0{,}9862a - 0{,}4745a \times \sin.\ t.$$

Divide the first by 0,9322, and the second by $-0{,}4745$, and

$$-183',44 = -1{,}8941a + a \times \sin.\ t.$$
$$214',47 = +2{,}0784a + a \times \sin.\ t.$$

Subtract the first from the second, and we get $397{,}91 = 3{,}9725a$; hence, $a = 100' = 1^\circ.\ 40'$; substitute this value of a into one of the other two equations, and we get $t = 3^\circ.\ 36'$; and these values of a and t substituted into one of the three first equations, give $b = 14^\circ.\ 33'$ the declination of Manilius. This value of t shows that the node of the lunar equator does not sensibly differ from the place of the node of the lunar orbit. This determination also gives the inclination of the moon's axis to the ecliptic $= 1^\circ.\ 40'$. Produce pP to meet the ecliptic and moon's equator in r and s; then $rs = 1^\circ.\ 40'$. Now the ascending node of the moon's orbit, and the descending node of its equator, are those which go together. Let therefore Nv be the situation of the moon's orbit in respect to the ecliptic Nr, then $vr = 5^\circ.\ 9'$ at the mean inclination of the lunar orbit; and as $rs = 1^\circ.\ 40'$, we have vs, or the angle vNs, equal to $6^\circ.\ 49'$ the inclination of the axis of the moon to the plane of its orbit. To have all the accuracy possible, the three latitudes observed should be very different, and NB about 90°; if two of the observations be towards the extreme latitudes, and the other near the node, the inclination will be determined with the greatest accuracy; and if two be near the node, and one near the greatest latitude, the node will be best determined.

To apply more than three observations to one operation, MAYER, having calculated the 27 observations in the Table, formed 27 equations similar to the three formed before; then he added nine of them together, and thus formed the following equations.

$$9b-118°. \quad 8' = +8{,}4987a - 0{,}7932a \times \sin.\ t.$$
$$9b-140. \quad 17 = -6{,}1404a + 1{,}7443a \times \sin.\ t.$$
$$9b-127. \quad 32 = +2{,}7977a + 7{,}9649a \times \sin.\ t.$$

In the forming of these equations, nine equations were taken for the first, so as to make the positive coefficient of a as great as possible; nine for the second, to make the negative coefficient the greatest; and the third was formed from the other nine. By this means, when we exterminate all but a, its coefficient will be the greatest, and will give the most accurate value of a. Proceeding therefore as before, we get $a = 89',9 = 1°.\ 30'$ very nearly, differing 10′ from the other determination, which cannot be considered so accurate as this; $b = 14°.\ 33'$, the same as before; $t = -3°.\ 45'$, giving the longitude of the node of the lunar equator about as much less as the other gave it greater. This value of a gives the inclination of the moon's axis to the plane of its orbit $=$ 6°. 39′. And as the longitude of the node of the moon's orbit at the beginning of 1748, was $10^s.\ 18°.\ 56'$, that of its equator was $10^s.\ 15°.\ 11'$.

In the year 1763, M. de la LANDE, in the month of October repeated these observations, and found the inclination to be 1°. 43′, and the declination of Manilius 14°. 35′; he thinks this determination is more to be depended upon than that from the observations of MAYER. He also found the distance of the nodes of the moon's orbit and equator to be about 2°, at a time when the distance of the node of the lunar orbit was 60° from the place where it was in 1748. We may therefore, with CASSINI, conclude, that *the nodes of the lunar equator agree with the mean place of the nodes of the lunar orbit, and consequently their mean motions are the same;* a very remarkable circumstance.

398. The values of ♈B and ♈N being known, we know NB the longitude of M; and its latitude MB being also known, together with the angle BNL, we can (393) find the right ascension NL of Manilius. Hence, compute the right ascension at any intervals of time, and it appears that the right ascension increases uniformly, therefore the rotation of the moon about its axis is uniform, and consequently is performed (355) in $27d.\ 7h.\ 43'.\ 11'',5$.

399. As L is a fixed point upon the moon's surface, if the right ascension of any other point estimated from L be found, and also its declination, the situation of that point will be known. Thus we might lay down the figure of the lunar disc.

On the Rotation of the Planets.

400. The *Georgian* is at so great a distance, that Astronomers, with their

best telescopes, have not been able to discover whether it has any revolution about its axis.

401. *Saturn* was suspected by Cassini and Fato, in 1683, to have a revolution about its axis; for they one day saw a bright streak, which disappeared the next, when another came into view near the edge of its disc; these streaks are called *Belts*. In 1719, when the ring disappeared, Cassini saw its shadow upon the body of the planet, and a belt on each side parallel to the shadow. When the ring was visible, he perceived their curvature was such as agreed with the elevation of the eye above the plane of the ring. He considered them as similar to our clouds floating in the atmosphere; and having a curvature similar to the exterior circumference of the ring, he concluded that they ought to be nearly at the same distance from the planet, and consequently the atmosphere of Saturn extends to the ring. Dr. Herschel found that the arrangement of the belts always followed the direction of the ring; thus, as the ring opened, the belts began to show an incurvature answering to it. And during his observations on June 19, 20 and 21, 1780, he saw the same spot in three different situations. He conjectured therefore, that Saturn revolved about an axis perpendicular to the plane of its ring. Another argument in defence of this is, that the planet is an oblate spheroid, having the diameter in the direction of the ring to the diameter perpendicular to it as about 11 : 10, according to Dr. Herschel; the measures were taken with a wire micrometer prefixed to his 20 feet reflector. The truth of his conjecture he has now verified, having determined that Saturn revolves about its axis in 10*h*. 16′. 0″,4. *Phil. Trans.* 1794. The rotation is according to the order of the signs.

402. *Jupiter* is observed to have belts, and also spots, by which the time of its rotation can be very accurately ascertained. M. Cassini found the time of rotation to be 9*h*. 56′, from a remarkable spot which he observed in 1665. In October 1691, he observed two bright spots almost as broad as the belts; and at the end of the month he saw two more, and found them to revolve in 9*h*. 51′; he also observed some other spots near Jupiter's equator, which revolved in 9*h*. 50′; and, in general, he found that the nearer the spots were to the equator, the quicker they revolved. It is probable therefore that these spots are not upon Jupiter's surface, but in its atmosphere; and for this reason also, that several spots which appeared round at first, grew oblong by degrees in a direction parallel to the belts, and divided themselves into two or three spots. M. Maraldi, from a great many observations of the spot observed by Cassini in 1665, found the time of rotation to be 9*h*. 56′; and concluded that the spots had a dependence upon the contiguous belt, as the spot had never appeared without the belt, though the belt had without the spot. It continued to appear and disappear till 1694, and was not seen any more till 1708; hence he concluded, that the spot was some effusion from the belt, upon a fixed place of

Jupiter's body, for it always appeared in the same place. Dr. HERSCHEL found the time of rotation of different spots to vary; and that the time of revolution of the same spot diminished; for the spot observed in 1778 revolved as follows. From February 25 to March 2, in 9*h*. 55'. 20''; from March 2 to the 14th, in 9*h*. 54'. 58''; from April 7 to the 12th, in 9*h*. 51'. 35''. Also, from a spot observed in 1779, its rotation was, from April 14 to the 19th, in 9*h*. 51'. 45''; from April 19 to the 23d, in 9*h*. 50'. 48''. This, he observes, is agreeable to the theory of equinoctial winds, as it may be some time before the spot can acquire the velocity of the wind; and if Jupiter's spots should be observed in different parts of its year to be accelerated and retarded, it would amount almost to a demonstration of its monsoons, and their periodical changes. M. SCHROETER makes the time of rotation 9*h*. 55'. 36'',6; he observed the same variations as Dr. HERSCHEL. The rotation is according to the order of the signs. This planet is observed to be flat at its poles. Dr. POUND measured the polar and equatorial diameters, and found them as 12 : 13. Mr. SHORT made them as 13 : 14. Dr. BRADLEY made them as 12,5 : 13,5. Sir I. NEWTON makes the ratio $9\frac{1}{2}$: $10\frac{1}{2}$ by theory. The belts of Jupiter are generally parallel to its equator, which is very nearly parallel to the ecliptic; they are subject to great variations, both in respect to their number and figure; sometimes eight have been seen at once, and at other times only one; sometimes they continue for three months without any variation, and sometimes a new belt has been formed in an hour or two. From their being subject to such changes, it is very probable, that they do not adhere to the body of Jupiter, but exist in its atmosphere.

403. GALILEO discovered the phases of *Mars*; after which, some Italians, in 1636, had an imperfect view of a spot. But in 1666, Dr. HOOK and M. CASSINI discovered some well defined spots; and the latter determined the time of the rotation to be 24*h*. 40'. Soon after, M. MARALDI observed some spots, and determined the time of rotation to be 24*h*. 39'. He also observed a very bright part near the southern pole, appearing like a polar zone; this, he says, has been observed for 60 years; it is not of equal brightness, more than one half of it being brighter than the rest; and that part which is least bright, is subject to great changes, and has sometimes disappeared. Something like this has been seen about the north pole. The rotation is made according to the order of the signs. Dr. HERSCHEL makes the time of a sidereal revolution to be 24*h*. 39'. 21'',67, without the probability of a greater error than 2'',34. He proposes to find the time of a sidereal revolution, in order to discover, by future observations, whether there is any alteration in the time of the revolution of the earth, or of the planets, about their axes; for a change of either would thus be discovered. He chose Mars, because its spots are permanent. See the *Phil. Trans.* 1781. From further observations upon Mars, which he published in the *Phil. Trans.* 1784, he makes its axis to be inclined to the

ecliptic 59°. 42′, and 61°. 18′. to its orbit; and the north pole to be directed to 17°. 47′ of Pisces upon the ecliptic, and 19°. 28′. on its orbit. He makes the ratio of the diameters of Mars to be as 16 : 15. Dr. MASKELYNE has carefully observed Mars at the time of opposition, but could not perceive any difference in its diameters. Dr. HERSCHEL observes, that Mars has a considerable atmosphere.

404. GALILEO first discovered the phases of *Venus* in 1611, and sent the discovery to WILLIAM de' MEDICI, to communicate it to KEPLER. He sent it in this cypher, *Hæc immaturæ a me frustra leguntur, o, y*, which put in order, is, *Cynthiæ figuras æmulatur mater amorum*, that is, *Venus emulates the phases of the moon.* He afterwards wrote a letter to him, giving an account of the discovery, and explaining the cypher. In 1666, M. CASSINI, at a time when Venus was dichotomised, discovered a bright spot upon it at the straight edge, like some of the bright spots upon the moon's surface; and by observing its motion, which was upon the edge, he found the sidereal time of rotation to be 23*h.* 16′. In the year 1726, BIANCHINI made some observations upon the spots of Venus, and asserted the time of rotation to be 24⅓ days; that the north pole answered to the 20th degree of Aquarius, and was elevated 15°. or 20°. above its orbit; and that the axis continued parallel to itself. The small angle which the axis of Venus makes with its orbit, is a singular circumstance; and must cause a very great variety in the seasons. M. CASSINI, the Son, has vindicated his Father, and shown from BIANCHINI's observations being interrupted, that he might easily mistake different spots for the same; and he concludes, that if we suppose the periodic time to be 23*h.* 20′, it agrees equally with their observations; but if we take it 24⅓ days, it will not at all agree with his Father's observations. M. SCHROETER has endeavoured to show that Venus has an atmosphere, from observing that the illuminated limb, when horned, exceeds a semicircle; this he supposes to arise from the refraction of the sun's rays through the atmosphere of Venus at the cusps, by which they appear prolonged. The cusps appeared sometimes to run 15°. 19′. into the dark hemisphere; from which he computes that the height of the atmosphere to refract such a quantity of light must be 15156 Paris feet. But this must depend on the nature and density of the atmosphere, of which we are ignorant. *Phil. Trans.* 1792. He makes the time of rotation to be 23*h.* 21′, and concludes, from his observations, that there are considerable mountains upon this planet. *Phil. Trans.* 1795. Dr. HERSCHEL agrees with M. SCHROETER, that Venus has a considerable atmosphere; but he has not made any observations, by which he can determine, either the time of rotation, or the position of the axis. *Phil. Trans.* 1793.

405. The phases of *Mercury* are easily distinguished to be like those of *Venus;* but no spots have yet been discovered by which we can ascertain whether it has any rotation.

406. There is reason to believe that the satellites of *Jupiter* and *Saturn* revolve about their axes; for the satellites of the former appear at different times to be of very different magnitudes and brightness. The fifth satellite of Saturn was observed by M. Cassini for several years as it went through the eastern part of its orbit to appear less and less, till it became invisible; and in the western part to increase again. These phænomena can hardly be accounted for, but by supposing some parts of the surfaces to be unfit to reflect light, and therefore when such parts are turned towards the earth, they appear to grow less, or to disappear. As the same appearances of this satellite returned again when it came to the same part of its orbit, it affords an argument that the time of the rotation about its axis is equal to the time of its revolution about its primary, a circumstance similar to the case of the moon and earth. See Dr. Herschel's account of this in the *Phil. Trans.* 1792. The appearance of this satellite of Saturn is not always the same, and therefore it is probable that the dark parts are not permanent.

CHAP. XX.

ON THE SATELLITES.

Art. 407. ON January 8, 1610, GALILEO discovered the four satellites of *Jupiter*, and called them *Medicea Sidera*, or *Medicean Stars*, in honor of the family of the MEDICI, his patrons. This was a discovery, very important in its consequences, as it furnished a ready method of finding the longitudes of places, by means of their eclipses; the eclipses led M. ROEMER to the discovery of the progressive motion of light; and hence Dr. BRADLEY was enabled to solve an apparent motion in the fixed stars, which could not otherwise have been accounted for.

408. The satellites of *Jupiter* in going from the west to the east are eclipsed by the shadow of Jupiter, and as they go from east to west are observed to pass over its disc; hence they revolve about Jupiter, and in the same direction as Jupiter revolves about the sun. The three first satellites are always eclipsed, when they are in opposition to the sun, and the lengths of the eclipses are found to be different at different times; but sometimes the fourth satellite passes through opposition without being eclipsed. Hence it appears, that the planes of the orbits do not coincide with the plane of Jupiter's orbit, for in that case, they would always pass through the center of Jupiter's shadow, and there would always be an eclipse, and of the same, or very nearly the same duration, at every opposition to the sun. As the planes of the orbits which they describe sometimes pass through the eye, they will then appear to describe straight lines passing through the center of Jupiter; but at all other times they will appear to describe ellipses, of which Jupiter is the center.

On the Periodic Times, and Distances of Jupiter's Satellites.

409. To get the times of their mean *synodic* revolutions, or of their revolutions in respect to the sun, observe, when Jupiter is in opposition, the passage of a satellite over the body of Jupiter, and note the time when it appears to be exactly in conjunction with the center of Jupiter, and that will be the time of conjunction with the sun. After a considerable interval of time, repeat the same observation, Jupiter being in opposition, and divide the interval of time by the number of conjunctions with the sun in that interval, and you get the

435d. 14*h*. 13′. Therefore after an interval of 437 days, the three first satellites return to their relative situation within nine minutes.

416. In the return of the satellites to their mean conjunction, they describe a revolution in their orbits together with the mean angle $a°$ described by Jupiter in that time; therefore to get the *periodic* time of each, we must say, 360° $+a°$: 360° :: time of a synodic revolution : the time of a periodic revolution; hence the *periodic* times of each are;

First	Second	Third	Fourth
$1^d. 18^h. 27'. 33''$	$3^d. 13^h. 13'. 42''$	$7^d. 3^h. 42'. 33''$	$16^d. 16^h. 32'. 8''$

417. The distances of the satellites from the center of Jupiter may be found at the time of their greatest elongations, by measuring, with a micrometer, at that time, their distances from the center of Jupiter, and also the diameter of Jupiter, by which you get their distances in terms of the diameter. Or it may be done thus. When a satellite passes over the middle of the disc of Jupiter, observe the whole time of its passage, and then, the time of a revolution : the time of its passage over the disc :: 360° : the arc of its orbit corresponding to the time of its passage over the disc; hence, the sine of half that arc : radius :: the semidiameter of Jupiter : the distance of the satellite. Thus M. CASSINI determined their distances in terms of the semidiameter of Jupiter to be, of the *first* 5,67, of the *second* 9, of the *third* 14,38, and of the *fourth* 25,3.

418. Or having determined the periodic times and the distance of one satellite, the distances of the other may be found from the proportion of the squares of the periodic times being as the cubes of their distance. Mr. POUND, with a telescope 15 feet long, found, at the mean distance of Jupiter from the earth, the greatest distance of the *fourth* satellite to be 8′. 16″; and by a telescope 123 feet long he found the greatest distance of the *third* to be 4′. 42″; hence, the greatest distance of the *second* appears to be 2′. 56″ 47‴, and of the *first* 1′. 51″. 6‴. Now the diameter of Jupiter, at its mean distance, was determined, by Sir I. NEWTON, to be $37''\frac{1}{4}$; hence, the distances of the satellites, in terms of the semidiameter of Jupiter, come out 5,965; 9,494; 15,141, and 26,63 respectively. *Prin. Math. Lib. ter. Phæn.*

Hence, by knowing the greatest elongations of the satellites in minutes and seconds, we get their distances from the center of Jupiter compared with the mean distance of Jupiter from the earth, by saying, the sine of the greatest elongation of the satellite : radius :: the distance of the satellite from Jupiter : the mean distance of Jupiter from the earth.

On the Equations of Jupiter's Satellites.

419. The conjunction of the satellites with the sun, and their eclipses, cannot (411) return at equal intervals of time, on account of the unequal motion of Jupiter, which constitutes the greatest inequality; because these intervals are equal to a revolution in their orbits increased by the time of describing an angle equal to that which Jupiter has described in these intervals, which angle is variable. The true conjunctions compared with the mean may therefore vary by twice the greatest equation of Jupiter's orbit, or by 11°. 8′. 2″ according to M. WARGENTIN; because Jupiter in one part of its orbit will be 5°. 34′. 1″ behind its mean place, and in another part 5°. 34′. 1″ before it. To find this inequality, or equation, in time, say, 360° : 5°. 34′. 1″ :: a synodic revolution : the equation answering to the greatest equation of Jupiter's orbit, which is found to be 39′. 22″, 1*h.* 19′. 13″, 2*h.* 39′. 42″, and 6*h.* 12′. 59″ for the first, second, third and fourth satellite respectively. This equation depending on Jupiter's anomaly, has (411) for its argument A the mean anomaly of Jupiter. But as the excentricity, and consequently the greatest equation of Jupiter's orbit, is subject to a change, this equation must also be variable. M. CASSINI first employed this equation in calculating the eclipses.

420. Another equation arises from the progressive motion of light. When the earth is at T and Jupiter in opposition at A, the eclipse begins sooner by 16′. 15″ than when the earth is at N, and Jupiter at A, light taking that time to move over the diameter of the earth's orbit*. If therefore we suppose Jupiter to revolve about the sun in a circle at its mean distance, and v and w be the places of the earth when at its mean distance from Jupiter, whilst the earth is in the part vNw of its orbit, the light from the satellite comes *later* to the earth, than when at its mean distance; and when the earth is in the part wTv, the light comes *sooner;* consequently the eclipse happens *later* in the *former* case, and *sooner* in the *latter*, than it would, if the earth were at its mean distance. This difference constitutes the *first* and greatest equation of light; it is nothing when Jupiter is at its mean distance from the

* M. CASSINI first suspected that light was progressive, from observing that the immersions of the *first* satellite, as they are observed from the conjunction of Jupiter to its opposition, took place sooner and sooner in respect to the computed time; and that the emersions, as they are observed from opposition to conjunction, took place later and later. But he perceived that if he admitted this for the first satellite, it must be admitted for the three others, which did not appear to him to require this equation; he therefore gave up the idea. M. ROEMER did not think that M. CASSINI's objection to the progressive motion of light was well founded; he therefore adopted the idea, and established the fact. Dr. HALLEY observed, that it was necessary to allow for the motion of light in the other satellites.

earth, and is at its maximum when Jupiter is in conjunction and opposition, at which time its quantity is half 16′. 15″, or 8′. 7″,5. This equation is subtractive in vNw, and additive in wTv; and has for its argument, the elongation of Jupiter from the sun. But Jupiter does not move in a circular orbit; and if A be the apogee and P the perigee, the difference between AS and PS is such, that light moves through PS in 4′. 5″ sooner than it does through AS. Now this equation beginning when Jupiter is at its mean distance, the half of 4′. 5″, or 2′. 2″,5, is the greatest equation arising from this cause, the excentricity of the orbit. Hence, the argument for this equation is the anomaly of Jupiter. This equation is *additive* when Jupiter is at a *less* than its mean distance, and *subtractive*, when at a *greater*. This is the *second* equation of light. These three equations, that is, the equation of Jupiter's orbit (419) and the two equations of light, are manifestly common to all the satellites, the apparent times at which the eclipses of all the satellites happen, being equally affected by them. But besides these equations, there are others which belong to each, the manner of determining which has generally been, to compare a great number of observations with the calculations, after taking into consideration the preceding equations, and the difference between such computations and the observations must give the equation required. Such an equation however may be the result of several inequalities, in which case it must be separated into several equations; and by trying one set of equations after another, and by increasing some and diminishing others, or adding new ones, Astronomers have made their Tables agree very well with observations. Equations thus introduced, are called *Empyric*. And this is the only way, where there is not proper *data* to compute their value from theory, or to separate them by. The uncertainty of the quantity of matter in the satellites, renders the theory, in estimating the effects of the disturbing forces upon each other, subject to the same degree of uncertainty.

421. M. Bailly, in his *Essai sur la Theorie des Satellites de Jupiter*, has shown, that the inequalities of the *first* satellite arises from the attraction of the *second*, which produces an equation of about 3′. 30″ in time, or of 29′. 30″. on the orbit, as was found by M. Wargentin. In the year 1719, Dr. Bradley found that in the years 1682, 1695, 1718, the eclipse of the first satellite lasted about 2h. 20′; but at the other node in 1677 and 1689, the duration was only 2h. 14′; this appeared to indicate, that the motion of the satellite was not uniform, and consequently that the orbit might be excentric; he nevertheless suspected that it arose from the attraction of the second, as the reader may see in the *Phil. Trans.* 1726. M. Wargentin's Tables, which agree very well with observations, contain this equation. M. Bailly and M. de la Grange examined this matter very fully, and found that all the irregularities of the first satellite arose from the attraction of the second, and produced an effect of about 3′. 30″

me. This equation is as the sine of the distance from the point where it is ing.

22. The *second* satellite is subject to the greatest irregularities. It appears observation, that the equation amounts to about 16′½ in time, of which the d is 437 days, which indicates that it is produced by the attractions of the and third, for in that time the three first satellites return to the same situ- in respect to Jupiter. M. BAILLY suspected an excentricity of the orbit, a motion of its apsides; but this he speaks of as a circumstance very tful.

3. The *third* satellite has its motion disturbed by the first, second and h; the whole effect of these, according to M. BAILLY, produces an equa- of 16′. 11″ of a degree. M. WARGENTIN makes it, from observation, to be 16″ in the Tables published in 1759; but in the last edition of his Tables, s employed three equations; one about 2′½ of time, of which the period is lays, which he determined from observation; the other two are 4′½ and f time, and which he determined also from observation; the periods of are about 12½ and 14 years. Perhaps, says he, the variation of the ex- icity of the orbit is subject to some change, which may produce the two quations. He afterwards doubted, whether it would not be better to sub- e one equation instead of these two. M. de la LANDE says, that the third ion may be suppressed, and the computations will then not deviate much observation. M. MARALDI suspected that this satellite had an equation center, and that the annual motion of its apside was 1°. 30′. M. BAILLY g calculated a great number of observations, and compared them with his ry, after allowing for all the other equations, found it necessary to assume r the equation of its center; he also found it necessary to give a motion to the es of about 2° in a year; but this motion appeared to him to be rather too to satisfy the observations. According to his Theory, the motion of the es is 2°. 12′. 3″, from the disturbing force of the sun, and the action of the satellites, without taking into consideration the figure of Jupiter, which lso cause a motion of the apsides. He joined to the equation of the cen- ve other equations; the first of 25″ from the action of the first satellite; cond of 4′. 10″ from the action of the second; the third of 1′. 19″ from the of the second, on account of the excentricity of the orbit; and lastly, thers of 17″ and 59″ from the action of the fourth. These equations, M. Y says, may in certain cases go as far as 16′. 11″, which is very nearly 16′. e value of the total equation which had been before determined by ob- ion.

4. Dr. BRADLEY found by observation, that the orbit of the *fourth* satellite lliptical, and made the greatest equation 0°. 48′. Before this was publish-

ed, M. MARALDI had observed, that M. CASSINI's Tables erred nearly two hours, and always the same way, when Jupiter returned to the same point of its orbit; and that the error was nothing, when Jupiter was at its mean distance. This might evidently arise from the excentricity of the orbit; for as Jupiter revolved about the sun and carried the orbit of its satellite with it, in one revolution of Jupiter, the apsides of the orbit of the satellite would have had every position in respect to the sun; so that the satellite would sometimes come into opposition to the sun when it was in its lower apside, where its motion was greatest, and therefore the eclipse would happen sooner than if its motion was uniform; sometimes the eclipse would happen when the satellite was in its higher apside, and then its motion being slowest, the eclipse would happen later; sometimes the eclipse would happen when the satellite was at its mean distance, and then the true motion being equal to its mean, the time of the eclipse would happen at the time by computation according to its mean motion. From a comparison of the true and mean place of the satellite in its orbit, M. MARALDI found the equation of the center to be $55'.\ 56''$. According to the Tables of M. WARGENTIN, this equation amounts to $1h.\ 0'.\ 30''$. The attractions of the other satellites do not sensibly affect its motion; but M. BAILLY found two or three small inequalities arising from the action of the sun; he fixed the equation of the center at $50'.\ 20''$, and the motion of the apsides $45'.\ 18''$ in a year. In the year 1717, Dr. BRADLEY found the place of the apside to be $11^s.\ 8°$; but the observations in 1671, 1676 and 1677, require the place in 1677 to be $10^s.\ 14°$; hence, he fixed the motion at about $36'$ in a year; and found this to agree very well with observations. M. MARALDI made the motion of the apsides $44'.\ 15''$ in a year; and the place of the apside $10^s.\ 29°.\ 22'$ for the beginning of 1700; and the mean longitude at that time $7^s.\ 17°.\ 18'.\ 2''$. Upon this hypothesis, he computed 152 observations, of which not above 30 differed more than $5\frac{1}{2}$ minutes from observations, amongst which, 4 only differed $10'$, and only 3 differed $13'$. This was nearer than could have been expected, considering that the disturbing force of *Saturn* was not considered. The motion of the apsides arises partly from the attraction of the sun, and partly from the figure of the body of Jupiter. But it being uncertain whether Jupiter be homogeneous, or what is the accurate ratio of its diameters, the part which arises from the figure of the planet must be very uncertain. M. de la PLACE found an equation of $1'.\ 54''$ of a degree, which depends on the action of the sun and on the distance of Jupiter from its aphelion; this is similar to the annual equation of the moon; and another of about $28''$, which answers to the evection of the moon.

425. M. MARALDI found the excentricity of the orbit, in the manner described in Article 340. In the conjunction on April 6, 1708, he found the place of the satellite on its orbit to be $5^s.\ 27°.\ 55'.\ 26''$, and on March 3, 1753,

to be $3^s. 15°. 51'. 7''$; hence, the true motion was $9^s. 17°. 55'. 41''$; but the mean motion in the same time was $9^s. 19°. 13'. 5''$, or $1°. 17'. 24''$ greater. Between the observation in 1708, and one on August 4, 1759, he found the true motion greatest by $34'. 28''$; hence, half the sum of $1°. 17'. 24''$ and $34'. 28''$, or $55'. 56''$, is the greatest equation of the orbit.

426. The reduction of the orbit of a satellite to the orbit of Jupiter, furnishes another equation. Let I be the center of the shadow of Jupiter, Nt the orbit, of the satellite, draw Iv perpendicular to NI the plane of Jupiter's orbit, and Ic perpendicular to Nt, and take $Na=NI$. The point a is here called the conjunction of the satellite, that point upon the orbit having (268) the same longitude as the point I, or Jupiter; at c is the middle of the eclipse, and ac is called the *Reduction;* when the satellite is at v it is in conjunction in respect to the orbit of Jupiter. The reduction is *subtractive* when the argument of latitude is between 0°. and 90°, and between 180° and 270°; and *additive* for the other two quadrants.

M. de la LANDE, in the last edition of his Astronomy, has given new Tables of Jupiter's satellites, computed by M. de la LAMBRE, from the theory of their mutual attractions, given by M. de la PLACE, in the *Mem. de l'Acad.* 1784, 1788; the theory gave the form of the equations; the values of the co-efficients were determined from observation. He also introduced the effect arising from the disturbing force of Jupiter. In these Tables there are no empyric equations, and M. de la LANDE says they give the times of the eclipses to a degree of accuracy, beyond what could be expected. These Tables are given in Vol. III.

On the Eclipses of Jupiter's Satellites.

427. Let S be the sun, EF the orbit of the earth, I Jupiter, abc the orbit of one of its satellites. When the earth is at E before the opposition of Jupiter, the spectator will see the immersion at a; but if it be the first satellite, upon account of its nearness to Jupiter the emersion is never visible, the satellite being then always behind the body of Jupiter; the other three satellites *may* have both their immersions and emersions visible; but this will depend upon the position of the earth. When the earth comes to F after opposition, we shall then see the emersion of the first, but can never see the immersion, and *may* see both the emersion and immersion of the other three. Draw EIr; then sr, the distance of the center of the shadow from the center of Jupiter referred to the orbit of the satellite, is measured at Jupiter by sr, or the angle $sIr=EIS$ the annual parallax. The satellite may be hidden behind the body at r without being eclipsed, which is called an *Occultation.* When the earth is at E, the con-

junction of the satellite happens *later* at the earth than at the sun; but when the earth is at F, it happens *sooner*.

428. The diameter of the shadow of Jupiter at the distance of any of the satellites, is best found by observing the time of an eclipse when it happens at the node, at which time the satellite passes through the center of the shadow; for the time of a synodic revolution : the time the satellite is passing through the center of the shadow :: 360° : the diameter of the shadow in degrees. But when the first and second satellites are in the nodes, the immersion and emersion cannot both be seen. Astronomers therefore compare the immersions some days *before* the opposition of Jupiter with the emersions some days *after*, and then knowing how many synodic revolutions have been made, they get the time of the transit through the shadow, and thence the degrees corresponding. But on account of the excentricity of some of the orbits, the time of the central transit must vary: for example, the second satellite is sometimes found to be $2h.\ 50'$ in passing through the center of the shadow, and sometimes $2h.\ 54'$; this indicates an excentricity.

429. The duration of the eclipses being very unequal, shows that the orbits are inclined to the orbit of Jupiter; sometimes the fourth satellite passes through opposition without suffering an eclipse. The duration of the eclipses must therefore depend upon the situation of the nodes in respect to the sun, just the same as in a lunar eclipse; when the line of the nodes passes through the sun, the satellite will pass through the center of the shadow; but as Jupiter revolves about the sun, the line of the nodes will be carried out of conjunction with the sun, and the time of the eclipse will be shortened, as the satellite will then describe only a chord of a section of the shadow instead of the diameter.

430. Let S be the sun, I Jupiter, $Nbnv$ the plane of Jupiter's orbit, $Ncan$ the orbit of one of its satellites, Nn the line of the nodes; draw Ia, Ib perpendicular to Nn, and ab perpendicular to the plane $Nbnv$; and let c be the point in opposition to the sun, and draw cd perpendicular to $Nbnv$. Now the angle aIb is the inclination of the orbit of the satellite, whose sine we will call s, to radius unity, and put $r=Ia$; then $1 : s :: r : ab = sr$; and if $v =$ the sine of Nc the distance of the node from opposition, $1 : sr :: v : cd = vsr$ the latitude of the satellite at the time of opposition. Let $AFBG$ be a section of the shadow of Jupiter where the satellite passes through, $NAIB$ the plane of the orbit of Jupiter, Nmt the orbit of the satellite, and draw Ic perpendicular to Nt; then $Ic = vsr$; put $R = IA$, $d = mc$; then $\sqrt{R^2 - d^2} = vsr$; hence, $s = \frac{\sqrt{R^2 - d^2}}{vr}$. But R, r, and d may be taken in time, that is, d may represent the half duration of the eclipse; call that time d'; and R may represent

half the greatest duration; call this R'. And to find the time the satellite is in passing through a space equal to r, put t=an arc of 57°. 17'. 45", which is equal in length to radius; hence, 360° : 57°. 17'. 45" :: the time of a synodic revolution : the time r' of describing a space equal to r; hence, $s=\frac{\sqrt{R'^2-d'^2}}{vr'}$. If therefore the semiduration be given, and the place of the node, the inclination of the orbit will be known; and if the inclination be given, we have $d'=\sqrt{R'^2-v^2s^2r'^2}$ the half duration. This will be a little affected by the disturbing forces of the satellites, and the excentricity of the orbits. M. Bailly estimates what this disturbing force is; but as it depends upon the quantity of matter in the satellites, which cannot be determined to a great degree of accuracy, any correction of that kind must be subject to a proportional degree of error.

Ex. On November 19, 1761, at 6 o'clock, the inclination of the orbit of the fourth satellite was 2°. 36', and the distance of the node from Jupiter 46°. 43'; also, the greatest duration was 2*h*. 23', according to M. de la Lande. Hence, $r'=2h.\ 23'$, $s=,04536$, $v=,72797$; therefore $d'=1h.\ 6'.\ 6''$ the half duration. When $Ic=IA$, or $vsr'=R'$, the satellite will not enter the shadow, but just touch it; hence, $v=\frac{R'}{sr'}$. Now by the Table, to Art. 466, it appears that R' may be represented by 2°. 8'. 2", r' being represented by 57°. 17'. 45". Hence, $v=,8209$ the sine of 55°. 11', within which distance must the node be from conjunction, in order that there may be an eclipse.

431. Draw Iv perpendicular to BN; then in the right angled triangle Icv, if we know Ic and the angle vIc (the complement of cIN), we shall know cv the distance from the middle of the eclipse to the conjunction of the satellite. The supposition that mt is a straight line, produces no error of any consequence.

432. Hitherto we have supposed the section of the shadow of Jupiter to be a circle; but as Jupiter is a spheroid, and not a sphere, and the plane of its equator very nearly coincides with its orbit, we should consider the section of the shadow as an ellipse and not a circle, the major axis of which is nearly coincident with the orbit. M. de la Lande therefore proposes the following correction. Let $AFBG$ be the section, supposed to be a circle, $AxBz$ the elliptical section of the shadow, and draw nm parallel, and nc', mc perpendicular to Ix. Let nc' be half the duration; then, upon supposition that the section was circular, the same half duration would be represented by mc; so that the distance Ic before computed : the true distance

Ic' :: (by the property of the ellipse) $IF : Ix$; hence, $Ic' = \frac{Ix}{IF} \times Ic = \frac{Ix}{IF} \times \sqrt{R^2 - d^2}$; consequently $\frac{Ix}{IF} \times \sqrt{R^2 - d^2} = vsr$, therefore $s = \frac{Ix}{IF} \times \frac{\sqrt{R'^2 - d'^2}}{vr'}$, R, r and d being expressed by R', r' and d' in time. M. de la LANDE puts $Ix : IF :: 13 : 14$, and therefore $s = \frac{13}{14} \times \frac{\sqrt{R'^2 - d'^2}}{vr'}$. To find the inclination of the orbit of the fourth satellite upon this supposition, M. WARGENTIN supposed the limit of the distance of the node from conjunction to be 55°. 11′. 10″; and upon supposition of a circular section, he found the inclination to be 2°. 36′; hence, by diminishing the sine of the inclination in the ratio of 14 : 13, he found the true inclination of the orbit to be 2°. 24′. 51″.

433. The orbit of the second satellite is found to change its inclination, the period of which change is 30 years. M. MARALDI found the least inclination at the beginning of the years 1672, 1702, 1732 and 1762 to be 2°. 48′; and at the beginning of the years 1687, 1717, 1747 and 1772 he found the greatest inclination to be 3°. 48′. The inclination of the orbit of the first satellite, upon which he made the motion of the node of the second depend, is 3°. 18′, calculated for a circular section, which is a mean between the greatest and least inclinations of the orbit of the second. This determination of M. MARALDI, combined with the libration of the node, made his calculation of the eclipses agree very well with observations; for of 122 which he calculated, only 12 differed more than 1 minute. According to the new Tables of M. WARGENTIN, the least inclination is 2°. 46′ and the greatest 3°. 46′, upon supposition that the section of the shadow is a circle.

434. This variation of the inclination of the orbit of the second satellite arises from the libration of its nodes. M. MARALDI, by an observation on October 18, 1714, found the place of the node to be 10ˢ. 21°. 21′. 45″; and by an observation on September 11, 1751, he found the place of the node to be 10ˢ. 0°. 54′. 9″, the difference of which is 20°. 27′. 36″ for the whole libration of the node, supposing that these were the extreme points; hence, its half, 10°. 13′. 48″ shows the libration from the mean place, which therefore is 10ˢ. 11°. 8′. M. WARGENTIN makes it 10ˢ. 12°. 15′. M. de la LANDE first pointed out this libration of the nodes, and the consequent change of the inclinations of the orbits. In consequence of this, M. BAILLY proposed to explain this motion of the nodes and variation of the inclination, in the following manner, similar to that by which M. de la LANDE explained the changes of the inclinations of the orbits of the planets.

435. Let AC be the orbit of Jupiter, CB the orbit of the satellite which is disturbed by the motion of another satellite moving in the orbit BA, so

that we may suppose the orbit BC first to have been in the situation AB'; the angle B is the mutual inclination of the two orbits, which is supposed to be constant; let AB be the movement of the node of the orbit CB which is disturbed, upon the other orbit AB, in any given time; then AC is the motion of the node upon the orbit of Jupiter. By Trigonometry (Prop. 45 and 43.) tan. $AC = \frac{\text{tan. } B \times \text{sin. } AB}{\text{cos. } AB \times \text{cos. } A \times \text{tan. } B + \text{sin. } A}$, and cos. $C = -\text{cos. } B \times \overline{\text{cos. } A - \text{sin. } A \times \text{tan. } B \times \text{cos. } AB}$. Now to determine when AC becomes a maximum, put $y = \text{tan. } AC$, $x = \text{sin. } AB$, $a = \text{tan. } B$, $b = \text{cos. } A$, $m = \text{sin. } BAC$; then $y = \frac{ax}{ab\sqrt{1-x^2}+m}$ = a maximum, whose fluxion being put $=0$ and reduced gives $x = \sqrt{1 - \frac{a^2 b^2}{m^2}} = \sqrt{1 - \frac{\text{tan. } B^2}{\text{tan. } A^2}}$, the sine of AB, when AC is a maximum, where AB is greater than 90°, for from the tan. of AC, it appears that AC increases till AB is greater than 90°. The motion of the node of the second satellite upon the orbit of the first is found by observation to be about 12° in a year, and therefore it completes its revolution in 30 years; hence, at the end of 30 years, the node of BC upon the orbit of Jupiter will return to the same situation, and to the same inclination. Hence, the node C has a movement of libration about A; if b be the utmost limit of the node of BC from A on one side, and a on the other, the node will librate between a and b.

436. The two inclinations A and C are not equal at the limits a and b; for as cos. $C = \text{cos. } B \times \overline{\text{cos. } A - \text{sin. } A \times \text{tan. } B \times \text{cos. } AB}$, therefore when the inclinations become equal, cos. $C = \text{cos. } B \times \overline{\text{cos. } C - \text{sin. } C \times \text{tan. } B \times \text{cos. } AB}$, hence, cos. $AB = \frac{\text{cos. } C \times \overline{\text{cos. } B - 1}}{\text{sin. } C \times \text{tan. } B}$, which being negative, shows that AB is greater than 90°. Also, sin. $AB = \sqrt{1 - \frac{\overline{\text{cos. } B - 1}^2}{\text{tan. } B^2 \times \text{tan. } C^2}}$; let us assume this $= \sqrt{1 - \frac{\text{tan. } B^2}{\text{tan. } A^2}}$, which is the sine of AB when AC is a maximum, and (supposing $A = C$) we deduce $1 = \text{cos. } B^3$, which is absurd, consequently the inclinations are not equal when IC is a maximum. Also, as $\sqrt{1 - \frac{\overline{\text{cos. } B - 1}^2}{\text{tan. } B^2 \times \text{tan. } C^2}}$ is greater than $\sqrt{1 - \frac{\text{tan. } B^2}{\text{tan. } C^2}}$, and both greater than 90°, the two inclinations become equal before the node comes to its limits.

437. From an eclipse of the third satellite on January 25, 1763, the half duration of which was 43′, M. MARALDI found the inclination 3°. 25′. 41″, sup-

posing the semidiameter of the shadow to be 1*h*. 47'. 10", and to be circular. In 1745, it was found to be greater by 7½'; but from 1763 it has appeared to decrease, for in 1769 it was found to be 3°. 23'. 33". M. de la GRANGE judged the period of its augmentation, to be 195 years; M. BAILLY made it 200 years. M. WARGENTIN made the least inclination to be in 1697, and the greatest in 1782. M. MARALDI made the period 132 years, finding the greatest inclination in the years 1633, 1765 to be 3°. 25'. 57"; and the least inclination 3°. 2' in the year 1697. Upon this he computed the inclination for every intermediate time, with the libration of the node arising from the attraction of the first satellite. But some of his computations make the duration of the eclipse err 6', which renders his period very uncertain. M. de la LANDE has found the inclination of the third satellite by Art. 435. The annual motion of the node B of the third upon the orbit AB of the first was found to be 2°. 43'. 38",2, and therefore it was 27°. 16'. 22" between the observations made in 1763 and 1773, a period of 10 years; let $AB = 27°. 16'. 22"$, the angle $A = 3°. 14'$, and the angle $B = 12'$; hence, the angle $C = 3°. 24'. 44"$, the inclination of the orbit of the third satellite in 1773. Also, $AC = 1°. 32'. 24"$, the libration in that interval.

438. The inclination of the orbit of the fourth satellite is 2°. 36' according to M. MARALDI, with very little, if any, variation. Dr. BRADLEY made it 2°. 42'. M. WARGENTIN, in 1781, found an increase of 1' or 2' in the five last years, and he estimated it at 2°. 38'. M. de la LANDE makes it 2°. 36' in a circular, and 2°. 24'. 51" in an elliptical shadow. The motion of the nodes of this satellite, which is 4'. 19" in a year according to M. WARGENTIN, ought to produce a change in the inclination, and M. BAILLY thought that in 1720 the inclination was a little diminished, the nodes of the first and fourth satellites then coinciding. M. MARALDI could not reconcile the semiduration of the eclipses with any variation of inclination, or motion of the node; yet in supposing the inclination to be constantly 2°. 36', and the semidiameter of the shadow to be 2°. 8'. 2", and the place of the node in 1745 to be 4^s. 16°. 11', with an annual progressive motion of 5'. 33', his computations have agreed very well with observation.

439. M. de la PLACE has shown, that the nodes of the fourth satellite have a retrograde motion in a plane which passes between Jupiter's equator and orbit, inclined to the former at about half a degree. The plane of the orbit of the fourth preserves a constant inclination of 14' or 15', and a retrograde motion of the node of about 35' in a year upon this plane. This theory will satisfy all the observations, and explain why the inclination is constant, and the motion of the nodes direct. This results from the action of the sun and of the other satellites, and from the flatness of Jupiter.

440. The inclination of the fourth satellite being considerable, it may be

found by finding the minor axis of the ellipse which it appears to describe when Jupiter is 90° from the node, which is done by observing its apparent distance from Jupiter in its conjunctions, which is the semi-minor axis, and the semi-major axis being the greatest elongation, the latter is to the former as radius to the sine of the inclination.

On the Nodes of the Orbits of Jupiter's Satellites.

441. The place of the node may be determined at the time of the greatest duration of an eclipse, for at that time the plane of the orbit of the satellite must pass through the sun, and therefore the place of Jupiter at that time gives the place of the node. Or the place of the node may be found by observing two eclipses of the same duration on each side of the node, in which case the place of the node will bisect the two situations of Jupiter. This method supposes that Jupiter has moved uniformly in the intermediate time, and that the nodes of the satellite remained fixed. On March 12, 1687, FLAMSTEAD observed the duration of an eclipse of the third satellite to be $2h.\ 33'$, Jupiter's heliocentric longitude at that time being $8^s.\ 11°.\ 58'$. On December 6, 1702, the duration was exactly the same, and the heliocentric longitude of Jupiter was $0^s.\ 15°.\ 21'$; half the difference of these longitudes added to the first gives $10^s.\ 13°.\ 29'$ for the place of the node nearly. Or the place of the node may be found when the satellite passes in a right line over the disc of Jupiter, which may be observed by its shadow upon Jupiter. This we may determine from the belts, as the motion of the satellites is very nearly in their direction.

442. In the year 1693, M. CASSINI, in his Astronomy, places the nodes of all the satellites in $10^s.\ 14°.\ 30'$. Dr. BRADLEY thought the place of the nodes of them all in 1718, to be $10^s.\ 11°,5$. From observations since, it appears that the nodes do not all coincide. The node of the *first* satellite is found to be in $10^s.\ 14°.\ 30'$, and observations show that it has no sensible motion.

443. According to M. WARGENTIN in his first Tables, the place of the node of the *second* was $10^s.\ 11°.\ 48'$, fixed; but in his new Tables he gives it a progressive motion upon the orbit of Jupiter of $1°.\ 42'$, in respect to the aphelion of Jupiter in 100 years. M. BAILLY gives the node a libration of $9°.\ 21'$. M. MARALDI makes it $8°.\ 42'\frac{1}{2}$. M. de la GRANGE makes it $11°.\ 27'$. The mean place of the ascending node is $10^s.\ 13°.\ 52'$ according to M. MARALDI. The nodes of the third and fourth satellite have a like libration about the nodes of the first, whilst the nodes of the first have a libratory motion about a point as the mean place.

444. The mean place of the node of the *third* satellite is constantly in $10^s.\ 14°.$

24′ according to M. WARGENTIN. M. MARALDI supposes it to have a motion of about 3′ in a year; and as we have seen (437) that the inclination is subject to a change, it may be necessary that the nodes should have a motion to account for it.

445. From the theory of attraction, Dr. BRADLEY thought that the nodes of the *fourth* satellite ought to be retrograde; the motion would be retrograde from the attraction of the sun only, but the attraction of the other satellites may make it direct; and observations show that it is direct. According to M. MARALDI, its place in 1745, was 4ˢ. 16°. 11′, with an annual motion of 5′. 33″. M. BAILLY finds it 5′. 15″. M. WARGENTIN placed the node in 1760 in 10ˢ. 16°. 39′, and gave the node an annual motion of 3′. 18″ in respect to the aphelion of Jupiter, which gives 4′. 15″ in respect to the equinoxes.

446. M. BAILLY, from his Theory, deduces the following conclusions respecting the motion of the nodes.—1. The node of the first has a libratory motion about its mean place of 18′, of which the period is 30 or 32 years.—2. The node of the second librates about the same point, about 9°. 37′, with a period of 30 or 32 years.—3. The node of the third has a libratory motion about the same point, of about 3°. 53′, of which the period is about 200 years.—4. The node of the fourth librates about the same point, about 12°. or 13°, with a period of 4 or 500 years.—5. This point, or the mean place of the node of the first satellite, about which the nodes of the other satellites librate, has a retrograde motion upon the orbit of Jupiter of 33′. 30″ in a year, from the disturbing force of the sun. M. BAILLY, from his Theory of the satellites, has computed a set of Tables of the motions of each.

447. The ascending nodes of the orbits of all the satellites we may consider in 10ˢ. 15°, in all cases where great accuracy is not required. When Jupiter therefore is in 10ˢ. 15° and 4ˢ. 15°, the planes of the orbits pass through the sun, and to a spectator, there situated, the satellites would appear to describe straight lines, as *AB*, in the direction of the belts; in any other situation of Jupiter, they would appear to describe an ellipse *AmBn*. When Jupiter comes to 1ˢ. 15° and 7ˢ. 15° the minor axis becomes the greatest, and in that situation, the major axis : minor :: rad. : sine of the inclination of the orbit of the planet; in any other situation of Jupiter, the minor axis is at the sine of the distance of Jupiter from the node. As Jupiter passes from 10ˢ. 15° to 4ˢ. 15° the *furthest* semicircle of the orbit of the satellite appears most to the north, or, as we may express it, highest, and therefore it will be represented by *AmB*; but as the planet passes from 4ˢ. 15° to 10ˢ. 15° the *nearest* semicircle will appear highest, and therefore it will be represented by *AmB*. Hence we may judge of the situation of the satellites in respect to the position of the belts, or to the line *AB*; a circumstance which we take into consideration in the configuration of the satellites; we will explain this by the example there given. The

heliocentric longitude of Jupiter is $9^s. 23°. 5'$, consequently AmB represents the *nearest* semicircle; hence, m is the inferior conjunction and n the superior, and therefore n has the same longitude seen from Jupiter, as Jupiter has from the sun; that is, the longitude of the point n of the orbit of the satellite is $9^s. 23°$, omitting the minutes; hence, the longitude of A is $6^s. 23°$, of B $0^s. 23°$, and of m $3^s. 23°$. Describe the circle $Am'Bn$; now the longitude of the first satellite is $2^s. 21°$; hence, set off $Ba = 2^s. 21° - 0^s. 23° = 1^s. 28°$, and draw ars perpendicular to AB, and s will be the place of the satellite, and sr the apparent elevation *above* the line of the belts; in like manner the situations of the others may be found. If four figures of a considerable size, and of the proper proportions, be thus described about I, and in each, $AB : mn ::$ rad. : sin. of the inclination of the orbit × sine of the distance of Jupiter from the nodes, and the ellipse be accurately drawn, and the orbits divided into every 5°, the situations of the satellites will, from inspection, appear sufficiently accurate. The spectator has here been supposed to be at the sun; if he be at the earth, the appearance will be very nearly the same; however, when Jupiter comes near to the nodes, it may be considered at what time the planes of the orbits pass through the earth instead of the sun.

On the Magnitudes of Jupiter's Satellites.

448. The satellites appear so small in the field of view of a telescope, that they cannot be measured by a micrometer; their magnitudes have therefore been determined from observing the times they are entering into the shadow of Jupiter, in a central eclipse; but this must always give their diameters too small, as we cannot tell the instant the satellite touches the shadow; a certain quantity of light must be lost before the eclipse appears to begin, and it must become invisible before it be wholly immersed in the shadow. Their magnitudes have also been found from measuring the diameters of their shadows upon the disc of Jupiter; or by observing how long they are entering upon the disc of Jupiter when they pass centrally over it. By the observations of Mr. LYNN (*Phil. Trans.*) Mr. WHISTON found that the first entered into the shadow in 1'. 10", the second in 2'. 20", the third in 3'. 40" and the fourth in 5'. 30", when they entered perpendicularly; hence their apparent diameters seen from the center of Jupiter become known. From this he deduced the magnitude of the third to be very nearly as big as the earth; the first nearly as big; the second a little less than the first; the fourth the least of all, and a little greater than the moon. M. WARGENTIN compared the shadows of the satellites upon the disc of Jupiter, from which he found the third and fourth to be five or six times greater than the first, and the second to be half as large as the first. M.

MARALDI having examined and calculated three observations of M. CASSINI made in 1695, found that the first satellite entered upon the disc of Jupiter in 7′, the second in 9′. 40″, the third in 12′. 6″; and their continuance upon the disc was 2*h*. 27′, 3*h*. 4′. 20″ and 3*h*. 43′. 38″ respectively; in respect to the fourth, he concluded from the Tables that it ought to be 15′ in entering upon the disc, and 5*h*. in its continuance upon it; hence he deduced the diameter of the third to be $\frac{1}{18}$ of that of Jupiter, and of the three others $\frac{1}{20}$. The difference of these conclusions shows that no great dependance can be placed upon them. The disappearance of a satellite will be later the better the telescope is, and it will appear sooner. M. de FOUCHY observed, that the disappearance and re-appearance of the satellites depended on the distance of Jupiter from the sun and earth. M. de BARROS observed, that different states of the atmosphere, different altitudes, and their distance from Jupiter, would influence the times of their appearance and disappearance. All these circumstances, so far as they cannot be considered, must tend to render the measures of their diameters very uncertain. Mr. WHISTON observes, that the comparison of the observations shows that the quantities are sometimes considerably larger than at others. *Longitude discovered by Jupiter's Planets*, page 5.

The following Table contains the diameters of the three first satellites as seen from Jupiter, according to CASSINI, WHISTON and BAILLY; the fourth as determined by M. de la LANDE.

Satellites.	CASSINI	WHISTON	BAILLY
I	59′. 4″	60′. 58″	60′. 20″
II	38. 1	28. 25	29. 42
III	24. 59	53. 40	22. 28
IV	13′. 32″	11′. 19″	9′. 39″

449. If their diameters could be ascertained to any great degree of certainty, their quantities of matter would still be very uncertain, because their densities are not known. Astronomers have endeavoured therefore to find out their quantities of matter from observing the quantities of the effects produced by their actions upon each other. From supposing the masses of the first and third equal, M. de la GRANGE found, from the inequalities which they produce

in the second, that their masses were 0,00006869, that of Jupiter being = 1; M. BAILLY found it 0,0000638 from the same supposition.

The mass of the second, from the inequalities which it produces in the first, of which it is principally the cause, is found by M. BAILLY to be 0,0000211; it is 0,00002417 according to M. de la GRANGE.

The mass of the third, from its effect upon the movement of the node of the second in conjunction with the first, is according to M. BAILLY, 0,00007624; but from its effect upon the inequalities of the motion of the second, supposing it equal to the first, it is 0,0000638; it is 0,0000687 according to M. de la GRANGE.

The mass of the fourth, from the small effect which it has upon the third, is not easily to be determined; M. BAILLY made it 0,00005.

These masses, M. BAILLY observes, represent very well the motions of the satellites, of their nodes, and the variation of their inclinations; we may therefore conclude, that they are pretty accurately established; and at the same time it proves that the variation of gravity according to the inverse square of the distance, will explain all the phænomena of the satellites. In respect to the motion of the apsides, that depends upon the figure of Jupiter, its density, and how the density may vary from the center to the surface; but as this is unknown, the theory cannot be here applied. M. BAILLY has, however, pointed out the method by which we may, from the observed motion of the apsides, deduce the law of the variation of the density.

M. de la PLACE has determined the masses of the satellites to be as follows: that of Jupiter being unity; the mass of the 1st, = 0,0000173281; 2nd, = 0,0000232355; 3rd, = 0,0000884972; 4th, = 0,0000426591.

On the Construction of the Epochs of the Mean Conjunctions of Jupiter's Satellites.

450. The epoch of the mean conjunction is the moment when the satellite arrives the first time every year at the mean place of Jupiter, reckoned upon the orbit of the satellite, diminished by the sum of the maxima of all the equations (the equations being expressed in time), in order to render the equations all additive; the maxima of the equations being added to the equations themselves, in order to make up for that subtraction. For example, the first equation of light, at its maximum, is $8'.\ 7'',5$, the time therefore is, in the epoch, diminished by this quantity. Now let us suppose that at the time we are making any computation, this equation of light is $\pm 4'$, then the equation, as we shall find it, is $8'.\ 7'',5 \pm 4' = 12'.\ 7'',5$ or $4'.\ 7'',5$, both additive, and this is manifestly the same as if $8'.\ 7'',5$ had not been subtracted at first, and the equation $\pm 4'$ applied, what was at first subtracted being now added; and as we add the

maximum, the quantity by which it is diminished can never render it negative. It is the same with all the other equations, except that depending on the excentricity of Jupiter's orbit, which being variable, does not admit of this method. All the epochs are thus put down; and the computations are rendered more easy and simple by making the equations additive.

451. To find the epoch for any year, we will take and explain that Example which is given by M. de la LANDE in the last edition of his Astronomy, Vol. III. pag. 185. On January 2, 1764, the first satellite was eclipsed, the emersion of which was at 10*h*. 27′. 44″ mean time, at Paris. Now to make the equation of time always additive, we must subtract 14′. 42″ which is the greatest equation subtractive, and we have 10*h*. 13′. 2″ for the time of the emersion, according to the construction of M. WARGENTIN's Tables.

The distance from the node was 60°. 17′, the semidiameter of the shadow 1*h*. 7′. 55″, and the inclination of the orbit 3°. 18′$\frac{2}{3}$; hence, (430) the semiduration of the eclipse was 1*h*. 4′. 51″; therefore the time of the middle of the eclipse was 9*h*. 8′. 11″. From this we must deduce the time of the mean conjunction, by applying all the equations for that time.

The mean anomaly of Jupiter was about 7ˢ. 8°,5, and the equation of its orbit was 4°. 51′. 30″ additive, which converted into time (419) according to the motion of the satellite, gives the equation 34′. 39″ to be subtracted from the middle of the eclipse, and hence there remains 8*h*. 33′. 32″.

From Art. 420, the maximum of the *first* part of the equation of light is 8′. 7″,5; but at the time of the eclipse the equation was found to be 7′. 0″,5 additive; hence, we have to subtract only 1′. 7″, which gives the time 8*h*. 32′. 25″.

The maximum (420) of the *second* part of the equation of light is 2′. 2″,5; but the equation was 59″,5 additive at the time; therefore we must subtract 1′. 3″, which gives 8*h*. 31′. 22″.

The maximum (421) of the equation, marked in the Tables *C*, is 3′. 30″; but that equation at the time of the eclipse was 0′. 27″ additive; hence, we have to subtract 3′. 3″, which gives 8*h*. 28′. 19″.

The small equations which come from the inequalities of Jupiter amounted at the same time to 15″ subtractive; and the maximum being 1′, we must subtract 1′. 15″, which gives 8*h*. 27′. 4″.

Lastly, we must subtract 17″ for the reduction (426), and we have 8*h*. 26′. 47″ on January 2; but it being bissextile, we must subtract one day, which gives January 1, 1764, 8*h*. 26′. 47″ for the epoch of the mean conjunction for that year, by M. WARGENTIN's construction of the Tables.

452. Having shown how the epochs for any year are established, we have only to show how they are carried on for any number of years. According to

5

M. WARGENTIN, the synodic revolution of the first satellite is 1*d.* 18*h.* 28′. 35″,947909; of the second, 3*d.* 13*h.* 17′. 53″,74893; of the third, 7*d.* 3*h.* 59. 35″,86754; and of the fourth, 16*d.* 18*h.* 5′. 7″,09174. Let us take for our example, the first satellite. If we multiply 1*d.* 18*h.* 28′. 35″,947909 by 207 it gives 366*d.* 8*h.* 40′. 1″,21716, which is a common year of 365 days, and 1*d.* 8*h.* 40′. 1″,21716 over. Therefore at the beginning of the next year the satellite will be forwarder than it was at the beginning of the preceding, by 1*d.* 8*h.* 40′. 1″,21716. If we add again 1*d.* 8*h.* 40′. 1″,21716, it gives 2*d.* 17*h.* 20′. 2″,43432, which being more than a revolution, by subtracting a revolution from it, we get 0*d.* 22*h.* 51′. 26″,48641, which is the quantity by which the satellite will be forwarder at the beginning of the second year. If to this we again add the same quantity, it gives 2*d.* 7*h.* 31′. 27″,70357, which being more than a revolution, by subtracting a revolution from it, we get 0*d.* 13*h.* 2′. 51″,75566, the quantity by which the satellite will be forwarder at the beginning of the third year. But as the fourth year is supposed to be bissextile, the epoch will take place on the first of January, therefore this year consists of 366 days, and consequently contains 207 revolutions and 0*d.* 8*h.* 40′. 1″,21716; this therefore added to 0*d.* 13*h.* 2′. 51″,75566 gives 0*d.* 21*h.* 42′. 52″,97282 the quantity by which the satellite is forwarder at the beginning of the fourth year. But as this year begins on the first of January instead of December 31, in the mean motions for days, in January and February, we must take the day of the month one less than it is, as will be further explained in the construction of the Tables. Hence it appears, that if we begin at the epoch of any leap-year, and add to it 1*d.* 8*h.* 40′. 1″,21716, 0*d.* 22*h.* 51′. 26″,48641, 0*d.* 13*h.* 2′. 51″,75566, and 0*d.* 21*h.* 42′. 52″,97282, the sums, rejecting a whole revolution when necessary, will be the epochs for the first, second, third and fourth years after. Thus we may continue the epochs as far as we please. In like manner we proceed with the arguments *A*, *B*, *C*, &c. of the equations, rejecting a revolution when the sum exceeds it.

To find the Configuration of Jupiter's Satellites at any Time.

453. 1. Find by Tables I, II, III, IV, V, the mean place of each satellite for the given time, which will be sufficiently near to the true place, except for the fourth satellite, for which we must apply the equation of the center; for that purpose we must, with its mean motion, take out the place of its apside, which subtracted from the mean longitude gives its mean anomaly, to which find the equation in Table IX, and apply it to the mean longitude, and it gives the true longitude.

2. From the place of each satellite thus found, subtract the geocentric lon-

gitude of Jupiter*, and in Table VI with that difference, find the corresponding numbers, which represent the apparent distance of each from the center of Jupiter in terms of its semidiameter. When the argument of this Table is *less* than six signs, the satellite will be to the *east* of Jupiter; when *greater*, to the *west*. This is Dr. HALLEY's method.

As the principal design of finding the configurations of the satellites is to distinguish one from another, the equation of light is commonly of but little importance, and may be neglected. Or if that accuracy be desired, compute the configuration to any hour mean time, and then add the equation of light to it, and you have the configuration as they appear at that time. The operation is rendered shorter by calculating to an hour; and it will be sufficient if the places be calculated to minutes of a degree.

454. To find the equation of light, from the sun's longitude subtract the heliocentric longitude of Jupiter, and with the remainder enter Table VII, and take out the first part of the equation; and with the anomaly of Jupiter enter Table VIII, and take out the second part. The anomaly is found by subtracting the place of the aphelion from the longitude.

Ex. To calculate the configuration of the satellites on April 6, 1795, at four o'clock in the morning, mean time, by the civil account; or on the *5d.* 16*h.* astronomical time.

	I.	II	III.	IV.	APS. IV.
1795,	1s. 25°. 8′	0s. 15°. 8′	2s. 25°. 11′	8s. 26°. 53′	1s. 10°. 49′
March	10. 14. 1	4. 3. 44	6. 28. 35	4. 21. 24	0. 0. 11
5 days	9. 27. 27	4. 26. 52	8. 11. 35	3. 17. 51	0. 0. 1
16 hours	4. 15. 40	2. 7. 35	1. 3. 33	0. 14. 23	
					1. 11. 1
	2. 22. 16	11. 23. 19	7. 8. 54	5. 20. 31	5. 20. 22
♃ Geo.	10. 3. 50	10. 3. 50	10. 3. 50	−39	
					4. 9. 21
	4. 18. 26	1. 19. 29	9. 5. 4	5. 19. 52	
				10. 3. 50	
				7. 16. 2	
	East 3,91	*East* 7,14	*West* 14,95	*West* 18,98	

* The method of finding this will be shown in the Introduction to the Tables in the Third Volume.

Hence, and by Article 447, this Configuration.

```
                       * 1                      * 4
A ──────────────────────────── O ─────────────────────────── B
                   2 *                      3 *
```

The line *AB* shows the direction of the belts.

Sun's longitude - -	0ˢ. 17°. 24′		
Jupiter's hel. long. -	9. 23. 3		
Difference - - -	2. 24. 21 - -	Equation of light	8′. 59″
Jupiter's aphelion -	6. 11. 4		
Jupiter's anomaly -	3. 11. 59 - -	Equation of light	1. 37
		Total equation of light	10. 36

This configuration therefore which is calculated for four o'clock mean time, is such as will appear at 10′. 36″ after four; but it will not sensibly differ from the appearance at four. To have computed the configurations as they appear at four o'clock, we must have computed for *3h.* 49′. 24″ mean time.

455. The configuration at the same hour, for a month, may be very readily determined, without repeating the whole operation, in this manner. The difference from day to day arises from the addition of the daily mean motions, and from the variation of the geocentric place of Jupiter, the mean daily variation of which for a month will be sufficiently accurate for our purpose. As the geocentric place of Jupiter is *subtracted* from the mean place of the satellite, if the geocentric motion be *direct*, *subtract* its mean daily variation from the mean daily motion of the satellite; but if the geocentric motion be *retrograde*, you must *add*, and you will have the whole daily motion to be applied to the calculation for any one day in order to get the situation of the satellites for the next day; and thus you may continue the process for a month, at the end of which, it may be proper to resume the first calculation, and then proceed for that month in like manner.

To take our example, I find Jupiter's geocentric motion is *direct*, at the mean rate of about 2′ in a day for the month; subtract therefore 2′ from the daily motions of the satellites, and we have 6ˢ. 23°. 27′, 3ˢ. 11°. 20′, 1ˢ. 20°. 17′ and

0ˢ. 21°. 32′, for the relative daily motions of the first, second, third and fourth satellites in respect to Jupiter; hence,

	I.	II.	III.	IV.
	4ˢ. 18°. 26′	1ˢ. 19°. 29′	9ˢ. 5°. 4′	7ˢ. 16°. 2′
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
7*d.*	11. 11. 53	5. 0. 49	10. 25. 21	8. 7. 34
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
8*d.*	6. 5. 20	8. 12. 9	0. 15. 38	8. 29. 6
	6. 23. 27	3. 11. 20	1. 20. 17	0. 21. 32
9*d.*	0. 28. 47	11. 23. 29	2. 5. 55	9. 20. 38

Hence by Table VI, we get the following configurations:

7*d.* - -	*West* 1,84	*East* 5,94	*West* 8,75	*West* 24,38
8*d.* - -	*West* 0,54	*West* 8,31	*East* 3,79	*West* 26,38
9*d.* - -	*East* 2,82	*West* 2,67	*East* 13,58	*West* 24,71

As there is the same motion of the satellites to be added every time, it will be best to put them down upon a strip of paper, and by laying it under, the addition may be made from it without the trouble of writing the motions down every time. In this manner we may lay down the configurations with great expedition, and with more accuracy than by the mechanical contrivances of Flamstead and Cassini. In the Example for the fourth satellite, the variation of the equation of the orbit is not considered, which, in general, is not necessary, as the configurations are put down, only that we may know which the satellites are; but if this satellite should be found very near another, it may be necessary to consider the equation of the orbit in Table IX.

The mean time at which these configurations are shown, may be reduced to apparent time, by applying the equation of time; thus the configuration on the sixth day at 10′. 36″ after four o'clock mean time, is 8′. 11″ after four apparent time; to have calculated therefore for four apparent time, we must have calculated for 3*h.* 51′. 49″.

456. The principle upon which the sixth Table is constructed is this. Let E be the earth, I Jupiter, a the place of a satellite in its orbit; join EI, and produce it to the heavens at m, and produce Ia to n, and draw ac perpendicular to Ib. Now m is the geocentric place of Jupiter in the heavens, and n the

true place of the satellite; if therefore from ♈ *n* the longitude of the satellite, we subtract ♈ *m* the geocentric longitude of Jupiter, we get *mn*, or the angle *bIa*, whose sine *ac* represents the apparent distance of the satellite from Jupiter; if therefore *ac* be expressed in terms of the semidiameter of Jupiter, we shall get the apparent distance in semidiameters, and in this manner the Tables are constructed.

When a satellite is *approaching* Jupiter, the figure is put *between* Jupiter and the point; when it is *receding*, the figure is put on the other side.

A TABLE

Of the apparent Distances of Jupiter's Satellites from its limb at the time of an Eclipse, for every tenth Day of Jupiter's Distance from Opposition or Conjunction; by the Rev. Dr. MASKELYNE, *Astronomer Royal; from the* British Mariner's Guide.

Distance of Jupiter from *opposition* to the ⊙.	Distance of the Satellites from Jupiter's limb at the Eclipses, in semidiameters of Jupiter and decimal parts.				Distance of Jupiter from *conjunction* with ⊙.	Distance of the Satellites from Jupiter's limb at the Eclipses, in semidiameters of Jupiter and decimal parts.			
Days.	I.	II.	III.	IV.	Days.	I.	II.	III.	IV.
10	0,20	0,33	0,50	0,85	10	0,15	0,25	0,35	0,55
20	0,40	0,66	1,05	1,66	20	0,30	0,45	0,70	1,25
30	0,60	0,95	1,50	2,65	30	0,40	0,67	1,05	1,70
40	0,75	1,20	1,90	3,35	40	0,55	0,90	1,40	2,50
50	0,90	1,40	2,25	3,95	50	0,70	1,00	1,80	3,20
60	1,00	1,60	2,50	4,40	60	0,80	1,25	2,00	3,50
70	1,05	1,70	2,66	4,70	70	0,90	1,40	2,25	3,95
80	1,10	1,75	2,75	4,85	80	1,00	1,55	2,45	4,33
90	1,10	1,75	2,75	4,85	90	1,05	1,66	2,60	4,60
100	1,10	1,70	2,70	4,80	100	1,10	1,75	2,70	4,80

The distances of the satellites from Jupiter's limb in this Table, are to be measured, either in a line with Jupiter's equator, or longer axis, or in a line parallel thereto; or, which is the same thing, to its belts; for the satellites generally appear a little to the north or south of this line.

In this Table, the apparent distances are those of the regular eclipses, that is, at the immersions before opposition and emersions after; but at the emersions which are visible before opposition, and immersions after, the distances from

Jupiter's limb will be less than in the Table, by a quantity which is in the same proportion to Jupiter's diameter, as the duration of the eclipse is to the longest duration when in the nodes.

Various Circumstances respecting the Phænomena of the Satellites.

457. To find when an immersion and emersion are visible, let s be the center of the shadow AB, r the center of the disc of Jupiter CD, the radius sn being expressed in minutes of the orbit of the satellite, and rn expressed in the same measure. Let rs be a portion of the orbit of the satellite equal to the annual parallax, expressed in minutes, which may be taken from the *Nautical Almanac*; and let AB and CD be represented as seen from the earth; let wcn be the path of the satellite, then the immersion at w being visible, the emersion at n will also be just visible, or rather it is the limit. In the triangle rns, we know all the sides, to find the perpendicular nm on rs, which, as the orbit wn is very nearly parallel to sr, is very nearly equal to sc; but (434) $sc=vsr$; or if we represent the radius r by unity, $sc=vs$; make therefore $mn=vs$, and we get v, the sine of the distance of Jupiter from the node. We have here supposed the earth in the plane of the orbit of Jupiter; but as the earth is not in that plane, it will make Jupiter appear a little higher or lower in the shadow, by the latitude of the earth seen from Jupiter; this when greatest is about $15'$, and varies as the sine of the distance of the earth from the node of Jupiter. If rs represent the orbit of Jupiter, in the first six months of the year, the center of the shadow will lie at t to the south of s. Hence, knowing sr and st, we can find the angle srt, and rt; consequently we know the three sides of nrt, to find the angle nrt; hence we know nrs; therefore in the triangle nrs, we know rn, rs and the angle nrs, to find nm. The latitude of the earth seen from Jupiter is nearly equal to one seventh part of the equation of the earth's orbit (*Mem. Acad.* 1765), at least when Jupiter is about quadratures. This is the method which is given by M. de la LANDE, to determine when an emersion will be visible before opposition, and an immersion, after opposition.

458. M. de la PLACE, in the *Mem. de l'Acad.* 1784, in his theory of the motions of the satellites, has deduced some very extraordinary conclusions, which are confirmed by observations. If p, q and r represent the mean motions of the first, second and third satellites, he has shown, that $p-q=2q-2r$, or $p-3q+2r=0$; and if x, y and z represent their mean longitudes, he has proved that $x-3y+2z=180°$. The Tables therefore must always satisfy these conditions. The last equation shows that the three satellites can never be eclipsed at the same time. But it may be observed, that the first equation is a consequence of the second; for the corresponding mean longitudes will always be represent-

ed by $x+p$, $y+q$, $z+r$, and hence $x+p-3\times\overline{y+q}+2\times\overline{z+r}=180°$, from which subtract $x-3y+2z=180°$, and we get $p-3q+2r=0$. M. de la PLACE makes the annual motion of the apside of the third to be 3°, and of the fourth to be 37′. M. BAILLY makes the former 2°, in his Tables, and the epoch for 1700, in 11^s. 13°; the latter he makes 45′. 18″. In the Tables which we have here added, the epochs of the apside of the fourth are those given by M. BAILLY, in his *Essai sur la Theorie des Satellites de Jupiter*.

459. The first satellite is most proper for finding the longitude, the Tables of that being the most correct; it is also the best on account of the greater velocity of the satellite, the instant of its appearing or disappearing being, on that account, more certain. It is better to compare an eclipse observed under one meridian with an eclipse observed under another, rather than with one computed, because of the imperfection of the Tables. The observers should also be furnished with the *same kind* of telescopes, as the time when a satellite becomes visible at an emersion, or invisible at an immersion, depends upon the quantity of light which the telescope receives, and its magnifying power; it depends also upon the proximity of the satellite to Jupiter, and its altitude above the horizon. M. BAILLY has given us some Rules to correct the difference arising from these circumstances; these we shall, in brief, here explain.

460. As the satellite enters the shadow of Jupiter, its light diminishes by degrees, until the satellite becomes invisible; and it is of great importance to ascertain how much of the satellite is immersed in the shadow at the time it disappears. M. FOUCHY first observed that this would depend upon the distance of the earth from Jupiter. Let *PR* be the shadow of Jupiter, *LM* the orbit of the satellite, and let *v* be the center of the satellite *mnrt* at the time it becomes invisible, then *mnr* is the part not yet immersed, and which is called the *invisible* segment; let *OQ* be perpendicular to *LM*, and join *Oc*, and draw *Ovsn*. Now if we know *sn*, subtract it from *vn*, and we get *vs*; hence we know *Os* − *vs* or *Ov*; and knowing *OQ*, we know *Qv*, which reduce into time; and as *OQ* and *Oc* are known, we can find *Qc*, and therefore we know the time of describing *Qc*; hence, having found the time of describing *Qc* and *Qv*, we know the time of describing *cv*, which subtracted from the time at which the center was at *v*, gives the time when it was at *c*, or the time of the immersion of the center, called the true time, which ought to be the same to all observers. This quantity *vc* is M. FOUCHY's equation, and when applied to the observed time, should give the same time for all observations. We have here supposed *sn* to be known; the idea how to find this was first suggested by M. FOUCHY, and afterwards improved upon by M. BAILLY; his method we shall here explain.

461. M. BAILLY, by diaphragms with a circular hole in the middle, diminished gradually the field of view of his telescope until the satellite disappeared; hence the aperture in the diaphragm at the time the satellite becomes invisible,

is to the whole aperture, as the quantity of light received from the satellite at the time it disappears, to the quantity of light in the whole aperture; let the whole aperture $=1$, the aperture of the diaphragm $=a$; then the light of the satellite when not eclipsed being represented by unity, the light when it disappears at an immersion will be $=a$; consequently $a : 1 ::$ segment mnr : whole surface $mnrt$; hence we know the segment mnr, and consequently its versed sine ns, for on account of the smallness of the arc msr, we may consider it as a straight line. In a telescope whose aperture was 24 lines, M. BAILLY found the fourth satellite, when at its greatest elongation, to vanish at an aperture of 5.5 lines; hence, $a = \overline{\frac{5.5}{24}}\Big|^2 = 0{,}0525$, which is equal to the segment mnr, the circle $mnrt$ being unity; hence the versed sine $ns = 0{,}4303$. For the third satellite, he found $a = 0{,}0156$; and for the first and second $a = 0{,}0646$. These determinations were made, when the distance of Jupiter from the sun was 5,2207, and the distance of the earth from Jupiter 4,8456, the earth's distance from the sun being unity; also, the altitude of Jupiter above the horizon was 15°, and the satellites were at their greatest elongations. To reduce the invisible segments to any other situations of the earth and Jupiter, and any other altitude, he takes the light received at Jupiter to vary inversely as the square of its distance from the sun, and the light received at the earth from the satellites to vary inversely as the square of the distance of the earth from Jupiter; and for the variation of the quantity of light at different altitudes, he takes that which is given by M. BOUGUER in his Optics. To find the allowance to be made for the different distances of the satellites from Jupiter, he proceeds thus:

On July 17 and 23, 1771, the following observations were made, and the invisible segments determined as above explained, by taking into consideration the distances of Jupiter from the sun and the earth, and the altitude of Jupiter.

	H. M.	Dist. of Sat. in semid. ♃	Segment
July 17,	9. 58	1,36	0,2485
	10. 17	1,62	0,1677
	10. 43	1,96	0,1361
23,	11. 5	1,49	0,1862
	11. 20	1,32	0,2357
	11. 28	1,21	0,2910
	11. 35	1,11	0,3201
	11. 39	1,6	0,3521

The law which the variation of these segments follow is nearly as $\frac{b}{x^2}+\frac{c}{x}$, x being the distance of the satellite from the center of Jupiter in semidiameters of Jupiter; M. BAILLY therefore assumes $\frac{b}{x^2}+\frac{c}{x}=y$, y being the segment; and by taking two values of y and the corresponding values of x, we get two equations, from which we can determine b and c; the two values of y which he assumed are 0,1862 and 0,3521, and taking the corresponding values of x, he found $b=0,3397$, and $c=0,0495$; hence, $\frac{0,3397}{x^2}+\frac{0,0495}{x}=y$; by applying this to other observations, he found the errors much smaller than could be expected. By proceeding thus for the second and third satellites, he found for the

Satellites

I . . . $\frac{0,3397}{x^2}+\frac{0,0495}{x}=y$

II . . . $\frac{0,3933}{x^2}+\frac{0,0375}{x}=y$

III . . . $\frac{0,0756}{x^2}+\frac{0,2157}{x}=y$

IV . . . $\frac{0,192}{x^2}+\frac{0,053}{x}=y$

The fourth was determined by M. de la LANDE, M. BAILLY not having sufficient observations upon the satellite, to determine the law of variation.

462. M. BAILLY, in the last place, considers the effects of different telescopes. The greater the quantity of light which a telescope receives, or the greater the aperture, the less will be the invisible segment, and that in the inverse ratio of the aperture, for in this case, the same quantity of light comes to the eye. Hence, by taking into consideration all the circumstances, he reduced the observations, and found, in general, a very near agreement after the reduction, compared with the agreement between the observations themselves. The calculation requires that we should know the diameters of the satellites; these he deduced in the following manner.

463. On June 30, 1771, he observed the immersion of the first satellite. At 24 minutes in time before the immersion, with an aperture of 10,5 lines he lost sight of the satellite, the whole aperture being 24 lines. With an aperture of 13 lines, he made his observation of the immersion; and taking off the dia-

phragm, he then observed it 4'. 54" longer. Now an aperture of 10,5 gives the invisible segment BFG=0,1914; but here we must take into c deration the proximity of the satellite to Jupiter. At the time when the s lite disappeared with an aperture of 10,5 lines, it was 1,57 distant from Ju and at the immersion it was 1,25; hence, if we put these for x, we shal the corresponding segments 0,2571 and 0,1695; hence 0,1695 : 0,25 0,1914 : 0,2903 the invisible segment BFG which corresponds to the v aperture at the distance 1,25, deduced from that which was observed a distance 1,75. But with an aperture of 13 lines, the segment BDE mu greater in proportion as the aperture is less, or in the ratio of $13^2 : 24^2$; h BDE=0,9895. Now the versed sine BK=0,664, and BH=1,99852, t fore KH=1,33452, the space passed over in 4'. 54"; here the satellite en obliquely into the shadow; but if it had entered perpendicularly, it would taken only 4'. 51" to have passed over the same part; hence, 1,33452 : 2 diameter) :: 4'. 51" : 7'. 16" the diameter in time, which answers to 1°. 1 the diameter of the satellite seen from Jupiter. If the reader wish for an ther satisfaction upon this subject, he may consult the *Mem. de l'Acad. des Scien.* 1771; or the *Phil. Trans.* Vol. LXIII.

FIG. 104.

464. Dr. MASKELYNE observes, that the method here proposed of corre the immersion and emersion of a satellite, must be subject to a certain d of inaccuracy from hence, that when you reduce the aperture of the tele so as to make the satellite disappear, you also diminish the quantity of from Jupiter in the same proportion, on which account the satellite will l sible with a less quantity of light than it would be if Jupiter continued o same brightness, and therefore the invisible segment will have a less ra the whole surface, than the quantity of light in the aperture when the sa is rendered invisible has to the quantity of light in the whole aperture. A rection therefore for this circumstance ought to be applied. We may als ther observe, that besides the circumstances which are here taken noti the twilight, the clearness of the air, the proximity of Jupiter to the r and the eye of the observer, all combine to affect the time at which the lite becomes invisible.

465. When Jupiter is so far distant from conjunction with the sun, as about 8° above the horizon when the sun is 8° below, an eclipse of the sat will be visible at any place; this may be determined near enough by a ce globe (*Nautical Almanac*). *Before* the oppositions of Jupiter to the su immersions and emersions happen to the *west* of Jupiter; *after* opposition happen to the *east*. If an astronomical telescope be used, which revers jects, the appearance will be contrary. The satellites in the configurati the *Nautical Almanac* are put down on their proper sides of Jupiter; s telescope therefore reverses their situation in respect to Jupiter, making

on the east appear to the west, and those on the west appear to the east. The immersions signify the instant of the disappearance of the satellite by entering the shadow of Jupiter; and the emersions signify the instant at which they first appear at the coming out of the same. For directions to the observer, I refer the Reader to my *Practical Astronomy*, page 186.

466. M. CASSINI suspected that the satellites had a rotation about their axes, as sometimes in their passage over Jupiter's disc they were visible, and at other times not; he conjectured therefore that they had spots upon one side and not on the other, and that they were rendered visible in their passage when the spots were next the earth. At different times also they appear of different magnitudes and of different brightness. The fourth appears generally the smallest, but sometimes the greatest; and the diameter of its shadow on Jupiter appears sometimes greater than the satellite. The third also appears of a variable magnitude, and the like happens to the other two. M. MARALDI also concluded, from his own observations, that they had a rotation. Mr. POUND also observed that they appeared more luminous at one time than another, and therefore he concluded that they revolved about their axes. This is confirmed by Dr. HERSCHEL, who has discovered that all the satellites of *Jupiter* have a rotatory motion about their axes, of the same duration with their periodic times about their primary. This he determined from the change of brightness in different parts of their orbits. He observes that the first is white, but sometimes more intensely than others. The second is white, bluish and ash-coloured. The third always white, but of different intensities. The fourth is dusky, dingy, inclining to orange, reddish and ruddy at different times. At the mean distance of Jupiter, he makes the diameter of the second satellite $0'',87$; the third to be considerably the greatest; the first a little larger than the second, and nearly of the size of the fourth; the second a little smaller than the first and fourth, or the smallest of them all.

THE FOLLOWING TABLE EXHIBITS THF ELEMENTS OF THE SATELLITES, AS GIVEN BY M. DE LA LANDE, FROM THE BEST OBSERVATIONS.

ELEMENTS.	I.	II.	III.	IV.
Periodic revolution	$1^d. 18^h. 27'. 33'',476$	$3^d. 13^h. 13'. 41'',929$	$7^d. 3^h. 42'. 32'',879$	$16^d. 16^h. 32'. 8'',491$
Synodic revolution	1. 18. 28. 36	3. 13. 17. 54	7. 3. 59. 36	16. 18. 5. 7
Dist. in semid. ♃ by CASSINI . .	5,67	9,00	14,38	25,30
——— Sir I. NEWTON	5,965	9,494	15,141	26,63
Mean dist. in minutes at mean dist. ♃	$1'. 51''$	$2'. 57''$	$4'. 42''$	$8'. 16''$
Semid. of shadow in deg. of the orbit	$9^\circ. 35. 37$	$6^\circ. 1. 53$	$3^\circ. 43. 58$	$2^\circ. 8. 2$
——— time . . .	$1^h. 7. 55$	$1^h. 25. 40$	$1^h. 47. 0$	$2^h. 23. 0$
——— that of ♃ = 1 .	0,9941	0,9967	0,9857	0,9913
Half duration of an eclipse 90°. from node when the inclination is least	$1^h. 3'. 45''$	$1^h. 16'. 5''$	$1^h. 3'. 40''$	$0^h. 0'. 0''$
——— greatest .	1. 3. 45	1. 6. 49	0. 38. 22	0. 0. 0
For circ. shadow greatest inclination	$3^\circ. 18. 38$	$3^\circ. 46'. 0''$	$3^\circ. 25. 57$	$2^\circ. 36. 0$
For circ. shadow mean ———	3. 18. 38	3. 16. 0	3. 13. 58	2. 36. 0
For circ. shadow least ———	3. 18. 38	2. 46. 0	3. 2. 0	2. 36. 0
For ellip. shadow greatest ———	3. 4. 27	3. 29. 42	3. 11. 14	2. 24. 51
For ellip. shadow least ———	3. 4. 27	2. 34. 0	2. 49. 0	2. 24. 51
Epoch of conj. 1760 for mer. Greenwich	$0^d. 10^h. 35'. 0''$	$1. 14^h. 49'. 36''$	$2^d. 5^h. 32'. 29''$	$1^d. 7^h. 20'. 50''$
Mean place of the node in 1760 .	$10^s. 14^\circ. 30'$	$10^s. 13^\circ. 45'$	$10^s. 14^\circ. 24'$	$10^s. 16^\circ. 39'$
——— annual motion of the node .	$0'. 0''$	$2'. 3''$	$0'. 0''$	$4'. 19''$
——— long. for 1700 for mer. Greenwich	$2^s. 12^\circ. 33. 26$	$2^s. 12^\circ. 55. 28$	$5^s. 13^\circ. 0. 48$	$7^s. 17^\circ. 20. 38$
——— diurnal motion	6. 23. 29. 20,3798	3. 11. 22. 29,14275	1. 20. 19. 3,5389	0. 21. 34. 16,0008
——— secular motion	7. 25. 31. 13	3. 23. 10. 39	1. 22. 9. 19	6. 29. 50. 29

On the Construction of the Epochs in the Tables.

467. The epoch of a satellite for any year is found from a conjunction of the satellite; this will be best explained by an example. By Article 451, it was found that the time of a mean conjunction of the first satellite in 1764, was January 1, 8*h*. 26′. 47″, as computed for the construction of the Tables; this, however, is not the true time of the mean conjunction, but it is that time diminished by the sum of the maxima of all the equations, except that arising from the equation of Jupiter's orbit, which sum is 29′. 22″; add this therefore to the above time, and it gives January 1, 8*h*. 56′. 9″ for the true time of the mean conjunction, or the time when the mean place of the satellite upon its orbit was the same as the mean place of Jupiter in its orbit; but, by computation, the mean place of Jupiter at that time was 2ˢ. 8°. 56′. 41″, this therefore is the mean place of the satellite at the same time; but the mean motion of the satellite in 1*d*. 8*h*. 56′. 9″ was 9ˢ. 9°. 14′. 10″; subtract this therefore from 2ˢ. 8°. 56′. 41″, and we have 4ˢ. 29°. 42′. 31″ for the mean place of the satellite at the beginning of 1764, or the epoch for that year, at Paris. The construction of the Tables here explained, has been for the mean distance of Jupiter from the earth, that is, to represent the satellites as seen from that distance, because we applied the equations of light as explained in Article 420, which reduces the time at the place where the earth is at the time of observation, to the time at which the same phænomenon of the satellite (its conjunction) would have appeared if the earth had been at its mean distance from Jupiter. As Greenwich is 9′. 20″. east of the Observatory at Paris, if 1°. 19′. 8″, the mean motion of the satellite for that time, be added to 4ˢ. 29°. 42′. 31″, it gives 5ˢ. 1°. 1′. 39″ for the epoch for Greenwich, the year at Greenwich beginning 9′. 20″ later than at Paris. But the epochs in these Tables are for the *least* distance of Jupiter from the earth, and consequently they are found from the epochs at the *mean* distance, by adding to them the mean motions of the satellites for 10′. 10″, that being the time (420) which light takes, in passing over a space equal to the difference between the least and mean distances. For as any situations of the satellites appear 10′. 10″ sooner at the least than at the mean distance of Jupiter from the earth, at any point of time they must appear forwarder in their orbits at the former than at the latter distance by their motions in that time. Now the mean motion of the satellite in 10′. 10″ is 1°. 26′. 12″; hence, the epoch for 1764, for the least distance of Jupiter, is 5ˢ. 2°. 27′. 51″; the Tables which are here given, were constructed from other observations. Having determined the epoch for any one year, the epochs for the following years are found by continually adding to it,

the mean motions for a year, as explained in Article 452; and the epochs for the preceding years are found by subtraction; thus we continue the Tables as far as we please.

468. The first Table contains the epochs of the satellites at the beginning of the year, that is, on December 31 of the preceding year by the civil account, at 12 o'clock at noon, mean time; except on leap-year, in which the place is put down for January 1, at 12 o'clock at noon, mean time.

Table the second contains the mean motions for months, showing at the end of each month, how much forwarder the satellites are than they were at the beginning of the year. If therefore to the place at the beginning of the year, you add the mean motion for any month, it gives the mean place for the end of that month, or for the beginning of the next. The month of February is here supposed to contain 28 days. Now in leap-year, the epoch being for the first of January at noon, mean time, when we add the motion for January, it gives the place on February 1, at noon; and adding the motion for February it gives the place for the last day at noon, because from January 1, to February 29, in leap-year, is the same as from December 31 to February 28, in the common years; hence, the mean motions for the other months added to the epochs, will give the mean places as well in leap-year as in the common years.

The third Table contains the mean motions for days, as far as 31, that being sufficient, as we have the mean motions for months. But in leap-year, in the months of January and February, we must take the motion for one day less than the day of the month, because (as above explained) the epoch is for the first of January, and the motion for a month being added gives the place on the first of February; the places therefore being thus obtained after one day in each month has passed, the motion from that time to any other day must be one day less than the number of days of the month.

The fourth Table contains the mean motions for hours; and the fifth Table contains the mean motions for minutes and seconds.

The sixth Table contains the apparent distances of the satellites from the center of Jupiter in terms of its semidiameters, according to their situations in their orbits, and the geocentric place of Jupiter.

The seventh Table contains the first equation of light; the eighth contains the second equation of light. These Tables are constructed to the nearest distance of Jupiter from the earth; and therefore at all other times, the satellites will appear to come later to the places found from the Tables than the time to which they were computed, by the equations in the Tables.

Table the ninth contains the equation of the center of the fourth satellite.

469. These Tables give only the *mean* places of the satellites, except for the fourth satellite, whose place may be corrected by the equation of the orbit.

This accuracy is sufficient for the purpose for which the Tables are here given, they being principally intended to find the configurations of the satellites. In the Tables for computing the eclipses, the epochs for each year are those of the first mean conjunction of the satellite after the commencement of the year; the construction of these Tables, and their uses, will be explained in the Third Volume.

470. If it be required to find the apparent positions of the satellites at any given apparent time, that time must be converted into mean time (the Tables being constructed to mean time) by applying the equation of time; and then from that mean time, the equation of light must be subtracted, and the computation made for that time; and from the places thus found, we must subtract the geocentric place of Jupiter, and proceed as already explained.

EPOCHS OF THE MEAN MOTIONS OF JUPITER'S SATELITES.

TABLE I.

YEARS, NEW STILE.	I SATELLITE	II. SATELLITE	III. SATELLITE	IV SATELLITE	IV. SAT APS
	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M.
1790	0. 4. 13. 37	10. 4. 57. 2	0. 5. 9. 5	3. 28. 1. 10	1. 7. 4
1791	3. 27. 42. 36	7. 16. 44. 19	0. 11. 5. 36	2. 11. 28. 30	1. 7. 49
*B.*1792	2. 14. 40. 55	8. 9. 54. 5	2. 7. 21. 12	1. 16. 31. 6	1. 8. 34
1793	6. 8. 9. 53	5. 21. 41. 22	2. 13. 17. 44	11. 29. 58. 26	1. 9. 19
1794	10. 1. 38. 52	3. 3. 28. 39	2. 19. 14. 15	10. 13. 25. 46	1. 10. 4
1795	1. 25. 7. 50	0. 15. 7. 56	2. 25. 10. 47	8. 26. 53. 6	1. 10. 49
*B.*1796	0. 12. 6. 9	1. 8. 17. 43	4. 21. 26. 23	8. 1. 54. 42	1. 11. 35
1797	4. 5. 35. 8	10. 20. 5. 0	4. 27. 22. 54	6. 15. 22. 2	1. 12. 20
1798	7. 29. 4. 7	8. 1. 52. 17	5. 3. 19. 26	4 28. 49. 22	1. 13. 5
1799	11. 22. 33. 5	5. 13. 39. 34	5. 9. 15. 58	3. 12. 17. 42	1. 13. 50
C. 1800	3. 16. 2. 4	2. 25. 26. 51	5. 15. 12. 30	1. 25. 45. 2	1. 14. 35
1801	7. 9. 31. 3	0. 7. 14. 8	5. 21. 9. 2	0. 9. 12. 22	1. 15 20
1802	11. 3. 0. 1	9. 19. 1. 25	5. 27. 5. 34	10 22. 39. 42	1. 16. 6
1803	2. 26. 29. 0	7. 0. 48. 42	6. 3. 2. 6	9. 6 7. 2	1. 16. 51
*B.*1804	1. 13. 27. 19	7. 23. 58. 29	7. 29. 17. 41	8. 11. 8. 38	1. 17. 36
1805	5. 6. 56. 18	5. 5. 45. 46	8. 5. 14. 13	6. 24. 35. 58	1. 18. 21
1806	9. 0. 25. 16	2. 17. 33. 5	8. 11. 10. 45	5. 8. 3. 18	1. 19. 6
1807	0. 23. 54. 16	11. 29. 20. 20	8. 17. 7. 16	3. 21. 30. 38	1. 19. 51
*B.*1808	11. 10. 52. 34	0. 22. 30. 6	10. 13. 22. 52	2. 26. 32 14	1. 20. 37
1809	3. 4. 21. 32	10. 4. 17. 23	10. 19. 19. 24	1. 9. 59. 34	1. 21. 22

THE FIRST TABLE CONTINUED.

YEARS, NEW STILE.	I. SATELLITE	II. SATELLITE.	III. SATELLITE	IV. SATELLITE	IV. S. APS.
	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. D. M. S.	S. M. D.
1810	6. 27. 50. 31	7. 16. 4. 40	10. 25. 15. 55	11. 23. 26. 54	1. 22. 7
1811	10. 21. 19. 30	4. 27. 51. 57	11. 1. 12. 27	10. 6. 54. 14	1. 22. 52
B. 1812	9. 8. 17. 49	5. 21. 1. 44	0. 27. 28. 3	9. 11. 55. 50	1. 23. 37
1813	1. 1. 46. 47	3. 2. 49. 1	1. 3. 24. 34	7. 25. 23. 10	1. 24. 22
1814	4. 25. 15. 46	0. 14. 36. 18	1. 9. 21. 6	6. 8. 50. 30	1. 25. 8
1815	8. 18. 44. 45	9. 26. 23. 35	1. 15. 17. 38	4. 22. 17. 50	1. 25. 53
B. 1816	7. 5. 43. 4	10. 19. 33. 21	3. 11. 33. 14	3. 27. 19. 26	1. 26. 38
1817	10. 29. 12. 2	8. 1. 20. 38	3. 17. 29. 45	2. 10. 46. 46	1. 27. 23
1818	2. 22. 41. 1	5. 13. 7. 55	3. 23. 26. 17	0. 24. 14. 6	1. 28. 8
1819	6. 16. 10. 0	2. 24. 55. 13	3. 29. 22. 49	11. 7. 41. 26	1. 28. 53
B. 1820	5. 3. 8. 18	3. 18. 4. 59	5. 25. 38. 24	10. 12. 43. 2	1. 29. 39

THE MEAN MOTION OF JUPITER'S SATELLITES FOR *MONTHS.*

TABLE II.

MONTHS.	I. SATELLITE	II. SATELLITE	III. SATELLITE	IV. SATELLITE	IV. APS.
31 January	6ˢ. 8°. 9′.32″	8ˢ.22°.37′. 3″	3ˢ.29°.50′.50″	10ˢ. 8°.42′.16″	3′. 50″
28 February	4. 5. 51. 2	7. 11. 6. 38	2. 28. 44. 29	6. 12. 41. 44	7. 18
31 March	10. 14. 0. 34	4. 3. 43. 40	6. 28. 35. 18	4. 21. 24. 0	11. 7
30 April	9. 28. 40. 46	9. 14. 58. 14	9. 8. 7. 4	2. 8. 32. 0	14. 50
31 May	4. 6. 50. 17	6. 7. 35. 16	1. 7. 57. 53	0. 17. 14. 16	18. 40
30 June	3. 21. 30. 29	11. 18. 49. 50	3. 17. 29. 40	10. 4. 22. 16	22. 22
31 July	9. 29. 40. 1	8. 11. 26. 52	7. 17. 20. 29	8. 13. 4. 32	26. 12
31 August	4. 7. 49. 32	5. 4. 3. 55	11. 17. 11. 19	6. 21. 46. 48	30. 2
30 Sept.	3. 22. 29. 44	10. 15. 18. 28	1. 26. 43. 5	4. 8. 54. 48	33. 45
31 October	10. 0. 39. 15	7. 7. 55. 31	5. 26. 33. 55	2. 17. 37. 4	37. 35
30 Nov.	9. 15. 19. 27	0. 19. 10. 4	8. 6. 5. 41	0. 4. 45. 4	41. 17
31 Dec.	3. 23. 28. 59	9. 11. 47. 7	0. 5. 56. 31	10. 13. 27. 20	45. 7

THE MEAN MOTION OF JUPITER's SATELLITES FOR *DAYS.*

TABLE III.

DAYS	I.				II.				III.				IV.				IV. APS.	
	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	M.	S.
1	6.	23.	29.	20	3.	11.	22.	29	1.	20.	19.	4	0.	21.	34.	16	0.	7
2	1.	16.	58.	41	6.	22.	44.	58	3.	10.	38.	7	1.	13.	8.	32	0.	15
3	8.	10.	28.	1	10.	4.	7.	27	5.	0.	57.	11	2.	4.	42.	48	0.	22
4	3.	3.	57.	22	1.	15.	29.	57	6.	21.	16.	14	2.	26.	17.	4	0.	30
5	9.	27.	26.	42	4.	26.	52.	26	8.	11.	35.	18	3.	17.	51.	20	0.	37
6	4.	20.	56.	2	8.	8.	14.	55	10.	1.	54.	21	4.	9.	25.	36	0.	44
7	11.	14.	25.	22	11.	19.	37.	24	11.	22.	13.	25	5.	0.	59.	52	0.	52
8	6.	7.	54.	43	3.	0.	59.	53	1.	12.	32.	28	5.	22.	34.	8	0.	59
9	1.	1.	24.	3	6.	12.	22.	22	3.	2.	51.	32	6.	14.	8.	24	1.	7
10	7.	24.	53.	23	9.	23.	44.	51	4.	23.	10.	35	7.	5.	42.	40	1.	14
11	2.	18.	22.	44	1.	5.	7.	21	6.	13.	29.	39	7.	27.	16.	56	1.	22
12	9.	11.	52.	4	4.	16.	29.	50	8.	3.	48.	42	8.	18.	51.	12	1.	29
13	4.	5.	21.	25	7.	27.	52.	19	9.	24.	7.	46	9.	10.	25.	28	1.	36
14	10.	28.	50.	45	11.	9.	14.	48	11.	14.	26.	50	10.	1.	59.	44	1.	44
15	5.	22.	20.	6	3.	20.	37.	17	1.	4.	45.	53	10.	23.	34.	0	1.	51
16	0.	15.	49.	26	6.	1.	59.	46	2.	25.	4.	57	11.	15.	8.	16	1.	59
17	7.	9.	18.	46	9.	13.	22.	15	4.	15.	23.	0	0.	6.	42.	32	2.	6
18	2.	2.	48.	6	0.	24.	44.	45	6.	5.	43.	4	0.	28.	16.	48	2.	13
19	8.	26.	17.	27	4.	6.	7.	14	7.	26.	2.	7	1.	19.	51.	4	2.	21
20	3.	19.	46.	47	7.	17.	29.	43	9.	16.	21.	11	2.	11.	25.	20	2.	28
21	10.	13.	16.	8	10.	28.	52.	12	11.	6.	40.	14	3.	2.	59.	36	2.	36
22	5.	6.	45.	29	2.	10.	14.	41	0.	26.	59.	18	3.	24.	33.	52	2.	43
23	0.	0.	14.	49	5.	21.	37.	10	2.	17.	18.	21	4.	16.	8.	8	2.	51
24	6.	23.	44.	9	9.	2.	59.	39	4.	7.	37.	25	5.	7.	42.	34	2.	58
25	1.	17.	13.	29	0.	14.	22.	9	5.	27.	56.	28	5.	29.	16.	50	3.	5
26	8.	10.	42.	50	3.	25.	44.	38	7.	18.	15.	32	6.	20.	50.	56	3.	13
27	3.	4.	12.	10	7.	7.	7.	7	9.	8.	34.	36	7.	12.	25.	12	3.	20
28	9.	27.	41.	30	10.	18.	29.	36	10.	28.	53.	39	8.	3.	59.	28	3.	28
29	4.	21.	10.	51	1.	29.	52.	5	0.	19.	12.	43	8.	25.	33.	44	3.	35
30	11.	14.	40.	11	5.	11.	14.	34	2.	9.	31.	46	9.	17.	8.	0	3.	42
31	6.	8.	9.	32	8.	22.	37.	3	3.	29.	50.	50	10.	8.	42.	16	3.	50

In leap-year, for *January* and *February* take one day less, for reasons already given.

THE MEAN MOTIONS OF JUPITER'S SATELLITES FOR *HOURS*.

TABLE IV.

HOURS	I.				II.				III.				IV.			
	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.
1	0.	8.	28.	43	0.	4.	13.	26	0.	2.	5.	48	0.	0.	53.	56
2	0.	16.	57.	27	0.	8.	26.	52	0.	4.	11.	35	0.	1.	47.	51
3	0.	25.	26.	10	0.	12.	40.	19	0.	6.	17.	23	0.	2.	41.	47
4	1.	3.	54.	53	0.	16.	53.	45	0.	8.	23.	11	0.	3.	35.	43
5	1.	12.	23.	36	0.	21.	7.	11	0.	10.	28.	58	0.	4.	29.	38
6	1.	20.	52.	20	0.	25.	20.	37	0.	12.	34.	46	0.	5.	23.	34
7	1.	29.	21.	3	0.	29.	34.	3	0.	14.	40.	33	0.	6.	17.	30
8	2.	7.	49.	46	1.	3.	47.	30	0.	16.	46.	21	0.	7.	11.	25
9	2.	16.	18.	30	1.	8.	0.	56	0.	18.	52.	9	0.	8.	5.	21
10	2.	24.	47.	13	1.	12.	14.	22	0.	20.	57.	56	0.	8.	59.	17
11	3.	3.	15.	56	1.	16.	27.	48	0.	23.	3.	44	0.	9.	53.	13
12	3.	11.	44.	40	1.	20.	41.	15	0.	25.	9.	32	0.	10.	47.	8
13	3.	20.	13.	24	1.	24.	54.	41	0.	27.	15.	19	0.	11.	41.	4
14	3.	28.	42.	7	1.	29.	8.	7	0.	29.	21.	7	0.	12.	35.	0
15	4.	7.	10.	51	2.	3.	21.	33	1.	1.	26.	55	0.	13.	28.	55
16	4.	15.	39.	34	2.	7.	34.	59	1.	3.	32.	42	0.	14.	22.	51
17	4.	24.	8.	17	2.	11.	48.	26	1.	5.	38.	30	0.	15.	16.	47
18	5.	2.	37.	0	2.	16.	1.	52	1.	7.	44.	18	0.	16.	10.	42
19	5.	11.	5.	43	2.	20.	15.	18	1.	9.	50.	5	0.	17.	4.	38
20	5.	19.	34.	27	2.	24.	28.	44	1.	11.	55.	53	0.	17.	58.	34
21	5.	28.	3.	10	2.	28.	42.	11	1.	14.	1.	41	0.	18.	52.	29
22	6.	6.	31.	53	3.	2.	55.	37	1.	16.	7.	28	0.	19.	46.	25
23	6.	15.	0.	37	3.	7.	9.	3	1.	18.	13.	16	0.	20.	40.	21
24	6.	23.	29.	20	3.	11.	22.	29	1.	20.	19.	4	0.	21.	34.	16

THE MEAN MOTIONS OF JUPITER's SATELLITES FOR *MINUTES* AND *SECONDS.*

TABLE V.

M. S.	D. M.	M. S.	S. T.	D. M.	M. S.	S. T.	D. M.	M. S.	S. T.	D. M.	M. S.	S. T.
	I.			II.			III.			IV.		
1	0.	8.	29	0.	4.	13	0.	2.	6	0.	0.	54
2	0.	16.	57	0.	8.	27	0.	4.	12	0.	1.	48
3	0.	25.	26	0.	12.	40	0.	6.	17	0.	2.	42
4	0.	33.	55	0.	16.	54	0.	8.	23	0.	3.	36
5	0.	42.	24	0.	21.	7	0.	10.	29	0.	4.	30
6	0.	50.	52	0.	25.	21	0.	12.	35	0.	5.	24
7	0.	59.	21	0.	29.	34	0.	14.	41	0.	6.	18
8	1.	7.	50	0.	33.	47	0.	16.	46	0.	7.	11
9	1.	16.	18	0.	38.	1	0.	18.	52	0.	8.	5
10	1.	24.	47	0.	42.	14	0.	20.	58	0.	8.	59
11	1.	33.	16	0.	46.	28	0.	23.	4	0.	9.	53
12	1.	41.	45	0.	50.	41	0.	25.	10	0.	10.	47
13	1.	50.	13	0.	54.	55	0.	27.	15	0.	11.	41
14	1.	58.	42	0.	59.	8	0.	29.	21	0.	12.	35
15	2.	7.	11	1.	3.	22	0.	31.	27	0.	13.	39
16	2.	15.	40	1.	7.	35	0.	33.	33	0.	14.	23
17	2.	24.	8	1.	11.	48	0.	35.	39	0.	15.	17
18	2.	32.	37	1.	16.	2	0.	37.	44	0.	16.	11
19	2.	41.	6	1.	20.	15	0.	39.	50	0.	17.	5
20	2.	49.	34	1.	24.	29	0.	41.	56	0.	17.	59
21	2.	58.	3	1.	28.	42	0.	44.	2	0.	18.	52
22	3.	6.	32	1.	32.	56	0.	46.	7	0.	19.	46
23	3.	15.	1	1.	37.	9	0.	48.	13	0.	20.	40
24	3.	23.	29	1.	41.	22	0.	50.	19	0.	21.	34
25	3.	31.	58	1.	45.	36	0.	52.	25	0.	22.	28
26	3.	40.	27	1.	49.	49	0.	54.	31	0.	23.	22
27	3.	48.	55	1.	54.	3	0.	56.	37	0.	24.	16
28	3.	57.	24	1.	58.	16	0.	58.	42	0.	25.	10
29	4.	5.	53	2.	2.	30	1.	0.	48	0.	26.	4
30	4.	14.	22	2.	6.	43	1.	2.	54	0.	26.	58

THE MEAN MOTIONS OF JUPITER's SATELLITES FOR *MINUTES* AND *SECONDS*.

THE FIFTH TABLE CONTINUED.

M. S.	D. M. S. M. S. T. I.	D. M. S. M. S. T. II.	D. M. S. M. S. T. III.	D. M. S. M. S. T. IV.
31	4. 22. 50	2. 10. 57	1. 5. 0	0. 27. 52
32	4. 31. 19	2. 15. 10	1. 7. 6	0. 28. 46
33	4. 39. 48	2. 19. 23	1. 9. 11	0. 29. 40
34	4. 48. 17	2. 23. 37	1. 11. 17	0. 30. 33
35	4. 56. 45	2. 27. 50	1. 13. 23	0. 31. 27
36	5. 5. 14	2. 32. 4	1. 15. 29	0. 32. 21
37	5. 13. 43	2. 36. 17	1. 17. 35	0. 33. 15
38	5. 22. 11	2. 40. 31	1. 19. 40	0. 34. 9
39	5. 30. 40	2. 44. 44	1. 21. 46	0. 35. 3
40	5. 39. 9	2. 48. 57	1. 23. 52	0. 35. 57
41	5. 47. 48	2. 53. 11	1. 25. 58	0. 36. 51
42	5. 56. 6	2. 57. 24	1. 28. 4	0. 37. 45
43	6. 4. 35	3. 1. 38	1. 30. 9	0. 38. 39
44	6. 13. 4	3. 5. 51	1. 32. 15	0. 39. 33
45	6. 21. 32	3. 10. 5	1. 34. 21	0. 40. 27
46	6. 30. 1	3. 14. 18	1. 36. 27	0. 41. 21
47	6. 38. 30	3. 18. 31	1. 38. 33	0. 42. 15
48	6. 46. 59	3. 22. 45	1. 40. 38	0. 43. 9
49	6. 55. 27	3. 26. 58	1. 42. 44	0. 44. 2
50	7. 3. 56	3. 31. 12	1. 44. 50	0. 44. 56
51	7. 12. 25	3. 35. 25	1. 46. 56	0. 45. 50
52	7. 20. 54	3. 39. 39	1. 49. 2	0. 46. 44
53	7. 29. 22	3. 43. 52	1. 51. 7	0. 47. 38
54	7. 37. 51	3. 48. 6	1. 53. 13	0. 48. 32
55	7. 46. 20	3. 52. 19	1. 55. 19	0. 49. 26
56	7. 54. 48	3. 56. 32	1. 57. 25	0. 50. 20
57	8. 3. 17	4. 0. 46	1. 59. 31	0. 51. 14
58	8. 11. 46	4. 4. 59	2. 1. 36	0. 52. 8
59	8. 20. 15	4. 9. 12	2. 3. 42	0. 53. 4
60	8. 28. 43	4. 13. 26	2. 5. 48	0. 53. 56

APPARENT DISTANCES OF THE SATELLITES FROM THE CENTER OF JUPITER, IN SEMIDIAMETERS OF JUPITER.

TABLE VI.

	DISTANCE OF THE SATELLITES FROM THE GEOCENTRIC PLACE OF JUPITER.												
Sig	O *East*			VI *West.*	I. *East.*			VII. *West*	II *East*			VIII. *West*	
Sat	I	II.	III	IV	I	II.	III.	IV	I.	II.	III.	IV.	
Deg	Semid	Semid	Semid.	Semid	Semid	Semid	Semid.	Semid	Semid	Semid.	Semid.	Semid.	Deg
0	0,0	0,0	0,0	0,0	2,95	4,70	7,50	13,19	5,12	8,14	12,99	22,85	30
1	0,10	0,16	0,26	0,46	3,04	4,84	7,73	13,59	5,17	8,22	13,12	23,07	29
2	0,21	0,33	0,52	0,92	3,13	4,98	7,95	13,98	5,22	8,30	13,24	23,29	28
3	0,31	0,49	0,78	1,38	3,22	5,12	8,17	14,37	5,27	8,38	13,36	23,51	27
4	0,41	0,66	1,05	1,84	3,30	5,26	8,39	14,75	5,31	8,45	13,48	23,71	26
5	0,51	0,82	1,31	2,30	3,39	5,39	8,60	15,13	5,36	8,52	13,59	23,91	25
6	0,62	0,98	1,57	2,76	3,47	5,53	8,82	15,51	5,40	8,59	13,70	24,10	24
7	0,72	1,14	1,83	3,22	3,56	5,66	9,03	15,88	5,44	8,66	13,81	24,28	23
8	0,82	1,31	2,09	3,67	3,64	5,79	9,24	16,24	5,48	8,72	13,91	24,46	22
9	0,92	1,47	2,34	4,13	3,72	5,92	9,44	16,60	5,52	8,78	14,00	24,63	21
10	1,03	1,63	2,60	4,58	3,80	6,04	9,64	16,96	5,55	8,84	14,10	24,79	20
11	1,13	1,79	2,86	5,03	3,87	6,17	9,84	17,31	5,59	8,89	14,18	24,95	19
12	1,23	1,95	3,11	5,48	3,95	6,29	10,04	17,65	5,62	8,94	14,27	25,09	18
13	1,33	2,12	3,37	5,93	4,03	6,41	10,23	17,99	5,65	8,99	14,34	25,23	17
14	1,43	2,27	3,63	6,38	4,10	6,53	10,42	18,32	5,68	9,04	14,42	25,36	16
15	1,53	2,43	3,88	6,83	4,18	6,65	10,61	18,65	5,71	9,08	14,49	25,48	15
16	1,63	2,59	4,13	7,27	4,25	6,76	10,79	18,98	5,73	9,12	14,56	25,60	14
17	1,73	2,75	4,38	7,71	4,32	6,88	10,98	19,30	5,76	9,16	14,62	25,71	13
18	1,83	2,91	4,63	8,15	4,39	6,99	11,15	19,61	5,78	9,20	14,67	25,81	12
19	1,92	3,06	4,88	8,59	4,46	7,09	11,32	19,91	5,80	9,23	14,72	25,90	11
20	2,02	3,22	5,13	9,02	4,53	7,20	11,49	20,21	5,82	9,26	14,77	25,98	10
21	2,12	3,37	5,37	9,45	4,59	7,31	11,66	20,50	5,83	9,29	14,81	26,06	9
22	2,22	3,52	5,62	9,88	4,66	7,41	11,82	20,79	5,85	9,31	14,85	26,13	8
23	2,31	3,67	5,86	10,31	4,72	7,51	11,98	21,07	5,87	9,33	14,89	26,19	7
24	2,40	3,82	6,11	10,73	4,78	7,61	12,14	21,34	5,88	9,35	14,92	26,24	6
25	2,50	3,97	6,34	11,15	4,84	7,70	12,29	21,61	5,89	9,3	14,94	26,28	5
26	2,59	4,12	6,57	11,57	4,90	7,80	12,44	21,87	5,89	9,38	14,96	26,32	4
27	2,68	4,27	6,81	11,98	4,96	7,89	12,58	22,13	5,90	9,39	14,98	26,35	3
28	2,77	4,41	7,04	12,39	5,01	7,97	12,72	22,37	5,91	9,40	14,99	26,37	2
29	2,86	4,56	7,27	12,79	5,07	8,06	12,86	22,61	5,91	9,40	15,00	26,38	1
30	2,95	4,70	7,50	13,19	5,12	8,14	12,99	22,85	5,91	9,40	15,00	26,38	0
Sig	XI. *West*			V *East*	X. *West.*			IV *East.*	IX. *West.*			III *East.*	

THE *FIRST* EQUATION OF LIGHT.

TABLE VII.

ARGUMENT. DISTANCE OF JUPITER FROM THE SUN.							
Deg.	Sig. O.	Sig. I.	Sig. II.	Sig. III.	Sig. IV.	Sig. V.	Deg.
0	16′. 15″	15′. 10″	12′. 12″	8′. 7″	4′. 3″	1′. 5″	30
1	16. 15	15. 6	12. 5	7. 59	3. 56	1. 1	29
2	16. 15	15. 1	11. 57	7. 51	3. 49	0. 57	28
3	16. 14	14. 56	11. 49	7. 42	3. 42	0. 53	27
4	16. 14	14. 52	11. 41	7. 34	3. 35	0. 49	26
5	16. 13	14. 47	11. 33	7. 25	3. 28	0. 45	25
6	16. 12	14. 42	11. 25	7. 16	3. 21	0. 42	24
7	16. 11	14. 37	11. 17	7. 8	3. 14	0. 38	23
8	16. 10	14. 32	11. 9	6. 59	3. 7	0. 36	22
9	16. 9	14. 27	11. 2	6. 51	3. 0	0. 33	21
10	16. 7	14. 21	10. 55	6. 42	2. 53	0. 30	20
11	16. 5	14. 15	10. 47	6. 34	2. 47	0. 27	19
12	16. 4	14. 9	10. 38	6. 26	2. 41	0. 24	18
13	16. 2	14. 3	10. 30	6. 18	2. 35	0. 22	17
14	16. 0	13. 58	10. 22	6. 10	2. 29	0. 19	16
15	15. 58	13. 52	10. 14	6. 1	2. 23	0. 17	15
16	15. 56	13. 46	10. 6	5. 53	2. 18	0. 15	14
17	15. 54	13. 40	9. 57	5. 45	2. 12	0. 13	13
18	15. 51	13. 34	9. 48	5. 37	2. 6	0. 11	12
19	15. 48	13. 28	9. 40	5. 29	2. 0	0. 10	11
20	15. 45	13. 22	9. 32	5. 21	1. 54	0. 8	10
21	15. 42	13. 15	9. 24	5. 13	1. 48	0. 6	9
22	15. 39	13. 8	9. 16	5. 6	1. 43	0. 5	8
23	15. 36	13. 1	9. 8	4. 58	1. 38	0. 4	7
24	15. 33	12. 54	8. 59	4. 50	1. 33	0. 3	6
25	15. 30	12. 47	8. 50	4. 42	1. 28	0. 2	5
26	15. 26	12. 40	8. 41	4. 34	1. 23	0. 1	4
27	15. 22	12. 33	8. 32	4. 26	1. 19	0. 1	3
28	15. 18	12. 26	8. 23	4. 18	1. 14	0. 0	2
29	15. 14	12. 19	8. 15	4. 11	1. 9	0. 0	1
30	15. 10	12. 12	8. 7	4. 3	1. 5	0. 0	0
	Sig. XI.	Sig. X.	Sig. IX.	Sig. VIII.	Sig. VII.	Sig. VI.	

THE *SECOND* EQUATION OF LIGHT.

TABLE VIII.

ARGUMENT. ANOMALY OF JUPITER							
Deg.	Sig. O.	Sig. I.	Sig. II.	Sig. III.	Sig. IV.	Sig. V.	Deg.
0	4′. 4″	3′. 47″	3′. 3″	2′. 2″	1′. 1″	0′. 16″	30
1	4. 4	3. 46	3. 1	2. 0	0. 59	0. 15	29
2	4. 4	3. 45	2. 59	1. 58	0. 57	0. 14	28
3	4. 4	3. 44	2. 57	1. 56	0. 55	0. 13	27
4	4. 4	3. 43	2. 55	1. 53	0. 54	0. 12	26
5	4. 3	3. 42	2. 53	1. 51	0. 52	0. 12	25
6	4. 3	3. 41	2. 51	1. 49	0. 51	0. 11	24
7	4. 3	3. 39	2. 49	1. 47	0. 49	0. 10	23
8	4. 3	3. 38	2. 47	1. 45	0. 47	0. 9	22
9	4. 2	3. 37	2. 45	1. 43	0. 46	0. 8	21
10	4. 2	3. 35	2. 44	1. 41	0. 44	0. 7	20
11	4. 2	3. 34	2. 42	1. 39	0. 43	0. 6	19
12	4. 1	3. 32	2. 40	1. 37	0. 41	0. 6	18
13	4. 0	3. 31	2. 38	1. 35	0. 40	0. 5	17
14	4. 0	3. 30	2. 36	1. 33	0. 38	0. 5	16
15	4. 0	3. 28	2. 34	1. 30	0. 36	0. 4	15
16	3. 59	3. 27	2. 32	1. 28	0. 35	0. 4	14
17	3. 59	3. 25	2. 29	1. 26	0. 33	0. 3	13
18	3. 58	3. 24	2. 27	1. 24	0. 32	0. 3	12
19	3. 57	3. 22	2. 25	1. 22	0. 30	0. 2	11
20	3. 56	3. 21	2. 23	1. 20	0. 29	0. 2	10
21	3. 55	3. 20	2. 21	1. 18	0. 27	0. 1	9
22	3. 55	3. 17	2. 19	1. 16	0. 26	0. 1	8
23	3. 54	3. 15	2. 17	1. 14	0. 25	0. 1	7
24	3. 53	3. 13	2. 15	1. 12	0. 23	0. 1	6
25	3. 52	3. 12	2. 13	1. 10	0. 22	0. 0	5
26	3. 51	3. 10	2. 10	1. 8	0. 21	0. 0	4
27	3. 50	3. 8	2. 8	1. 7	0. 20	0. 0	3
28	3. 49	3. 6	2. 6	1. 5	0. 19	0. 0	2
29	3. 48	3. 5	2. 4	1. 3	0. 17	0. 0	1
30	3. 47	3. 3	2. 2	1. 1	0. 16	0. 0	0
	Sig. XI	Sig. X.	Sig. IX.	Sig. VIII	Sig. VII.	Sig. VI.	

EQUATION OF THE CENTER OF THE *FOURTH* SATELLITE.

TABLE IX.

ARGUMENT. MEAN ANOMALY.				
Deg.	+ VI − O.	+ VII − I.	+ VIII − II.	Deg.
0	0′. 0″	24′. 57″	43′. 22″	30
1	0. 52	25. 42	43. 48	29
2	1. 44	26. 26	44. 14	28
3	2. 37	27. 10	44. 39	27
4	3. 29	27. 54	45. 3	26
5	4. 21	28. 37	45. 26	25
6	5. 13	29. 20	45. 48	24
7	6. 5	30. 2	46. 9	23
8	6. 57	30. 44	46. 30	22
9	7. 48	31. 25	46. 50	21
10	8. 39	32. 6	47. 9	20
11	9. 31	32. 46	47. 26	19
12	10. 22	33. 25	47. 42	18
13	11. 13	34. 4	47. 58	17
14	12. 4	34. 43	48. 13	16
15	12. 55	35. 21	48. 28	15
16	13. 45	35. 47	48. 42	14
17	14. 36	36. 33	48. 56	13
18	15. 26	37. 8	49. 7	12
19	16. 15	37. 43	49. 18	11
20	17. 4	38. 18	49. 28	10
21	17. 53	38. 52	49. 37	9
22	18. 41	39. 25	49. 46	8
23	19. 29	39. 57	49. 54	7
24	20. 17	40. 28	50. 1	6
25	21. 5	40. 59	50. 6	5
26	21. 52	41. 29	50. 9	4
27	22. 39	41. 58	50. 14	3
28	23. 26	42. 27	50. 17	2
29	24. 12	42. 55	50. 19	1
30	24. 57	43. 22	50. 20	0
	+ XI − V.	+ X − IV.	+ IX − III.	

On the Satellites of Saturn.

471. In the year 1655, HUYGENS discovered the fourth satellite of *Saturn*; and published a Table of its mean motion in 1659. In 1671, M. CASSINI discovered the fifth, and the third in 1672; and in 1684, the first and second; and afterwards published Tables of their motions. He called them *Sidera Lodoicea*, in honour of LOUIS le GRAND, in whose reign, and observatory, they were first discovered. Dr. HALLEY found by his own observations in 1682, that HUYGENS's Tables had considerably run out, they being about 15°. in 20 years too forward, and therefore he composed new Tables from more correct elements. He also reformed M. CASSINI's Tables of the mean motions; and about the year 1720, published them a second time, corrected from Mr. POUND's observations. He observes, that the four innermost satellites describe orbits very nearly in the plane of the ring, which he says is, as to sense, parallel to our equator; and that the orbit of the fifth is a little inclined to them. The following Table contains the periodic times of the five satellites, and their distances in semidiameters of the ring, as determined by Mr. POUND, by a micrometer fitted to the telescope given by HUYGENS to the Royal Society. Mr. POUND first measured the distance of the fourth, and then deduced the rest from the proportion between the squares of the periodic times and cubes of their distances, and these are found to agree with observations.

Satellites.	Periodic Times by POUND.	Dist. in semid. of Ring by POUND.	Dist in semid of Saturn by POUND.	Dist. in semid. of Ring by CASSINI.	Dist. at the mean dist. of Saturn.
I	1d. 21h. 18′. 27″	2,097	4,893	$1\frac{4}{5}$	0′. 43″,5
II	2. 17. 41. 22	2,686	6,286	$2\frac{1}{2}$	0. 56
III	4. 12. 25. 12	3,752	8,754	$3\frac{1}{2}$	1. 18
IV	15. 22. 41. 12	8,698	20,295	8	3. 0
V	79. 7. 49. 0	25,348	59,154	23	8. 42,5

The last column is from CASSINI; but Dr. HERSCHEL makes the distance of the fifth to be 8′. 31″,97, which is probably more exact. In this and the two next Tables, the satellites are numbered from Saturn as they were before the discovery of the other two.

On June 9, 1749, at 10*h*. Mr. POUND found the distance of the fourth satel-

lite to be 3'. 7" with a telescope of 123 feet and an excellent micrometer fixed to it; and the satellite was at that time very near its greatest eastern digression. Hence, at the mean distance of the earth from Saturn, that distance becomes 2'. 58",21; Sir I. NEWTON makes it 3'. 4".

472. The periodic times are found as for the satellites of Jupiter (409.) To determine these, M. CASSINI chose the time when the semi-minor axes of the ellipses which they describe were the greatest, as Saturn was then 90° from their node, because the place of the satellite in its orbit is then the same as upon the orbit of Saturn; whereas in every other case it would be necessary to apply the reduction (426) in order to get the place in its orbit.

473. As it is difficult to see Saturn and the satellites at the same time in the field of view of a telescope, their distances have sometimes been measured by observing the time of the passage of the body of Saturn over a wire adjusted as an hour circle in the field of the telescope, and the interval between the times when Saturn and the satellite passed. From comparing the periodic times and distances, M. CASSINI observed that KEPLER's Rule (218) agreed very well with observations.

474. By comparing the satellites with the ring in different points of their orbits, and the greatest minor axes of the ellipses which they appear to describe compared with the major axes, the planes of the orbits of the first four are found to be very nearly in the plane of the ring, and therefore are inclined to the orbit of Saturn about 30°; but the orbit of the fifth, according to M. CASSINI the Son, makes an angle with the ring of about 15°.

475. M. CASSINI places the node of the ring, and consequently those of the four first satellites, in 5^s. 22° upon the orbit of Saturn, and 5^s. 21° upon the ecliptic. M. HUYGENS had determined it to be in 5^s. 20°. 30'. M. MARALDI in 1716 determined the longitude of the node of the ring upon the orbit of Saturn to be 5^s. 19°. 48'. 30"; and upon the ecliptic to be 5^s. 16°. 20'. The node of the fifth satellite is placed by M. CASSINI in 5^s. 5°. upon the orbit of Saturn. M. de la LANDE makes it 5^s. 0°. 27'. From the observation of M. BERNARD at Marseilles in 1787, it appears that the node of this satellite is retrograde.

476. Dr. HALLEY discovered that the orbit of the fourth satellite was excentric. For having found its mean motion, he discovered that its place by observation was at one time 3° forwarder than by his calculations, and at other observations it was 2°. 30' behind; this indicated an excentricity; and he placed the line of the apsides in 10^s. 22°. *Phil. Trans.* N°. 145.

TABLES OF THEIR REVOLUTIONS AND MEAN MOTIONS, ACCORDING TO M. DE LA LANDE.

Satel	Diurnal Motion	Motion in 365 days
I	$6^s. 10^\circ. 41'. 53''$	$4^s. 4^\circ. 44'. 42''$
II	4. 11. 32 .6	4. 10 .15. 19
III	2. 19. 41. 25	9. 16. 57. 5
IV	0. 22. 34. 38	10. 20. 39. 37
V	0. 4. 32. 17	7. 6. 23. 37

Satel.	Periodic Revolution	Synodic Revolution
I	$1^d. 21^h. 18'. 26'',222$	$1^d. 21^h. 18'. 54'',778$
II	2. 17. 44. 51,177	2. 17. 45. 51,013
III	4. 12. 25. 11,100	4. 12. 27. 55,239
IV	15. 22. 41. 16,022	15. 23. 15. 23,153
V	79. 7. 53. 42,772	79. 22. 3. 12,883

477. M. Cassini observed that the fifth satellite disappeared regularly for about half its revolution, when it was to the east of Saturn; from which he concluded, that it revolved about its axis; he afterwards however doubted of this. But Sir I. Newton in his *Principia*, Lib. III. Prop. 17, concludes from hence, that it revolves about its axis, and in the same time that it revolves about Saturn; and that the variable appearance arises from some parts of the satellite not reflecting so much light as others. Dr. Herschel has confirmed this, by tracing regularly the periodical change of light through more than 10 revolutions, and finding it, in all appearances, to be cotemporary with the return of the satellite to the same situation in its orbit. This is further confirmed by some observations of M. Bernard at Marseilles in 1787; and is a remarkable instance of analogy among the secondary planets.

478. These are all the satellites which were known to revolve about Saturn till the year 1789, when Dr. HERSCHEL, in a Paper in the *Phil. Trans.* for that year, announced the discovery of a *sixth* satellite, interior to all the others, and promised a further account in another paper. But in the intermediate time he discovered a *seventh* satellite, interior to the sixth; and in a Paper upon Saturn and its ring, in the *Phil. Trans.* 1790, he has given an account of the discovery, with some of the elements of their motions. He afterwards added Tables of their motions.

479. After his observations upon the ring, he says, he cannot quit the subject without mentioning his own surmises, and that of several other Astronomers, of a supposed roughness of the ring, or inequality in the planes and inclinations of its flat sides. This supposition arose, from seeing luminous points on its boundaries projecting like the moon's mountains; or from seeing one arm brighter or longer than another; or even from seeing one arm when the other was invisible. Dr. HERSCHEL was of this opinion, when he saw one of these points move off the edge of the ring in the form of a satellite. With his 20 feet telescope he suspected that he saw a sixth satellite; and on August 19, 1787, marked it down as probably being one; and having finished his telescope of forty feet focal length, he saw six of its satellites the moment he directed his telescope to the planet. This happened on August 28, 1789. The retrograde motion of Saturn was then nearly 4'. 30'' in a day, which made it very easy to ascertain, whether the stars he took to be satellites were really so, and in about two hours and an half after, he found that the planet had visibly carried them all away from their places. He continued his observations, and on September 17, he discovered the seventh satellite. These two satellites lie within the orbits of the other five. Their distances from the center of Saturn are 36'',7889, and 28'',6689; and their periodic times are 1*d.* 8*h.* 53'. 8'',9 and 22*h.* 37'. 22'',9. The orbits of these satellites lie so near to the plane of the ring, that the difference cannot be perceived.

480. As soon as he had made observations sufficient to construct Tables of their mean motions, he calculated their places backwards, and found that his suspicions of the existence of these satellites, in the shape of protuberant points on the arms of the ring, were confirmed, and this served to correct the Tables. He has also constructed Tables of the motions of the other five satellites; the epochs he deduced from his own observations, which differ considerably from those given by M. de la LANDE, in the *Connoissance des Temps*, for 1791; but he assumed the mean motions the same as there given. The following Tables of the epochs and mean motions are given by Dr. HERSCHEL in the *Phil. Trans.* for 1790. The satellites are here numbered in their order from Saturn.

EPOCHS OF THE MEAN LONGITUDES OF SATURN'S SATELLITES.

* *	VII.	VI.	V.	IV.	III.	II.	I.
Years.	Deg. dec	Deg. dec.	Deg dec	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1787	335,91	149,16	87,21	272,18	176,46	269,31	307,07
1788	196,84	132,41	93,86	173,95	131,91	307,48	65,02
1789	53,23	93,09	20,82	304,19	256,66	82,92	161,00
1790	269,63	53,77	307,78	74,43	21,41	218,36	256,98
1791	126,02	14,45	234,74	204,68	146,16	353,81	352,97

THE MOTION OF THE SATELLITES ABOUT SATURN IN *MONTHS.*

* * *	VII.	VI.	V.	IV.	III.	II.	I.
Months	Deg. dec.	Deg. dec.	Deg dec	Deg dec	Deg dec	Deg. dec.	Deg dec.
January	000,00	000,00	000,00	000,00	000,00	000,00	000,00
February	140,68	339,89	310,40	117,58	151,64	224,54	320,81
March	267,75	252,05	21,73	200,56	91,18	20,91	215,73
April	48,43	231,95	332,13	318,14	242,81	245,45	176,54
May	184,57	189,26	202,84	304,19	203,75	207,27	115,39
June	325,25	169,16	153,24	61,77	355,39	71,81	76,20
July	101,39	126,47	23,94	47,82	316,33	33,63	15,05
August	242,07	106,37	334,34	165,40	107,96	258,17	335,86
September	22,75	86,26	284,74	282,98	259,60	122,72	296,67
October	158,89	43,58	155,45	269,03	220,54	84,54	235,52
November	299,57	23,47	105,85	26,61	12,17	309,08	196,33
December	75,71	340,78	336,56	12,66	333,11	270,90	135,17

In the months *January* and *February* of a bissextile year, subtract 1 from the number of days given.

THE MOTION OF SATURN'S SATELLITES IN *DAYS.*

DAYS	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1	4,54	22,58	79,69	131,53	190,70	262,73	21,96
2	9,08	45,15	159,38	263,07	21,40	165,45	43,92
3	13,61	67,73	239,07	34,60	212,09	68,18	65,88
4	18,15	90,31	318,76	166,14	42,79	330,91	87,85
5	22,69	112,89	38,45	297,67	233,49	233,64	109,81
6	27,23	135,46	118,14	69,21	64,19	136,36	131,77
7	31,77	158,04	197,83	200,74	254,89	39,09	153,73
8	36,30	180,62	277,52	332,28	85,58	301,82	175,69
9	40,84	203,19	357,21	103,81	276,28	204,55	197,65
10	45,38	225,77	76,90	235,35	106,98	107,27	219,62
11	49,92	248,35	156,59	6,88	297,68	10,00	241,58
12	54,46	270,93	236,28	138,42	128,38	272,73	263,54
13	58,99	293,50	315,97	269,95	319,07	175,45	285,50
14	63,53	316,08	35,66	41,49	149,77	78,18	307,46
15	68,07	338,66	115,35	173,02	340,47	340,91	329,42
16	72,61	1,24	195,04	304,56	171,17	243,64	351,39
17	77,15	23,81	274,74	76,09	1,87	146,36	13,35
18	81,69	46,39	354,43	207,63	192,56	49,09	35,31
19	86,22	68,97	74,12	339,16	23,26	311,82	57,27
20	90,76	91,54	153,81	110,70	213,96	214,54	79,23
21	95,30	114,12	233,50	242,23	44,66	117,27	101,19
22	99,84	136,70	313,19	13,77	235,35	20,00	123,16
23	104,38	159,28	32,88	145,30	66,05	282,73	145,12
24	108,91	181,85	112,57	276,84	256,75	185,45	167,08
25	113,45	204,43	192,26	48,37	87,45	88,18	189,04
26	117,99	227,01	271,95	179,91	278,15	350,91	211,00
27	122,53	249,58	351,64	311,44	108,84	253,64	232,96
28	127,07	272,16	71,33	82,98	299,54	156,36	254,92
29	131,60	294,74	151,02	214,51	130,24	59,09	276,89
30	136,14	317,32	230,71	346,05	320,94	321,82	298,85
31	140,68	339,89	310,40	117,58	151,64	224,54	320,81

THE MOTION OF SATURN'S SATELLITES IN *HOURS*.

HOURS	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec	Deg dec.	Deg. dec	Deg. dec	Deg. dec.
1	0,19	0,94	3,32	5,48	7,95	10,95	15,92
2	0,38	1,88	6,64	10,96	15,89	21,89	31,83
3	0,57	2,82	9,96	16,44	23,84	32,84	47,75
4	0,76	3,76	13,28	21,92	31,78	43,79	63,66
5	0,95	4,70	16,60	27,40	39,73	54,73	79,58
6	1,13	5,64	19,92	32,88	47,67	65,68	95,49
7	1,32	6,58	23,24	38,36	55,62	76,63	111,41
8	1,51	7,53	26,56	43,84	63,57	87,58	127,32
9	1,70	8,47	29,88	49,33	71,51	98,52	143,24
10	1,89	9,41	33,20	54,81	79,46	109,47	159,15
11	2,08	10,35	36,52	60,29	87,40	120,42	175,07
12	2,27	11,29	39,84	65,77	95,35	131,36	190,98
13	2,46	12,23	43,17	71,25	103,29	142,31	206,90
14	2,65	13,17	46,49	76,73	111,24	153,26	222,81
15	2,84	14,11	49,81	82,21	119,19	164,20	238,73
16	3,03	15,05	53,13	87,69	127,13	175,15	254,64
17	3,21	15,99	56,45	93,17	135,08	186,10	270,56
18	3,40	16,93	59,77	98,65	143,02	197,05	286,47
19	3,59	17,87	63,09	104,13	150,97	207,99	302,39
20	3,78	18,81	66,41	109,61	158,91	218,94	318,30
21	3,97	19,75	69,73	115,09	166,86	229,89	334,22
22	4,16	20,70	73,05	120,57	174,81	240,83	350,13
23	4,35	21,64	76,37	126,05	182,75	251,78	6,05
24	4,54	22,58	79,69	131,53	190,70	262,73	21,96

THE MOTION OF SATURN'S SATELLITES IN *MINUTES.*

MINUTES	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.	Deg. dec.
1	0,00	0,02	0,06	0,09	0,13	0,18	0,27
2	0,01	0,03	0,11	0,18	0,26	0,36	0,53
3	0,01	0,05	0,17	0,27	0,40	0,55	0,80
4	0,01	0,06	0,22	0,37	0,53	0,73	1,06
5	0,02	0,08	0,28	0,46	0,66	0,91	1,33
6	0,02	0,09	0,33	0,55	0,79	1,09	1,59
7	0,02	0,11	0,39	0,64	0,93	1,28	1,86
8	0,03	0,13	0,44	0,73	1,06	1,46	2,12
9	0,03	0,14	0,50	0,82	1,19	1,64	2,39
10	0,03	0,16	0,55	0,91	1,32	1,82	2,65
11	0,04	0,17	0,61	1,00	1,46	2,01	2,92
12	0,04	0,19	0,66	1,10	1,59	2,19	3,18
13	0,04	0,20	0,72	1,19	1,72	2,37	3,45
14	0,05	0,22	0,77	1,28	1,85	2,55	3,71
15	0,05	0,24	0,83	1,37	1,99	2,74	3,98
16	0,05	0,25	0,89	1,46	2,12	2,92	4,24
17	0,06	0,27	0,94	1,55	2,25	3,10	4,51
18	0,06	0,28	1,00	1,64	2,38	3,28	4,78
19	0,06	0,30	1,05	1,73	2,52	3,47	5,04
20	0,07	0,31	1,11	1,83	2,65	3,65	5,31
21	0,07	0,33	1,16	1,92	2,78	3,83	5,57
22	0,07	0,34	1,22	2,01	2,91	4,01	5,84
23	0,08	0,36	1,27	2,10	3,05	4,20	6,10
24	0,08	0,38	1,33	2,19	3,18	4,38	6,37
25	0,08	0,39	1,38	2,28	3,31	4,56	6,63
26	0,09	0,41	1,44	2,37	3,44	4,74	6,90
27	0,09	0,42	1,49	2,47	3,57	4,93	7,16
28	0,09	0,44	1,55	2,56	3,71	5,11	7,43
29	0,10	0,45	1,60	2,65	3,84	5,29	7,69
30	0,10	0,47	1,66	2,74	3,97	5,47	7,96

THE MOTION OF SATURN'S SATELLITES IN *MINUTES.*

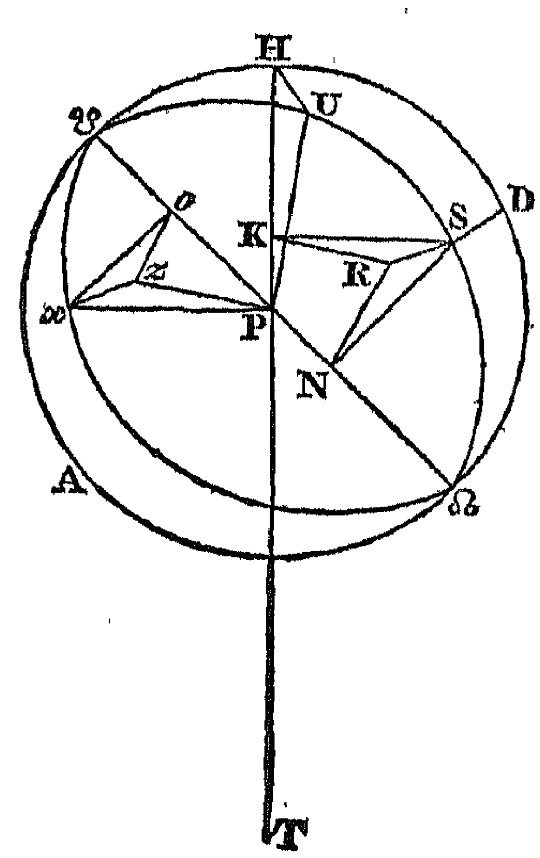
MINUTES	VII.	VI.	V.	IV.	III.	II.	I.
	Deg. dec.	Deg. dec	Deg. dec.	Deg. dec.	Deg dec	Deg dec.	Deg dec
31	0,10	0,49	1,72	2,83	4,10	5,66	8,22
32	0,11	0,50	1,77	2,92	4,24	5,84	8,49
33	0,11	0,52	1,83	3,01	4,37	6,02	8,75
34	0,11	0,53	1,88	3,10	4,50	6,20	9,02
35	0,12	0,55	1,94	3,20	4,63	6,39	9,29
36	0,12	0,56	1,99	3,29	4,77	6,57	9,55
37	0,12	0,58	2,05	3,38	4,90	6,75	9,82
38	0,13	0,60	2,10	3,47	5,03	6,93	10,08
39	0,13	0,61	2,16	3,56	5,16	7,12	10,35
40	0,13	0,63	2,21	3,65	5,30	7,30	10,61
41	0,14	0,64	2,27	3,74	5,43	7,48	10,88
42	0,14	0,66	2,32	3,83	5,56	7,66	11,14
43	0,14	0,67	2,38	3,93	5,69	7,85	11,41
44	0,15	0,69	2,43	4,02	5,83	8,03	11,67
45	0,15	0,71	2,49	4,11	5,96	8,21	11,94
46	0,15	0,72	2,55	4,20	6,09	8,39	12,20
47	0,16	0,74	2,60	4,29	6,22	8,58	12,47
48	0,16	0,75	2,66	4,38	6,36	8,76	12,73
49	0,16	0,77	2,71	4,47	6,49	8,94	13,00
50	0,17	0,78	2,77	4,57	6,62	9,12	13,27
51	0,17	0,80	2,82	4,66	6,75	9,30	13,53
52	0,17	0,82	2,88	4,75	6,88	9,49	13,80
53	0,17	0,83	2,93	4,84	7,02	9,67	14,06
54	0,18	0,85	2,99	4,93	7,15	9,85	14,33
55	0,18	0,86	3,04	5,02	7,28	10,03	14,59
56	0,18	0,88	3,10	5,11	7,41	10,22	14,86
57	0,19	0,89	3,15	5,20	7,55	10,40	15,12
58	0,19	0,91	3,21	5,30	7,68	10,58	15,39
59	0,19	0,93	3,27	5,39	7,81	10,76	15,65
60	0,20	0,94	3,32	5,48	7,94	10,95	15,92

For the motion in *Seconds*, for Deg. dec. read Min. dec.

On these Tables, Dr. Herschel makes the following observations. "I have not attempted to extend them farther than a few years backwards or forwards, as I am not in possession of any observations that could authorize me to undertake such a work. On the contrary, I am well convinced, that no Tables will give us the situation of the satellites accurately, till we have at least established the dimensions of their elliptical orbits, and the motion as well as the situation of their aphelia. The epochs for 1789, therefore, must be looked upon not as *mean* ones, but such as respect the orbits of these satellites in their situation during the time of the following observations; and the two preceding, and two following years, must be already a little affected with those errors which are the necessary consequence of our not knowing the required elements."

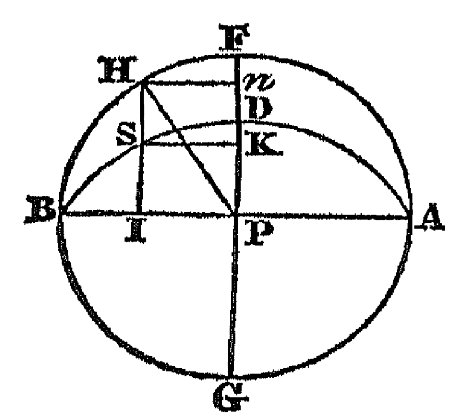
Dr. Maskelyne's *Method of investigating the Configuration of the Satellites, is as follows:*

481. Let T be the earth, P the primary planet, ☋ P ☊ the line of the nodes of the satellite, ☊ S ☋ x its orbit, the plane of the paper representing the plane of the ecliptic, and therefore here may be taken for that of the orbit of the planet; and on this plane, describe the circle ☋ HD ☊ A with the center P; produce TP to H called the apogee point, draw HU perpendicular to the satellites' orbit, and U is the apogee point in that orbit. Assume the point S the place



of the satellite, and take $Ux = 90°$, join HS, draw SD perpendicular to H☊,

and SR, xz, perpendicular to the ecliptic; also RN, xo, zo, perpendicular to ☊P, and RK to HP. Then 1 (rad.) : cos. S☊D :: tan. S☊ : tan. ☊D, and $HD = H$☊ − ☊D; also, cos. ☊D : cos. HD :: cos. S☊ : cos. SH, whose sine is the apparent distance of the satellite at S from the primary; and sin. HD : sin. ☊D :: tan. S☊H (inclin. of orbit) : tan. SH☊ the inclination of the distance of the satellite from the primary to a parallel to the ecliptic. Now SK the sine of SH rises above or falls below the plane of the ecliptic, as ☊S is less or greater than 180°; and it will appear east or west as HPR is less or greater than 180°. Also, 1 : cos. H☊U :: tan. H☊ : tan. U☊ the distance of the apogee point of the satellite's orbit from ☊, and 1 : sin. H☊U :: sin. H☊ : sin. HU the elevation (E) of the eye above the plane of the satellite's orbit, or the minor axis of the satellite's orbit on the cœlestial sphere. Now Table I. col. *Red.* serves to find the apogee point, and col. *Lat.* to find the inclination of the visual ray from I to the planet in respect to the plane of the Ring of Saturn, or orbit of the satellites, the sine of which measures the minor axis, the major being unity. The first of the two col. *Lat.* serves also to find the inclination of the line of the ansæ of the Ring to the ecliptic. Let $ADSB$ represent the satellite's orbit on the cœlestial sphere, S the satellite, AP, PD the semi-major and minor axes, then SK (perpendicular



to PD) = the apparent distance of the satellite from the primary in a line parallel to AB, SI its distance north of the primary in a line perpendicular to the major axis. Now $SK = Hn = AP \times$ sin. $FPH = AP \times$ sin. SU (first fig.) $= AP \times$ sin. dist. sat. in orbit from apog. point $= AP \times$ sin. (long. ♄ in its orbit − long. ap. point) $= AP \times$ sin. (long. ♄ − $\overline{\text{long. ♄ + red.}}$); also, $SI = AP$ × sin. E × cos. $HF = AP \times$ sin. $HU \times$ cos. SU (first fig.) $= AP \times$ sin. I (inc. of Sat. orb.) × sin. ☊H (N) × cos. SU (A) $= AP \times$ sin. $I \times \frac{1}{2}$ sin. $\overline{N+A} + \frac{1}{2}$ sin. $\overline{N-A}$, whence $N \pm A$ = long. ♄ − long. sat. node on its orbit ± (long. sat. − long. apog.), and these arguments correspond to those in Table III. Take Ux (first fig.) = 90° eastward of U, then xPz = inclin. of the transverse axis to the ecliptic, and xoz the inclination of the orbit; also, sin. xPz : sin. xoz :: xo : Px :: sin. xPo (UP☊ + 90°) or cos. UP☊ : 1; but sin. UH : sin. inclin.

sat. orb. :: sin. $H☊$: 1, and the inclination of the eye above the plane of the sat. orb. in Table I. is hence constructed. Therefore, xPz = lat. taken from Table I. with arg. $UP☊ + 90°$; and the eastward of the transverse axis will rise above or fall below the plane of the ecliptic, as the said arg. is less or greater than six signs.

TABLE I.

The Latitude and Reduction of Saturn's Satellites.

ARGUMENT.—{ Geocentric Longitude of ♄ — ☊ of the Satellite's orbit. Or the Distance of the Apogee from ☊ + 3 Signs.

S.	O. *North.*		VI. *South.*		I. *North.*		VII. *South.*		II. *North.*		VIII. *South.*		S
	Satellites. I. II. III. IV. V. VI.		Satellite VII.		Satellites. I. II. III. IV. V. VI		Satellite VII.		Satellites. I. II. III. IV. V. VI.		Satellite VII.		
D	Lat.	Red.	Lat.	Red.	Lat.	Red.	Lat.	Red	Lat.	Red.	Lat.	Red	D.
	D. M.	D. M.	D. M.	D. M	D. M.	D. M.	D. M.	D. M	D. M.	D. M.	D. M	D. M	
0	0. 0	0. 0	0. 0	0. 0	14. 29	3. 26	7. 26	0. 51	25 39	3. 41	12 57	0 52	30
1	0. 30	0. 8	0. 15	0. 2	14. 55	3. 31	7. 40	0. 52	25. 56	3. 37	13 5	0. 51	29
2	1. 0	0. 16	0. 31	0. 4	15. 22	3. 35	7. 53	0. 53	26. 12	3. 33	13 13	0 50	28
3	1. 30	0. 24	0. 46	0. 6	15. 48	3. 39	8. 6	0. 54	26. 27	3. 28	13. 20	0. 49	27
4	2. 0	0. 32	1. 2	0. 8	16. 14	3. 42	8. 19	0. 55	26. 42	3. 23	13. 27	0. 47	26
5	2. 30	0. 40	1. 17	0. 10	16. 40	3. 46	8. 32	0. 55	26. 57	3. 18	13. 34	0 46	25
6	3. 0	0. 48	1. 33	0. 12	17. 5	3. 49	8. 45	0. 56	27. 11	3. 12	13. 41	0. 45	24
7	3. 30	0. 56	1. 48	0. 14	17. 31	3. 52	8. 58	0. 57	27. 24	3 7	13 47	0. 44	23
8	3. 59	1. 4	2. 4	0. 16	17. 56	3. 55	9 10	0. 57	27. 37	3 1	13 53	0 42	22
9	4. 29	1. 12	2. 19	0. 18	18. 21	3. 58	9. 2	0. 58	27. 49	2 55	13. 59	0 41	21
10	4. 59	1. 19	2. 34	0. [illegible]0	18. 45	4. 0	9. 35	0. 58	28. 2	2 48	14 4	0. 39	20
11	5. 28	1. 27	2 5[illegible]	0. 22	19. 9	4. 2	9. 47	0. 59	28. 13	2 41	14. 10	0. 37	19
12	5. 58	1. 34	3. [illegible]	0. 21	19. 33	4. 3	9. 58	0 59	28 21	2 34	14 15	0. 3[illegible]	18
13	6. 27	1. 42	3. 2[illegible]	0. 26	19. 56	4. 5	10. 10	0. 59	28. 31	2 27	14. 20	0. 34	17
14	6. 57	1. 49	3. 3[illegible]	0 27	20. 20	4. 6	10. 21	0. 59	28. 44	2. 19	14. 24	0. 32	16
15	7. 26	1. 56	3. 5[illegible]	0. 2[illegible]	20. 43	4. 6	10. 33	0. 59	28. 53	2 12	14 28	0. 30	15
16	7. 55	2. 3	4. 5	0. 31	21. 5	4. 7	10. 44	0. 59	29. 1	2. 4	14 32	0. 28	14
17	8. 24	2. 10	4. 20	0. 33	21. 27	4. 7	10. 55	0. 59	29. 9	1 56	14. 36	0. 27	13
18	8. 53	2. 17	4. 35	0. 34	21. 49	4. 7	11. 6	0. 59	29. 17	1. 47	14 40	0. 25	12
19	9. 22	2. 24	4. 50	0. 36	22. 10	4. 6	11. 16	0. 59	29. 24	1. 39	14. 43	0 23	11
20	9. 51	2. 30	5. 5	0. 38	22. 31	4. 6	11. 26	0. 59	29. 30	1. 30	14 46	0. 21	10
21	10. 19	2. 37	5. 19	0. 39	22. 52	4. 5	11. 36	0. 58	29. 35	1. 22	14. 49	0. 19	9
22	10. 48	2. 43	5. 34	0. 41	23. 12	4. 3	11. 46	0. 58	29. 41	1. 13	14. 51	0. 17	8
23	11. 16	2. 49	5. 48	0. 43	23. 32	4. 2	11. 56	0. 57	29. 45	1. 4	14. 53	0. 15	7
24	11. 44	2. 55	6. 2	0. 44	23. 51	4. 0	12. 5	0. 57	29. 49	0. 55	14. 55	0 13	6
25	12. 12	3. 1	6. 17	0. 45	24. 11	3. 58	12. 14	0 56	29. 52	0 46	14 56	0. 11	5
26	12. 40	3. 6	6. 31	0. 46	24. 29	3. 55	12. 23	0. 56	29. 55	0 37	14 57	0. 9	4
27	13. 7	3. 11	6. 45	0. 48	24. 47	3. 52	12. 32	0. 55	29. 57	0 28	14. 58	0. 7	3
28	13. 34	3. 16	6. 59	0. 49	25. 5	3. 49	12. 41	0. 54	29. 59	0 19	14 59	0 4	2
29	14. 2	3. 21	7. 12	0. 50	25. 22	3. 45	12. 49	0. 53	30. 0	0 10	15 0	0. 2	1
30	14. 29	3. 26	7. 26	0. 51	25. 39	3. 41	12. 57	0. 52	30. 0	0. 0	15. 0	0 0	0
S.	XI. *South* +		V. *North* +		X. *South.* +		IV. *North.* +		IX. *South.* +		III. *North.* +		S

TABLE II.

The apparent Distances of Saturn's first, second, third, fourth, fifth *and* sixth *Satellites from its Center, in lines parallel to the line of the Ansæ of the Ring; and of the* seventh *Satellite in lines parallel to the longer axis of its apparent orbit, in semidiameters of the Ring, and hundredths of the same.*

Argument.—The Distance of the Satellite from the Apogee.																						
S	O. *East.*						VI. *West.*	I. *East.*						VII. *West.*	II. *East.*						VIII. *West.*	S
	Distance of the Satellites.							Distance of the Satellites.							Distance of the Satellites.							
D	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII.	I.	II.	III.	VI.	V.	VI.	VII.	D
	semid	semi	semid	semid	semid	semid	semid.	semid	semid	semid	semid	semid	semid	semid.	semid	semid	semi	semid	semid	semid	semid.	
0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,72	0,93	1,05	1,34	1,87	4,35	12,67	1,24	1,59	1,82	2,33	3,25	7,53	21,95	30
1	0,02	0,03	0,04	0,05	0,06	0,15	0,44	0,74	0,96	1,08	1,38	1,93	4,48	13,05	1,25	1,61	1,84	2,36	3,28	7,61	22,17	29
2	0,05	0,06	0,07	0,09	0,13	0,30	0,88	0,76	0,98	1,11	1,42	1,99	4,61	13,42	1,26	1,62	1,85	2,38	3,31	7,68	22,38	28
3	0,08	0,10	0,11	0,14	0,20	0,45	1,32	0,78	1,01	1,14	1,46	2,05	4,74	13,79	1,27	1,64	1,87	2,40	3,34	7,75	22,58	27
4	0,10	0,13	0,14	0,19	0,26	0,60	1,76	0,80	1,03	1,17	1,50	2,10	4,86	14,16	1,28	1,65	1,89	2,42	3,37	7,82	22,78	26
5	0,12	0,16	0,18	0,23	0,32	0,75	2,20	0,82	1,06	1,20	1,54	2,16	4,99	14,53	1,28	1,67	1,91	2,44	3,40	7,89	22,97	25
6	0,15	0,19	0,22	0,28	0,39	0,91	2,64	0,84	1,08	1,23	1,58	2,21	5,11	14,89	1,30	1,68	1,92	2,46	3,43	7,95	23,16	24
7	0,18	0,22	0,25	0,33	0,46	1,06	3,08	0,86	1,11	1,26	1,62	2,26	5,24	15,25	1,31	1,70	1,94	2,48	3,46	8,01	23,33	23
8	0,20	0,26	0,29	0,38	0,52	1,21	3,52	0,88	1,13	1,29	1,65	2,31	5,36	15,60	1,32	1,71	1,95	2,50	3,48	8,07	23,50	22
9	0,22	0,29	0,32	0,42	0,58	1,36	3,96	0,90	1,16	1,32	1,69	2,36	5,48	15,94	1,33	1,72	1,96	2,52	3,51	8,13	23,66	21
10	0,25	0,32	0,36	0,47	0,65	1,51	4,40	0,92	1,18	1,35	1,73	2,41	5,59	16,28	1,34	1,73	1,97	2,53	3,53	8,18	23,82	20
11	0,28	0,35	0,40	0,52	0,72	1,66	4,83	0,94	1,21	1,38	1,77	2,46	5,71	16,62	1,35	1,74	1,98	2,55	3,55	8,23	23,97	19
12	0,30	0,38	0,44	0,56	0,78	1,81	5,26	0,96	1,23	1,40	1,80	2,51	5,82	16,96	1,36	1,75	1,98	2,56	3,57	8,27	24,11	18
13	0,33	0,42	0,48	0,61	0,85	1,96	5,70	0,98	1,26	1,43	1,84	2,56	5,93	17,28	1,37	1,76	2,00	2,58	3,59	8,32	24,24	17
14	0,35	0,45	0,51	0,65	0,91	2,11	6,13	1,00	1,28	1,46	1,87	2,61	6,04	17,60	1,37	1,77	2,02	2,59	3,60	8,36	24,37	16
15	0,38	0,48	0,55	0,70	0,97	2,26	6,56	1,02	1,30	1,49	1,91	2,66	6,15	17,92	1,38	1,78	2,03	2,60	3,62	8,40	24,48	15
16	0,40	0,51	0,58	0,74	1,03	2,40	6,98	1,03	1,32	1,51	1,94	2,70	6,26	18,23	1,39	1,79	2,04	2,61	3,64	8,44	24,59	14
17	0,42	0,54	0,62	0,79	1,10	2,55	7,41	1,05	1,35	1,54	1,97	2,75	6,36	18,53	1,40	1,80	2,05	2,62	3,66	8,48	24,70	13
18	0,44	0,57	0,65	0,83	1,16	2,69	7,83	1,06	1,37	1,56	2,00	2,79	6,46	18,83	1,40	1,80	2,05	2,63	3,67	8,51	24,80	12
19	0,47	0,60	0,69	0,88	1,22	2,83	8,25	1,08	1,39	1,59	2,03	2,83	6,56	19,13	1,40	1,81	2,06	2,64	3,68	8,54	24,88	11
20	0,49	0,63	0,72	0,92	1,28	2,97	8,66	1,09	1,41	1,61	2,06	2,87	6,66	19,42	1,41	1,81	2,07	2,65	3,69	8,56	24,96	10
21	0,51	0,66	0,76	0,97	1,34	3,12	9,08	1,11	1,43	1,64	2,09	2,91	6,76	19,70	1,41	1,82	2,08	2,66	3,70	8,59	25,03	9
22	0,53	0,69	0,79	1,01	1,40	3,26	9,50	1,13	1,45	1,66	2,12	2,95	6,85	19,97	1,41	1,82	2,08	2,66	3,71	8,61	25,10	8
23	0,56	0,72	0,83	1,06	1,46	3,40	9,90	1,15	1,47	1,68	2,15	2,99	6,95	20,24	1,42	1,83	2,08	2,67	3,72	8,63	25,16	7
24	0,58	0,75	0,86	1,10	1,52	3,54	10,30	1,16	1,49	1,70	2,18	3,03	7,04	20,51	1,42	1,83	2,09	2,67	3,73	8,65	25,21	6
25	0,61	0,78	0,89	1,14	1,58	3,68	10,71	1,18	1,51	1,72	2,21	3,07	7,13	20,76	1,43	1,84	2,09	2,68	3,74	8,67	25,25	5
26	0,63	0,81	0,92	1,18	1,64	3,82	11,11	1,19	1,53	1,74	2,23	3,11	7,21	21,01	1,43	1,84	2,09	2,68	3,74	8,68	25,29	
27	0,65	0,84	0,96	1,22	1,70	3,96	11,50	1,20	1,55	1,76	2,26	3,15	7,30	21,26	1,43	1,84	2,10	2,69	3,74	8,69	25,32	3
28	0,67	0,87	0,99	1,26	1,76	4,10	11,89	1,21	1,56	1,78	2,28	3,18	7,38	21,50	1,43	1,84	2,10	2,69	3,75	8,69	25,33	2
29	0,69	0,90	1,02	1,30	1,82	4,23	12,28	1,23	1,58	1,80	2,31	3,22	7,46	21,73	1,43	1,84	2,10	2,69	3,75	8,70	25,35	1
30	0,72	0,93	1,05	1,34	1,87	4,35	12,67	1,24	1,59	1,82	2,33	3,25	7,53	21,95	1,43	1,84	2,10	2,69	3,75	8,70	25,35	0
S	XI. *West.*						V. *East.*	X. *West.*						IV. *East.*	IX. *West.*						III. *East.*	S

TABLE III.

The apparent Distances of Saturn's first, second, third, fourth, fifth *and* sixth *Satellites from the line of the Ansæ of the Ring; and of the* seventh *Satellite from the longer axis of its apparent orbit, in semidiameters of the Ring, and hundredths of the same.*

ARGUMENT.—Argument of Latitude ± distance of the Satellite from the Apogee.

S	O. *North.*						VI. *South.*	I. *North.*						VII. *South.*	II. *North.*						VIII. *South.*	S
	Distance of the Satellites.							Distance of the Satellites.							Distance of the Satellites.							
D	I.	II.	III.	IV.	V	VI.	VII.	I.	II.	III.	IV.	V.	VI.	VII	I.	II.	III.	IV.	V.	VI.	VII.	D
	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	semid	
0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,18	0,23	0,26	0,33	0,47	1,09	1,64	0,31	0,40	0,45	0,58	0,81	1,88	2,84	30
1	0,00	0,01	0,01	0,01	0,01	0,04	0,06	0,18	0,24	0,27	0,34	0,48	1,12	1,69	0,31	0,40	0,46	0,59	0,82	1,90	2,87	29
2	0,01	0,01	0,02	0,02	0,03	0,07	0,11	0,19	0,24	0,28	0,35	0,50	1,15	1,74	0,31	0,40	0,46	0,59	0,83	1,92	2,90	28
3	0,02	0,02	0,03	0,03	0,05	0,11	0,17	0,19	0,25	0,28	0,36	0,51	1,18	1,79	0,32	0,41	0,47	0,60	0,83	1,92	2,93	27
4	0,02	0,03	0,03	0,05	0,06	0,15	0,23	0,20	0,26	0,29	0,37	0,52	1,21	1,83	0,32	0,41	0,47	0,60	0,84	1,94	2,95	26
5	0,03	0,04	0,04	0,06	0,08	0,19	0,28	0,20	0,26	0,30	0,38	0,54	1,25	1,88	0,32	0,42	0,48	0,61	0,85	1,97	2,98	25
6	0,04	0,05	0,05	0,07	0,10	0,23	0,34	0,21	0,27	0,31	0,39	0,55	1,28	1,93	0,32	0,42	0,48	0,61	0,86	1,99	3,00	24
7	0,04	0,05	0,06	0,08	0,11	0,26	0,40	0,21	0,28	0,31	0,40	0,56	1,31	1,98	0,33	0,42	0,48	0,62	0,86	2,00	3,02	23
8	0,05	0,06	0,07	0,09	0,13	0,30	0,46	0,22	0,28	0,32	0,41	0,58	1,34	2,02	0,33	0,43	0,49	0,62	0,87	2,02	3,04	22
9	0,05	0,07	0,08	0,10	0,14	0,34	0,51	0,22	0,29	0,33	0,42	0,59	1,37	2,07	0,33	0,43	0,49	0,63	0,88	2,03	3,06	21
10	0,06	0,08	0,09	0,12	0,16	0,38	0,57	0,23	0,29	0,34	0,43	0,60	1,40	2,11	0,33	0,43	0,49	0,63	0,88	2,04	3,08	20
11	0,07	0,09	0,10	0,13	0,18	0,41	0,63	0,23	0,30	0,34	0,44	0,61	1,43	2,15	0,34	0,43	0,49	0,64	0,89	2,06	3,10	19
12	0,07	0,09	0,11	0,14	0,19	0,45	0,68	0,24	0,31	0,35	0,45	0,63	1,45	2,19	0,34	0,44	0,49	0,64	0,89	2,07	3,12	18
13	0,08	0,10	0,12	0,15	0,21	0,49	0,74	0,24	0,31	0,36	0,46	0,64	1,48	2,24	0,34	0,44	0,50	0,64	0,90	2,08	3,14	17
14	0,09	0,11	0,13	0,16	0,23	0,53	0,79	0,25	0,32	0,36	0,47	0,65	1,51	2,28	0,34	0,44	0,50	0,65	0,90	2,09	3,15	16
15	0,09	0,12	0,14	0,17	0,24	0,56	0,85	0,25	0,32	0,37	0,48	0,66	1,54	2,32	0,34	0,44	0,51	0,65	0,90	2,10	3,17	15
16	0,10	0,13	0,14	0,18	0,26	0,60	0,90	0,26	0,33	0,38	0,48	0,67	1,56	2,36	0,35	0,45	0,51	0,65	0,91	2,11	3,18	14
17	0,10	0,13	0,15	0,20	0,27	0,64	0,96	0,26	0,34	0,38	0,49	0,69	1,59	2,40	0,35	0,45	0,51	0,65	0,91	2,12	3,20	13
18	0,11	0,14	0,16	0,21	0,29	0,67	1,01	0,26	0,34	0,39	0,50	0,70	1,61	2,44	0,35	0,45	0,51	0,66	0,92	2,13	3,21	12
19	0,12	0,15	0,17	0,22	0,30	0,71	1,07	0,27	0,35	0,40	0,51	0,71	1,64	2,48	0,35	0,45	0,51	0,66	0,92	2,13	3,22	11
20	0,12	0,16	0,18	0,23	0,32	0,74	1,12	0,27	0,35	0,40	0,51	0,72	1,66	2,51	0,35	0,45	0,52	0,66	0,92	2,14	3,23	10
21	0,13	0,16	0,19	0,24	0,33	0,78	1,18	0,28	0,36	0,41	0,52	0,73	1,69	2,55	0,35	0,45	0,52	0,66	0,92	2,15	3,24	9
22	0,13	0,17	0,20	0,25	0,35	0,81	1,23	0,28	0,36	0,41	0,53	0,74	1,71	2,58	0,35	0,45	0,52	0,66	0,93	2,15	3,25	8
23	0,14	0,18	0,21	0,26	0,36	0,85	1,29	0,29	0,37	0,42	0,54	0,75	1,74	2,62	0,35	0,46	0,52	0,67	0,93	2,16	3,26	7
24	0,14	0,19	0,21	0,27	0,38	0,88	1,34	0,29	0,37	0,42	0,54	0,76	1,76	2,65	0,35	0,46	0,52	0,67	0,93	2,16	3,26	6
25	0,15	0,19	0,22	0,28	0,39	0,92	1,39	0,29	0,38	0,43	0,55	0,77	1,78	2,69	0,36	0,46	0,52	0,67	0,93	2,17	3,27	5
26	0,16	0,20	0,23	0,29	0,41	0,95	1,44	0,30	0,38	0,43	0,56	0,78	1,80	2,72	0,36	0,46	0,52	0,67	0,93	2,17	3,27	4
27	0,16	0,21	0,24	0,30	0,42	0,99	1,49	0,30	0,39	0,44	0,56	0,79	1,82	2,75	0,36	0,46	0,52	0,67	0,93	2,17	3,28	3
28	0,17	0,22	0,25	0,31	0,44	1,02	1,54	0,30	0,39	0,44	0,57	0,79	1,84	2,78	0,36	0,46	0,52	0,67	0,94	2,17	3,28	2
29	0,17	0,22	0,25	0,32	0,45	1,06	1,59	0,31	0,39	0,45	0,58	0,80	1,86	2,81	0,36	0,46	0,52	0,67	0,94	2,17	3,28	1
30	0,18	0,23	0,26	0,33	0,47	1,09	1,64	0,31	0,40	0,45	0,58	0,81	1,88	2,84	0,36	0,46	0,52	0,67	0,94	2,17	3,28	0
S	XI. *South.*						V. *North.*	X. *South.*						IV. *North.*	IX. *South.*						III. *North.*	S

The Use of the three foregoing Tables for finding the apparent Positions of Saturn's Satellites *at any Time, with respect to Saturn and the Line of the Ansæ of the Ring.*

482. From Saturn's geocentric longitude, taken out of the Nautical Almanac, subtract the place of the satellite's ascending node (at present $5^s. 20°. 54'$ for all but the *seventh*, and for that $5^s. 6°. 4'$), and there will remain the argument of latitude; with which take the reduction out of Table I, and subtract it from the argument of latitude in the first or third quadrant, but add it to the same in the second or fourth quadrant; that is, according to the sign put at the top or bottom of the Table in the column of reduction, and you will have the distance of the apogee* point of the satellite's orbit from the node. Apply the reduction also, and with the same sign as before, to Saturn's geocentric longitude, and you will have the longitude of the apogee point on the satellite's orbit. Subtract the longitude of the apogee thus found from the satellite's longitude, and there will remain the distance of the satellite from the apogee; with which in Table II. find the apparent distances of the satellites from Saturn's center, measured in lines parallel to the longer axis of the ring; except of the seventh satellite, which will be measured in lines parallel to the longer axis of its apparent elliptical orbit; and if the argument be under six signs, the satellite will be east of Saturn's center; but if the argument be greater than six signs, the satellite will be west of Saturn's center; as is marked by the side of the signs of the argument in the Table.

Add and subtract the distance of the satellite from the apogee to and from the argument of latitude, and you will have two arguments, with which enter Table III. separately, and take out the correspondent latitudes, with their proper titles north or south, standing by the signs of the arguments; of which, if both of the same kind, the sum with the common title; but if of different kinds, the difference with the title of the greater, will be the latitude of the satellite, as seen from the earth, measured by its apparent distance from the line of the ansæ of the ring; except in the seventh satellite, which is measured by its distance from the longer axis of its apparent ellipses, in semidiameters of the ring, and hundredths of the same.

Add three signs to the distance of the apogee from the node, with which take out the latitude from Table I. with its proper title, north or south, adjoin-

* The satellite will be in apogee, when its longitude in its orbit, is equal to Saturn's longitude, corrected by reduction by Table I; and it is in its perigee, when its longitude in its orbit is opposite to Saturn's longitude corrected by reduction; and it is at its greatest elongation, when its longitude is 90° from the longitudes of the apogee or perigee.

ing to the sign of the argument, which will be the apparent inclination of the line of the ansæ of Saturn's ring, or of the longer axis of the seventh satellite's orbit, to Saturn's orbit; and the line of the ansæ, or longer axis of the seventh satellite's orbit, will ascend from west to east northward above Saturn's apparent orbit in the heavens, or descend from west to east southward below it, according as the title adjoining to the sign of the argument in the Table is north or south. Change the title of the inclination of the line of the ansæ of the ring to Saturn's orbit to the contrary; and if the inclination of the line of the ansæ of the ring with the title thus changed, and the inclination of the longer axis of the seventh satellite's orbit, be of the same kind, their sum with the common title; but if of different kinds, their difference, with the title of the greater, will be the inclination of the longer axis of the seventh satellite's orbit to the line of the ansæ of the ring. And the longer axis of the seventh satellite's orbit will ascend from west to east northward above, or descend from west to east southward below the line of the ansæ, according as the resulting title is north or south.

EXAMPLE.

To compute the apparent Positions of Saturn's Satellites,

on September 25, 1791, at 11h. mean time.

Geoc. long. of ♄	$0^s.\ 16°,47$
☊ of Ring - -	$-5.\ 20,9$
Arg. Lat. -	$6.\ 25,57$
Table I. Red.	$-3,07$
Dist. apog. à ☊	$6.\ 22,50$
D° - - $+3^s$ =	$9.\ 22,50$
Table I. Inclin. of line of ansæ to ♄'s orbit	$27°.\ 30',5$S.

Geoc. long. of ♄	$0^s.\ 16°,47$
☊ of 7th Sat. orb.	$-5.\ 6,1$
Arg. Lat. -	$7.\ 10,37$
Table I. Red. -	$-0,97$
Dist. apog. à ☊	$7.\ 9,40$
D° - $+3^s$ =	$10.\ 9,40$
Incl. of longer axis of 7th Sat. orb. to ♄ orb.	$11°.\ 32'$ S.

Geoc. long. of ♄	$0^s.\ 16°,47$
Reduc. - - - -	$-3,07$
Long. apog. of all the Satellites except 7th	$0.\ 13,40$

Geoc. long. of ♄	$0^s.\ 16°,47$
Reduc. - -	$-0,97$
Long. apog. of 7th Satellite	$0.\ 15,50$

Hence, $27°.\ 30'$N. $-$ $11°.\ 32'$S. $=$ $15°.\ 58'$N.; or the longer axis of the seventh satellite's orbit rises so much above or north of the line of the ansæ to the east.

	I.	II.	III.	IV.	V.	VI.	VII.
	S. D.	S. D.	S. D.	S. D.	S. D.	S. D.	S. D.
1791 - -	11. 22,97	11. 23,81	4. 26,16	6. 24,68	7. 24,74	0. 14,45	4. 6,02
September - -	9. 26,67	4. 2,72	8. 19,60	9. 12,98	9. 14,74	2. 26,26	0. 22,75
25D. - -	6. 9,04	2. 28,18	2. 27,45	1. 18,37	6. 12,26	6. 24,43	3. 23,45
11H. - -	5. 25,07	4. 0,42	2. 27,40	2. 0,29	1. 6,52	0. 10,35	0. 2,08
Long. of satellite -	9. 23,75	10. 25,13	7. 10,61	7. 26,32	0. 28,26	10. 15,49	8. 24,30
Long. of apogee -	−0. 13,40	− 0. 13,40	−0. 13,40	−0. 13,40	−0. 13,40	− 0. 13,40	−0. 15,50
Dist. sat. à apog. = Arg. II.	±9. 10,35	±10. 11,73	±6. 27,21	±7. 12,92	±0. 14,86	±10. 2,09	±8. 8,80
Arg. lat. - -	6. 25,57	6. 25,57	6. 25,57	6. 25,57	6. 25,57	6. 25,57	7. 10,37
Arg. of Tab. III. {	4. 5,92	5. 7,30	1. 22,78	2. 8,49	7. 10,43	4. 27,66	3. 19,17
	9. 15,22	8. 13,84	11. 28,36	11. 12,65	6. 10,71	8. 3,48	11. 1,57
Dist. sat. à ♄ E. or W.	1,41 W.	1,06 W.	0,97 W.	1,84 W.	0,96 E.	7,37 W.	23,63 W.
Numbers from Tab. III. {	0,29 N.	0,18 N.	0,42 S.	0,62 N.	0,60 S.	1,16 N.	3,10 N.
	0,34 S.	0,44 S.	0,01 S.	0,20 S.	0,17 S.	2,16 S.	1,56 S.
Dist. sat. à ♄ N. or S.	0,05 S.	0,26 S.	0,43 S.	0,42 N.	0,77 S.	1,00 N.	1,54 N.

The epochs of the satellites which have been here given, are for the meridian of *Slough*, which is about 2′. 18″ west of *Greenwich*.

On the Satellites of the Georgian.

483. On January 11, 1787, as Dr. HERSCHEL was observing the *Georgian*, he perceived, near its disc, some very small stars, whose places he noted. The next evening, upon examining them, he found that two of them were missing. Suspecting therefore that they might be satellites which had disappeared in consequence of having changed their situation, he continued his observations, and in the course of a month discovered them to be satellites, as he had at first conjectured. Of this discovery he gave an account in the *Phil. Trans.* 1787.

484. In the *Phil. Trans.* 1788, he published a further account of this discovery, containing their periodic times, distances, and positions of their orbits, so far as he was then able to ascertain them. The most convenient method of determining the periodic time of a satellite is either from its eclipses, or from taking its position in several successive oppositions of the planet; but no eclipses have yet happened since the discovery of these satellites, and it would be waiting a long time to put in practice the other method. Dr. HERSCHEL therefore took their situations whenever he could ascertain them with some degree of precision, and then reduced them by computation to such situations as were necessary for his purpose. In computing the periodic times, he has taken the synodic revolution, as the positions of their orbits, at the times when their situations were taken, were not sufficiently known to get a very accurate sidereal revolution. The mean of several results gave the synodic revolution of the first satellite $8d.\ 17h.\ 1'.\ 19'',3$, and of the second $13d.\ 11h.\ 5'.\ 1'',5$. The results, he observes, of which these are a mean, do not much differ among themselves, and therefore the mean is probably tolerably accurate. The epochs from which their situations may at any time be computed are, for the first, October 19, 1787, at $19h.\ 11'.\ 28''$, and for the second, at $17h.\ 22'.\ 40''$, at which times they were $76°.\ 43'$ north following the planet.

485. The next thing to be determined in the elements of these satellites, was their distances from the planet; to obtain which, he found one distance by observation, and then the other from the periodic times (218). Now in attempting to discover the distance of the second, the orbit was seemingly elliptical. On March 18, 1787, at $8h.\ 2'.\ 50''$, he found the elongation to be $46'',46$, this being the greatest of all the measures he had taken. Hence at the mean distance of the Georgian from the earth, this elongation will be $44'',23$. Admitting therefore, for the present, says Dr. HERSCHEL, that the satellite moves in a circular orbit, we may take $44'',23$ for the true distance without much error; hence, as the squares of the periodic times are as the cubes of the distances, the distance of the first satellite comes out $33'',09$. The synodic revolutions were here used instead of the sidereal, which will make but a small error.

486. The last thing to be done was to determine the inclination of the orbits, and places of their nodes. And here a difficulty presented itself which could not be got over at the time of his first observation, for it could not then be determined which part of the orbit was inclined *to* the earth, and which *from* it. On the two different suppositions therefore Dr. HERSCHEL has computed the inclinations of the orbits, and the places of the nodes, and found them as follows. The orbit of the second satellite is inclined to the ecliptic 99°. 43′. 53″,3, or 81°. 6′. 4″,4; its ascending node upon the ecliptic is in 5^s. 18°, or 8^s. 6°; and when the planet comes to the ascending node of this satellite, which will happen about the year 1799, or 1818, the northern half of the orbit will be turned towards the east, or west, at the time of its meridian passage. M. de LAMBRE makes the ascending node in 5^s. 21°, or 8^s. 9°, from Dr. HERSCHEL's observations. The situation of the orbit of the first satellite does not materially differ from that of the second. The light of the satellites is extremely faint; the second is the brightest, but the difference is small. The satellites are probably not less than those of *Jupiter*. There will be eclipses of these satellites about the year 1799, or 1818, when they will appear to ascend through the shadow of the planet, in a direction almost perpendicular to the ecliptic. In the *Phil. Trans.* for 1798, Dr. HERSCHEL announced the discovery of four other satellites of the Georgian; and that the motions of all the satellites are retrograde.

CHAP. XXI.

ON THE RING OF SATURN.

Art. 487. GALILEO was the first person who observed any thing extraordinary in *Saturn*. The planet appeared to him like a large globe between two small ones. In the year 1610 he announced this discovery. He continued his observations till 1612, when he was surprised to find only the middle globe; but sometime after he again discovered the globes on each side, which, in process of time, appeared to change their form; sometimes appearing round, sometimes oblong like an acorn, sometimes semicircular, then with horns towards the globe in the middle, and growing by degrees so long and wide as to encompass it, as it were with an oval ring. Upon this HUYGENS set about improving the art of grinding object glasses; and made telescopes which magnified two or three times more than any which had been before made, with which he discovered very clearly the ring of Saturn; and having observed it for some time, he published the discovery in 1656. He made the space between the globe and the ring equal to, or rather bigger than the breadth of the ring; and the greatest diameter of the ring to that of the globe as 9 to 4. But Mr. POUND, with a micrometer applied to HUYGENS's telescope of 123 feet long, determined the ratio to be as 7 to 3. Mr. WHISTON, in his Memoirs of the Life of Dr. CLARK, relates, that the Doctor's Father once saw a fixed star between the ring and the body of Saturn. In the year 1675, M. CASSINI saw the ring, and observed upon it a dark elliptical line, dividing it as it were into two rings, the inner of which appeared brighter than the outer. He also observed a dark belt upon the planet, parallel to the major axis of the ring. Mr. HADLEY observed, that the outer part of the ring seemed narrower than the inner part; and that the dark line was fainter towards its upper edge; he also saw two belts, and observed the shadow of the ring upon Saturn. In October 1714, when the plane of the ring very nearly passed through the earth, and was approaching to it, M. MARALDI observed, that while the arms were decreasing both in length and breadth, the eastern arm appeared a little larger than the other for three or four nights, and yet it vanished first, for after two nights interruption by clouds, he saw the western arm alone. This inequality of the ring made him suspect that it was not bounded by exactly parallel planes, and that it turned about its axis. But the best description of this singular phænomenon is that given by Dr. HERSCHEL in the *Phil. Trans.* 1790, who, by his extraordinary telescopes, has discovered many circumstances which had escaped

FIG. 105. all other observers. We shall here give the substance of his account. Figure 105, is a view of Saturn and its ring, as they appeared on June 20, 1778.

488. The black disc, or belt, upon the ring of Saturn is not in the middle of its breadth; nor is the ring subdivided by many such lines, as has been represented by some astronomers; but there is one* single, dark, considerably broad line, belt, or zone, as in this Figure, which he has constantly found on the north side of the ring. As this dark belt is subject to no change whatever, it is probably owing to some permanent construction of the surface of the ring. This construction cannot be owing to the shadow of a chain of mountains, since it is visible all round on the ring; for at the ends of the ring there could be no shade; and the same argument will hold against any supposed caverns. It is moreover pretty evident, that this dark zone is contained between two concentric circles, as all the phænomena answer to the projection of such a zone. The substance of the ring is undoubtedly no less solid than the planet itself; and it is observed to cast a strong shadow upon the planet. The light of the ring is also generally brighter than that of the planet; for the ring appears sufficiently bright, when the telescope affords scarcely light enough for Saturn. Dr. HERSCHEL next takes notice of the extreme thinness of the ring. He frequently saw the first, second, third, fourth and fifth satellites pass before and behind the ring in such a manner, that they served as an excellent micrometer to measure its thickness by. It may be proper to mention a few instances, as they serve also to solve some phænomena observed by other Astronomers, without having been accounted for in any manner that could be admitted consistently with other known facts. July 18, 1789, at 19*h*. 41′. 9″ sidereal time, the third satellite seemed to hang upon the following arm, declining a little towards the north, and was seen gradually to advance upon it towards the body of Saturn; but the ring was not so thick as the lucid point. July 23, at 19*h*. 41′. 8″, the fourth satellite was a very little preceding the ring, but the ring appeared to be less than half the thickness of the satellite. July 27, at 20*h*. 15′. 12″, the fourth satellite was about the middle, upon the following arm of the ring, and towards the south; and the second at the farther end, towards the north; but the arm was thinner than either. August 29, at 22*h*. 12′. 25″, the fifth satellite was upon the ring, near the end of the preceding arm, and the thickness of the arm seemed to be about $\frac{1}{3}$ or $\frac{1}{4}$ of the diameter of the satellite, which, in the situation it then was, he took to be less than one second in diameter. At the same time, the first appeared at a little distance following the fifth, in the shape of a bead upon a thread, projecting on both sides of the same arm; hence the arm is thinner

* In a Paper in the *Phil. Trans.* 1792, Dr HERSCHEL observes that, "since the year 1774 to the present time, I can find only four observations where any other black division of the ring is mentioned than the one which I have constantly observed; these were all in June, 1780."

than the first, which is considerably smaller than the second, and a little less than the third. October 16, he followed the first and second satellites up to the very disc of the planet; and the ring, which was extremely faint, did not obstruct his seeing them gradually approach the disc. These observations are sufficient to show the extreme thinness of the ring. But Dr. HERSCHEL further observes, that there may be a refraction through an atmosphere of the ring, by which the satellites may be lifted up and depressed, so as to become visible on both sides of the ring, even though the ring should be equal in thickness to the smallest satellite, which may amount to 1000 miles. From a series of observations upon luminous points of the ring, he has discovered that it has a rotation about its axis, the time of which is 10*h*. 32′. 15″,4.

489. The ring is invisible* when its plane passes through the sun, the earth, or between them; in the first case, the sun shines only upon its edge, which is too thin to reflect sufficient light to render it visible; in the second case, the edge only being opposed to us, it is not visible for the same reason; in the third case, the dark side of the ring is exposed to us, and therefore the edge being the only luminous part which is towards the earth, it is invisible on the same account as before. Observers have differed 10 or 12 days in the time of its becoming invisible, owing to the difference of the telescopes, and of the state of the atmosphere. Dr. HERSCHEL observes, that the ring was seen in his telescope when we were turned towards the unenlightened side; so that he either saw the light reflected from the edge, or else the reflection of the light of Saturn upon the dark side of the ring, as we sometimes see the dark part of the moon. He cannot however say which of the two might be the case; especially as there are very strong reasons to think, that the edge of the ring is of such a nature as not to reflect much light. M. de la LANDE thinks that the ring is just visible with the best telescopes in common use, when the sun is elevated 3′ above its plane, or 3 days before its plane passes through the sun; and when the earth is elevated 2′. 30″ above the plane, or one day from the earth's passing it.

490. In a paper in the *Phil. Trans.* 1790, Dr. HERSCHEL ventured to hint at a suspicion that the ring was divided; this conjecture was strengthened by future observations, after he had had an opportunity of seeing both sides of the ring. His reasons are these: 1. The black division upon the southern side of the ring, is in the same place, of the same breadth, and at the same distance from the outer edge, that it always appeared upon the northern side. 2. With his seven feet reflector and an excellent speculum, he saw the division on the ring, and the open space between the ring and the body, equally dark, and of

* The disappearance of the Ring is only with the telescopes in common use among Astronomers; for Dr. HERSCHEL, with his large telescopes, has been able to see it in every situation. He thinks the edge of the Ring is not flat, but spherical or spheroidical.

the same colour with the heavens about the planet. 3. The black division is equally broad on each side of the ring. From these observations, Dr. HERSCHEL thinks himself authorized to say that Saturn has two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet. The dimensions of the two rings are in the following proportions, as nearly as they could be ascertained.

	Parts.
Inside diameter of the smaller ring . .	5900
Outside diameter	7510
Inside diameter of the larger ring . .	7740
Outside diameter	8300
Breadth of the inner ring	805
Breadth of the outer ring	280
Breadth of the space between the rings .	115

In the *Mem. de l'Acad.* at Paris 1787, M. de la PLACE supposes that the ring may have many divisions; but Dr. HERSCHEL remarks, that no observations will justify this supposition.

491. From the mean of a great many measures of the diameter of the larger ring, Dr. HERSCHEL makes it 46″,677 at the mean distance of Saturn. Hence, its diameter : the diameter of the earth :: 25,8914 : 1. From the above proportions therefore, the diameter of this ring must be 204883 miles; and the distance of the two rings 2839 miles.

492. The ring being a circle, appears elliptical from its oblique position; and it appears most open when Saturn is 90° from the nodes of the ring upon the orbit of Saturn, or when Saturn's longitude is about 2ˢ. 17°, and 8ˢ. 17°. In such a situation, the minor axis is extremely nearly equal to half the major, when the observations are reduced to the sun; consequently the plane of the ring makes an angle of about 30° with the orbit of Saturn; it will also be shown that it continues parallel to itself. The situation of the nodes of the ring, and all its other phænomena, may be determined as follows.

FIG. 106. 493. Let S be the sun, AB the orbit of the earth, $nlrzv$ the orbit of Saturn, vSn the line of the nodes; transfer these circles to the sphere of the fixed stars, and let them be represented by NZ, NX; and let RV be the plane of the ring extended to the same sphere; then F is the place of the node of the ring upon Saturn's orbit, and V the node upon the ecliptic; draw FtS, and t is the place of Saturn when the plane of the ring passes through the sun; also let r be the place of Saturn when the plane passes through the earth at c, and draw crH; let z be any other position of Saturn, and e the corresponding place of the earth,

and draw Szd, ezm to the abovementioned sphere; join mV by a great circle, and let fall the perpendiculars dp, mw upon VR, and mG, HI and FT upon ZN.

494. The place F of the node upon the orbit of Saturn may be immediately found, from observing the heliocentric place of Saturn in its orbit when the plane passes through the sun, for that place is the place of the node. Now as the ring is invisible a few days before, and as many after it passes through the sun, to get the time when it passes, observe the time when it disappears and the time when it becomes visible, and the middle point of time is the time when the plane passed through the sun; and the place of Saturn at that time is the place of the node of the ring. According to M. de la LANDE, on January 8, 1774, the plane of the ring passed through the sun, at which time the longitude of Saturn upon its orbit was $5^s. 20°. 38'$, which therefore was the place of the node F of the ring. Now the node N at that time was in $3^s. 21°. 43'$; hence, $FN = 58°. 55'$, the distance of the node of the ring upon the orbit of Saturn from the node of Saturn; and to find the distance VN upon the ecliptic, we have in the right angled triangle FTN, $FN = 58°. 55'$ and the angle $FNT = 2°. 29'. 50''$; hence we find $TN = 58°. 53'. 33''$, $TF = 2°. 8'. 19''$, and the angle $NFT = 88°. 42'. 36''$; from which subtract $NFV = 30°$ (492) the angle which the plane of the ring makes with the orbit of Saturn, and we have $VFT = 58°. 42'. 36''$; hence, in the right angled triangle VFT, we have $FT = 2°. 8'. 19''$, and $VFT = 58°. 42'. 36''$, therefore $VT = 3°. 30'. 49''$, which subtracted from TN, leaves $VN = 55°. 22'. 44''$ the distance of the node of the ring on the ecliptic from Saturn's node; and the angle $FVT = 31°. 21'. 19''$ the inclination of the ring to the ecliptic. M. MARALDI made it $31°. 20'$; HEINSIUS made it $31°. 23'. 17''$. Also, as the longitude of N was $3^s. 21°. 43'$, we have the longitude of the node V of the ring upon the ecliptic $5^s. 17°. 5'. 44''$. MARALDI found it $5^s. 16°. 17'$ in 1715; and if from that time to 1774 we allow $49'$ for the precession of the equinoxes, it makes the place in 1774 to be $5^s. 17°. 6'$, within $16''$ of what it was found from observation. Hence it appears, that the plane of the ring is fixed. The place of the node of the ring upon the orbit of Saturn was, according to HUYGENS, in the middle of the last century $5^s. 20°. 30'$; CASSINI made it $5^s. 22°$. This is the ascending node.

495. The place of the node may also be found from the time when the plane of the ring passes through the earth, observing, at that time, the geocentric latitude HI, and the longitude, from which we get IN, knowing the place of the node N. Hence, in the right angled triangle NIv, we know IN and the angle N, to find Nv, Iv and the angle NvI; therefore in the triangle HvF, we know Hv, HvF and HFv, to find vF, which taken from vN leaves FN. Or if we suppose the angle HVI known, then knowing HI, we find IV, therefore if we know the longitude of I, we shall know that of V. Now according to M. de la LANDE, the ring passed through the earth on April 3, 1774, at

which time the geocentric place I of Saturn was $5^s.\ 21^\circ.\ 7'.\ 38''$, and latitude HI was $2^\circ.\ 27'$; if therefore we suppose the angle HVI to be $31^\circ.\ 21'.\ 19''$, we have $IV = 4^\circ.\ 1'.\ 35''$, which subtracted from $5^s.\ 21^\circ.\ 7'.\ 38''$, leaves the longitude of the node V on the ecliptic $5^s.\ 17^\circ.\ 6'.\ 3''$, which is within $19''$ of what it was found (494) from the passage of the plane through the sun. When the earth and Saturn are moving in opposite directions, the place of the node may be more accurately determined by this method, than by the passage of the plane through the sun; because the disappearance takes place quicker, and therefore you can determine the time more accurately.

496. In determining the nodes of the ring, we supposed the inclination known, whereas the inclination is found from knowing the place of the nodes, by observing the ring when Saturn is 90° from the nodes. But by constantly observing the opening of the ring about the time when it is the greatest, we shall get very nearly its inclination, and a small error in that will make but a very little alteration in the place of the node.

497. When the plane of the ring passes through the earth, we have $\frac{\tan.\ IH}{\sin.\ IV} = \tan.\ IVH = \tan.\ 31^\circ.\ 20' = 0{,}6088$. As this must take place when I is within a few degrees of the node V, it will be very easy to compute when the passage happens, by assuming some time, and taking at that time the geocentric latitude and longitude of Saturn from the *Nautical Almanac*, and thence finding IV, and dividing the tangent of the latitude HI by the sine of IV; assume two times as near as you can conjecture to the time required, and the two results will direct you to find, by proportion, a time very near to that required; thus you will very easily get the time.

Ex. On May 3, 1789, the geocentric longitude of Saturn was $11^s.\ 20^\circ.\ 23'$, and latitude $1^\circ.\ 54'.\ 20''$; and taking the place of the descending node to be $11^s.\ 17^\circ.\ 18'$, we have $IV = 3^\circ.\ 5'$; hence, $\frac{\tan.\ 1^\circ.\ 54'.\ 20''}{\sin.\ 3^\circ.\ 5'} = 0{,}619$. On May 4, we find the value to be 0,602; hence, the ring passed through the earth between May 3 and 4, agreeing with Dr. Maskelyne's computation (501).

498. To determine the appearance of the ring when the earth is at any point e, and Saturn at z, it is manifest, that e is at the same angular distance from the plane of the ring at Saturn that m is, the angles at z being vertical; but the angular distance of m from the plane of the ring at z is measured by mw, which therefore measures the elevation of the eye at e above the plane. Now to find mw, let there be given mG the geocentric latitude of Saturn, and its geocentric longitude, or the point G on the ecliptic; then, as the point V is known, we shall know GV; hence we can find mV, and the angle mVG, which subtracted from wVG gives wVm; therefore in the triangle mwV, we can find

mw the elevation of the eye above the plane of the ring. Hence, rad. : sin. mw :: major axis : minor axis of the ring.

Ex. On July 12, 1784, Mr. BUGGE observed the geocentric longitude of Saturn to be $9^s. 20°. 34'. 48''$, and latitude $3'. 35''$ N. he also determined the ascending node of Saturn's orbit to be $3^s. 21°. 50'. 8''$; hence, $GV = 4^s. 3°. 29'. 48''$, and as $Gm = 3'. 35''$, we have $mV = 123°. 29'. 48''$, and the angle $mVG = 4'. 18''$; and if we take $wVG = 31°. 21'. 19''$, we have $mVw = 31°. 17'. 1''$; hence $mw = 25°. 38'. 37''$; consequently the major axis : minor axis :: rad. : sin. $25°. 38'. 37''$ :: 100 : 43.

499. The arc dp measures the elevation of the sun above the plane of the ring; hence, knowing the heliocentric longitude of Saturn on its orbit, or of the point d, and the longitude of F the node of the ring upon its orbit, we know dF; and we know also the angle dFp which the plane of the ring makes with the orbit; hence, in the right angled triangle Fpd, we find dp the elevation required.

500. In the same manner as we have determined the inclination of the ring and position of the nodes, the inclination of the orbit of the seventh satellite and place of its node may be determined, by measuring the minor axis of the orbit which it appears to describe at the time when it is greatest, and comparing it with the major axis, or twice the greatest elongation. The semi-minor axis is determined by measuring the distance of the satellite from the planet, when that distance is the least in the whole revolution.

501. In the *Nautical Almanac* for 1791, Dr. MASKELYNE has computed the disappearances and re-appearances of the ring in 1789 and 1790; assuming the place of the descending node on the ecliptic to be $11^s. 17°. 18'$, the ascending node of Saturn $3^s. 21°. 51'$; inclination of its orbit $2°. 30'. 20''$; inclination of the ring to the ecliptic $31°. 20'$, and place of the ascending node of the ring on Saturn's orbit $5^s. 20°. 52'$, all according to M. de la LANDE; and supposing, with him, that the ring is just visible with the best telescopes in common use when the sun is elevated $3'$ above its plane, or three days before the plane passes through the sun; and when the earth is elevated $2'. 30''$ above the plane, or one day from the earth's passing it.

May 3, 1789, the ring passes through the earth, the earth passing from the northern side which is enlightened, to its southern side which is dark.

August 26, the ring repasses to the northern or enlightened side.

October 11, the ring passes through the sun; when it will change its enlightened side, from the northern to the southern one; consequently the dark side will then be towards the earth.

January 29, 1790, the ring passes through the earth, and the earth passing

from the northern or dark, to the southern, or enlightened side of the ring, the ring will become visible, and continue so till 1803.

The phænomena may happen five days sooner or later than here set down, if the Tables should err 10′ in the geocentric longitude of Saturn.

By observation with an achromatic telescope of five feet focal length, Dr. Maskelyne concluded that the ring repassed through the earth on August 28, at 11 hour.

The following Tables, taken from the *Recueil de Tables Astronomiques, Berlin*, 1776, are calculated to show the apparent figure of the Ring, and of the orbits of the satellites, as seen either from the sun or the earth.

For the Ring, and Six first Satellites.				
Arg. Long. ♄ + 13°. 43′. 30′				
Deg.	O. VI. − +	I. VII. − +	III. VIII. − +	Deg.
0	0,000	0,260	0,451	30
3	0,027	0,284	0,464	27
6	0,054	0,306	0,476	24
9	0,081	0,328	0,486	21
12	0,108	0,348	0,495	18
15	0,135	0,368	0,503	15
18	0,161	0,384	0,509	12
21	0,187	0,405	0,514	9
24	0,212	0,421	0,518	6
27	0,236	0,437	0,520	3
30	0,260	0,451	0,521	0
	+ − XI. V.	+ − X. IV.	+ − IX. III.	

For the Seventh Satellite.				
Arg. Long. ♄ + 24°. 50′.				
Deg.	O. VI. − +	I. VII. − +	III. VIII − +	Deg.
0	0,000	0,129	0,224	30
3	0,014	0,141	0,230	27
6	0,027	0,152	0,236	24
9	0,041	0,163	0,242	21
12	0,054	0,174	0,246	18
15	0,064	0,183	0,250	15
18	0,080	0,192	0,253	12
21	0,093	0,201	0,256	9
24	0,105	0,209	0,257	6
27	0,117	0,217	0,258	3
30	0,129	0,224	0,259	0
	+ − XI. V.	+ − X. IV.	+ − IX. III	

To the quantity taken from the Tables, apply the latitude of Saturn expressed in minutes divided by 4000, with the sign − when the latitude is north, and + when it is south; but for the seventh satellite, we must multiply this last

quantity by $\frac{10}{9}$; and the result gives the minor axis of the ring, or of the orbits, the major axis being unity.

Ex. On April 22, 1767, the geocentric latitude of Saturn was $1^\circ.\ 10'$ south, and longitude $5^s.\ 16^\circ.\ 55'$; hence,

For the Ring, and Six first Satellites.

$2^s.\ 16^\circ.\ 55'$	
$13.\ 43$	
$3.\ 0.\ 38$ - - - -	$-0{,}521$
$\frac{70}{4000}$ - - - - -	$+0{,}017$
Minor axis - -	$-0{,}504$

For the Seventh Satellite.

$2^s.\ 16^\circ.\ 55'$	
$24.\ 50$	
$3.\ 11.\ 45$ - -	$-0{,}253$
$\frac{70}{4000} \times \frac{10}{9}$ - -	$+0{,}019$
Minor axis - -	$-0{,}234$

The sign + shows that that half of the ring, or of the orbits, which is most distant, is more *north* than the center of Saturn, and the sign − shows it to be more *south.*

The *geocentric* latitude and longitude being here taken, we get the appearance as seen from the earth; the *heliocentric* latitude and longitude being assumed, gives the appearance at the sun.

CHAP. XXII.

ON THE ABERRATION OF LIGHT.

Art. 502. IN the year 1725, Mr. MOLYNEUX, assisted by Dr. BRADLEY, fitted up a zenith sector at Kew, in order to discover whether the fixed stars had any
FIG. 107. sensible parallax*, that is, whether a star s would appear to have changed its place whilst the earth moved from one extremity A of the diameter of its orbit to the other extremity C; or which is the same, to determine whether the diameter AC of the earth's orbit subtends any sensible angle AsC at the star s. The very important discovery arising from their observations is so clearly and fully related by Dr. BRADLEY in a Letter to Dr. HALLEY, that I cannot do better than give it the reader in his own words. *Phil. Trans.* N°. 406.

503. " Mr. MOLYNEUX's apparatus was completed and fitted for observing about the end of November 1725, and on the third day of December following, the bright star in the head of *Draco* (marked γ by BAYER) was for the first time observed as it passed near the zenith, and its situation carefully taken with the instrument. The like observations were made on the 5th, 11th and 12th of the same month; and there appearing no material difference in the place of the star, a farther repetition of them at this season seemed needless, it being a part of the year wherein no sensible alteration of parallax in this star could soon be expected. It was chiefly therefore curiosity that tempted me (being then at Kew where the instrument was fixed) to prepare for observing the star on December 17, when having adjusted the instrument as usual, I perceived that it passed a little more southwardly this day than when it was observed before. Not suspecting any other cause of this appearance, we first concluded that it was owing to the uncertainty of the observations, and that either this or the foregoing were not so exact as we had before supposed; for which reason we purposed to repeat the observation again, in order to determine from whence this difference proceeded; and upon doing it on December 20, I found that the star passed still more southwardly than in the former observations. This sensible alteration the more surprised us, in that it was the contrary way from what it would have been, had it proceeded from an annual parallax of the star:

* Dr. HOOK was the inventor of this method, and in the year 1669 he put it in practice at Gresham College, with a telescope 36 feet long. His first observation was July 6, at which time he found the bright star in the head of *Draco*, marked γ by BAYER, passed about 2′. 12″ northward from the zenith; on July 9, it passed at the same distance; on August 6, it passed 2′. 6″ northward from the zenith; on October 21, it passed 1′. 48″ or 50″ north from the zenith, according to his observations. See his *Cutlerian* Lectures.

but being now pretty well satisfied that it could not be entirely owing to the want of exactness in the observations, and having no notion of any thing else that could cause such an apparent motion as this in the star, we began to think that some change in the materials, &c. of the instrument itself might have occasioned it. Under these apprehensions we remained some time, but being at length fully convinced by several trials of the great exactness of the instrument, and finding by the gradual increase of the star's distance from the pole, that there must be some regular cause that produced it, we took care to examine nicely at the time of each observation how much it was; and about the beginning of March 1726, the star was found to be 20″ more southwardly than at the time of the first observation. It now indeed seemed to have arrived at its utmost limit southward, because in several trials made about this time no sensible difference was observed in its situation. By the middle of April it appeared to be returning back again towards the north; and about the beginning of June it passed at the same distance from the zenith as it had done in December when it was first observed.

"From the quick alteration of this star's declination about this time (it increasing a second in three days) it was concluded that it would now proceed northward, as it before had gone southward of its present situation; and it happened as was conjectured, for the star continued to move northward till September following, when it again became stationary, being then near 20″ more northwardly than in June, and no less than 39″ more northwardly than it was in March. From September the star returned towards the south, till it arrived in December to the same situation it was in at that time twelve months, allowing for the difference of declination on account of the precession of the equinox.

"This was a sufficient proof that the instrument had not been the cause of this apparent motion of the star, and to find one adequate to such an effect seemed a difficulty. A nutation of the earth's axis was one of the first things that offered itself upon this occasion; but it was soon found to be insufficient; for though it might have accounted for the change of declination in γ Draconis, yet it would not at the same time agree with the phænomena in other stars, particularly in a small one almost opposite in right ascension to γ Draconis, at about the same distance from the north pole of the equator; for though this star seemed to move the same way as a nutation of the earth's axis would have made it, yet it changing its declination but about half as much as γ Draconis in the same time, (as appeared upon comparing the observations of both made upon the same days at different seasons of the year,) this plainly proved that the apparent motion of the stars was not occasioned by a real nutation, since if that had been the cause, the alteration in both stars would have been nearly equal.

"The great regularity of the observations left no room to doubt but that there

was some regular cause that produced this unexpected motion, which did not depend on the uncertainty or variety of the seasons of the year. Upon comparing the observations with each other, it was discovered that in both the forementioned stars the apparent difference of declination from the maxima was always nearly proportional to the versed sine of the sun's distance from the equinoctial points. This was an inducement to think that the cause, whatever it was, had some relation to the sun's situation with respect to those points. But not being able to frame any hypothesis at that time sufficient to solve all the phænomena, and being very desirous to search a little farther into this matter, I began to think of erecting an instrument for myself at Wanstead, that having it always at hand I might with the more ease and certainty enquire into the laws of this new motion. The consideration likewise of being able by another instrument to confirm the truth of the observations hitherto made with Mr. Molyneux's was no small inducement to me; but the chief of all was the opportunity I should thereby have of trying in what manner other stars were affected by the same cause, whatever it was. For Mr. Molyneux's instrument being originally designed for observing γ Draconis (in order, as I said before, to try whether it had any sensible parallax) was so contrived as to be capable of but little alteration in its direction, not above seven or eight minutes of a degree; and there being few stars within half that distance from the zenith of Kew bright enough to be well observed, he could not with his instrument thoroughly examine how this cause affected stars differently situated with respect to the equinoctial and solstitial points of the ecliptic.

"These considerations determined me; and by the contrivance and direction of the very ingenious person Mr. Graham, my instrument was fixed up August 19, 1727. As I had no convenient place where I could make use of so long a telescope as Mr. Molyneux's, I contented myself with one of but little more than half the length of his (viz. of about $12\frac{1}{2}$ feet, his being $24\frac{1}{4}$) judging from the experience which I had already had, that this radius would be long enough to adjust the instrument to a sufficient degree of exactness, and I have had no reason since to change my opinion; for from all the trials I have yet made, I am very well satisfied that when it is carefully rectified, its situation may be securely depended upon to half a second. As the place where my instrument was to be hung in some measure determined its radius, so did it also the length of the arch or limb on which the divisions were made to adjust it; for the arch could not conveniently be extended farther than to reach to about 6°. 15′ on each side my zenith. This indeed was sufficient, since it gave me an opportunity of making choice of several stars very different both in magnitude and situation, there being more than two hundred inserted in the British Catalogue that may be observed with it. I needed not to have extended the limb so far, but that I was willing to take in Capella, the only star of the first magnitude that comes so near my zenith.

"My instrument being fixed, I immediately began to observe such stars as I judged most proper to give me light into the cause of the motion already mentioned. There was variety enough of small ones, and not less than twelve that I could observe through all the seasons of the year, they being bright enough to be seen in the day time when nearest the sun. I had not been long observing before I perceived that the notion we had before entertained of the stars being farthest north and south when the sun was about the equinoxes, was only true of those that were near the solstitial colure; and after I had continued my observations a few months, I discovered what I then apprehended to be a general law, observed by all the stars, viz. that each of them became stationary or was farthest north or south when they passed over my zenith at six o'clock either in the morning or evening. I perceived likewise that whatever situation the stars were in with respect to the cardinal points of the ecliptic, the apparent motion of every one tended the same way when they passed my instrument about the same hour of the day or night; for they all moved southward while they passed in the day, and northward in the night; so that each was farthest north when it came about six o'clock in the evening, and farthest south when it came about six in the morning.

"Though I have since discovered that the maxima in most of these stars do not happen exactly when they come to my instrument at those hours, yet not being able at that time to prove the contrary, and supposing that they did, I endeavoured to find out what proportion the greatest alterations of declination in different stars bore to each other; it being very evident that they did not all change their declinations equally. I have before taken notice that it appeared from Mr. MOLYNEUX's observations that γ Draconis altered its declination about twice as much as the fore-mentioned small star almost opposite to it; but examining the matter more particularly, I found that the greatest alteration of declination in these stars was as the sine of the latitude of each respectively. This made me suspect that there might be the like proportion between the maxima of other stars; but finding that the observations of some of them would not perfectly correspond with such an hypothesis, and not knowing whether the small difference I met with might not be owing to the uncertainty and error of the observations, I deferred the farther examination into the truth of this hypothesis till I should be furnished with a series of observations made in all parts of the year; which might enable me not only to determine what errors the observations are liable to, or how far they may safely be depended upon; but to judge whether there had been any sensible change in the parts of the instrument itself.

"Upon these considerations I laid aside all thoughts at that time about the cause of the fore-mentioned phænomena, hoping that I should the easier discover it when I was better provided with proper means to determine more precisely what they were.

"When the year was completed I began to examine and compare my observations, and having pretty well satisfied myself as to the general laws of the phænomena, I then endeavoured to find out the cause of them. I was already convinced that the apparent motion of the stars was not owing to a nutation of the earth's axis. The next thing that offered itself, was an alteration in the direction of the plumb-line with which the instrument was constantly rectified; but this upon trial proved insufficient. Then I considered what refraction might do, but here also nothing satisfactory occurred. At last I conjectured that all the phænomena hitherto mentioned proceeded from the progressive motion of light and the earth's annual motion in its orbit. For I perceived if light was propagated in time, the apparent place of a fixed object would not be the same when the eye is at rest, as when it is moving in any other direction than that of the line passing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different."

This is Dr. BRADLEY's account of this very important discovery; we shall therefore proceed to show that his principle will solve all the phænomena.

504. The situation of any object in the heavens is determined by the position of the axis of the telescope annexed to the instrument with which we measure; for such a position is given to the telescope, that the rays of light from the object may descend down the axis, and in that situation the index shows the angular distance required. Now if light be progressive, when a ray from any object descends down the axis, the position of the telescope must be different from what it would have been if light had been instantaneous, and therefore the place which is measured in the heavens will be different from the true place. For let S' be a fixed star, VF the direction of the earth's motion, $S'F$ the direction of a particle of light, entering the axis ac of a telescope at a, and moving through aF while the earth moves from c to F; then if the telescope keep parallel to itself, the light will descend in the axis. For let the axis nm, Fw continue parallel to ac; then, considering each motion* as uniform, the spaces described in the same time will continue in the same proportion; but $cF : aF :: cn : av$, and by supposition cF, aF are described in the same time, therefore cn, av, are described in the same time; hence when the telescope comes into the situation nm, the particle of light will be in the axis at v; and this being true for every instant, in this position of the telescope the ray will descend down the axis, and consequently the place measured by the telescope at F is s', and the angle $S'Fs'$ is the *aberration*, or *the difference between the true place of the star and the place measured by the instrument.* Hence, if we take any line $FS : Ft ::$ velocity of light : the velocity of the earth,

* The motion of the spectator arising from the rotation of the earth about its axis is not here taken into the calculation, it being so small as not to produce any sensible effect.

and join St, and complete the parallelogram $FtSs$, the aberration will be equal to FSt. Also S represents the true place of the star, and s the place measured by the instrument.

505. As the place measured by the instrument differs from the true place, let us next consider how the progressive motion of light may affect the place of the star seen by the naked eye. If a ray of light fall upon the eye in motion, its relative motions in respect to the eye will be the same as if you were to impress equal motions in the same direction upon each at the instant of contact; for equal motions in the same direction impressed upon two bodies will not affect their relative motions, and therefore the effect of one upon the other will not be altered. Let VF be a tangent to the earth's orbit at F which will represent the direction of the earth's motion at F, S' the star, join $S'F$ and produce it to G, and take $FG : Fn ::$ the velocity of light : the velocity of the earth in its orbit, and complete the parallelogram $nFGH$, and draw the diagonal FH. Now as FG, nF represent the motions of light and of the earth in its orbit, conceive a motion Fn equal and opposite to nF to be impressed upon the eye at F and upon the ray of light, then the eye will be at rest, and the ray of light, by the two motions FG, Fn, will describe the diagonal FH; this therefore is the relative motion of the ray of light in respect to the eye itself. Hence, the object appears in the direction HF, and consequently its apparent place differs from its true place by the angle $GFH = FSt$. It appears therefore, that the apparent place of the object to the naked eye is the same as the place measured by the instrument. We may therefore call the place measured by the instrument, the apparent place. Many writers have given the explanation in this article, as the proof of the aberration, not considering that the aberration is the difference between the true place and that measured by the instrument, or, as you may call it, the *instrumental* error; indeed, in this case, the apparent place to the naked eye coincides with the place measured by the instrument, and therefore no error has been produced by considering it in that point of view; but it introduces a wrong idea of the subject; the correction which we apply, or the aberration, is the correction of the place measured by the instrument, and therefore the investigation ought to proceed upon this principle; how much does the measured place differ from the true place?

506. By Trigonometry, sin. FSt : sin. $FtS :: Ft : FS ::$ velocity of the earth : velocity of light; hence, sine of aberration $=$ sin. $FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$; therefore if we consider the velocity of the earth and of light to be constant, the sine of aberration, or the aberration itself as it never exceeds $20''$, varies as sin. FtS, and therefore is greatest when that angle is a right angle; if therefore s be put for the sine of FtS, we have 1 (rad.) $: s :: 20'' : s \times 20''$ the aberration.

and join St, and complete the parallelogram $FtSs$, the aberration will be equal to FSt. Also S represents the true place of the star, and s the place measured by the instrument.

505. As the place measured by the instrument differs from the true place, let us next consider how the progressive motion of light may affect the place of the star seen by the naked eye. If a ray of light fall upon the eye in motion, its relative motions in respect to the eye will be the same as if you were to impress equal motions in the same direction upon each at the instant of contact; for equal motions in the same direction impressed upon two bodies will not affect their relative motions, and therefore the effect of one upon the other will not be altered. Let VF be a tangent to the earth's orbit at F which will represent the direction of the earth's motion at F, S' the star, join $S'F$ and produce it to G, and take $FG : Fn ::$ the velocity of light : the velocity of the earth in its orbit, and complete the parallelogram $nFGH$, and draw the diagonal FH. Now as FG, nF represent the motions of light and of the earth in its orbit, conceive a motion Fn equal and opposite to nF to be impressed upon the eye at F and upon the ray of light, then the eye will be at rest, and the ray of light, by the two motions FG, Fn, will describe the diagonal FH; this therefore is the relative motion of the ray of light in respect to the eye itself. Hence, the object appears in the direction HF, and consequently its apparent place differs from its true place by the angle $GFH = FSt$. It appears therefore, that the apparent place of the object to the naked eye is the same as the place measured by the instrument. We may therefore call the place measured by the instrument, the apparent place. Many writers have given the explanation in this article, as the proof of the aberration, not considering that the aberration is the difference between the true place and that measured by the instrument, or, as you may call it, the *instrumental* error; indeed, in this case, the apparent place to the naked eye coincides with the place measured by the instrument, and therefore no error has been produced by considering it in that point of view; but it introduces a wrong idea of the subject; the correction which we apply, or the aberration, is the correction of the place measured by the instrument, and therefore the investigation ought to proceed upon this principle; how much does the measured place differ from the true place?

506. By Trigonometry, sin. FSt : sin. $FtS :: Ft : FS ::$ velocity of the earth : velocity of light; hence, sine of aberration $=$ sin. $FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$, therefore if we consider the velocity of the earth and of light to be constant, the sine of aberration, or the aberration itself as it never exceeds $20''$, varies as sin. FtS, and therefore is greatest when that angle is a right angle; if therefore s be put for the sine of FtS, we have 1 (rad.) : $s :: 20'' : s \times 20''$ the aberration.

Hence, when Ft coincides with FS', or the earth be moving directly to or from a star, there is no aberration.

507. As (by observation) the angle $FSt = 20''$ when $FtS = 90°$, we have, *the velocity of the earth : velocity of light :: sin.* $20''$ *: radius* :: 1 : 10314.

508. The aberration $S's'$ lies from the true place of the star in a direction parallel to the direction of the earth's motion, and towards the same part.

509. Whilst the earth makes one revolution in its orbit, the curve, parallel to the ecliptic, described by the apparent place of a fixed star, is a circle. For let $AFBQ$ be the earth's orbit, K the focus in which the sun is, H the other focus; on the major axis AB describe a circle; draw a tangent YFZ to the point F, and KY, HZ perpendicular to it; then, by Conics, the points Y and Z will be always in the circumference of the circle. Let S be the true place of the star, any where out of the plane of the ecliptic, which therefore must be conceived to be elevated above the plane $AFBQ$, and take $tF : FS$ as the velocity of the earth : the velocity of light, and complete the parallelogram $FtSs$, and s will (504) be the apparent place of the star. Draw FL perpendicular to AB, and let $WsVx$ be the curve described by the point s, and WSV be parallel to FL. Now (as will be proved when we come to the physical causes of the planets' motions) the velocity of the earth varies as $\frac{1}{KY}$, or as HZ; but tF, or Ss, represents the velocity of the earth; hence, Ss varies as HZ. Also, as Ss, SV are parallel to FY, FL, the angle $sSV =$ the angle YFL which is $=$ the angle ZHL, because the angle LFZ added to each makes two right angles, for in the quadrilateral figure $LFZH$ the angles L and Z are right ones. Hence, as Ss varies as HZ, and the angle $sSV = ZHA$, the figures described by the points s and Z must be similar; but Z describes a circle in the time of one revolution of the earth in its orbit, hence, s must describe a circle about s in the same time. And as Ss is always parallel to tF which lies in the plane of the ecliptic, the circle $WsVx$ is parallel to the ecliptic. Also, as S and H are two points similarly situated in WV and AB, it appears that the true place of the star divides that diameter which, although in a different plane, we may consider as perpendicular to the major axis of the earth's orbit, in the same ratio as the focus divides the major axis. But as the earth's orbit is very nearly a circle, we may consider S in the center of the circle without any sensible error.

510. As we may, for the purposes which we shall here want to consider, conceive the earth's orbit $AFBQ$ to be a circle, if from the center C we draw Cs' parallel to Ss, or YF, s' will be the point in that circle corresponding to s in the circle $WsVx$, and as $FCs' = 90°$, the apparent place of the star in the circle of aberration is always 90° *before* the place of the earth in its orbit, and consequently the apparent angular velocity of the star and earth about their respec-

tive centers are always equal. It is further supposed, that the star S' is at an indefinitely great distance; for the situation of the star is not supposed to be altered from the motion of the earth, and considering FS as always parallel to itself, it will always be directed to S' as a fixed point in the heavens. Hence also, as the apparent place of the sun is opposite to that of the earth, the apparent place of the star in the circle of aberration is 90°. *behind* that of the sun.

511. As a small part of the heavens may be conceived to be a plane perpendicular to a line joining the star and eye, it follows from the principles of orthographic projection, that the circle parallel to the ecliptic described by the apparent place of the star projected upon this plane will be an ellipse; the apparent path of the star in the heavens will therefore be an ellipse, and the major axis will be to the minor as radius to the sine of the star's latitude. For let CE be the plane of the ecliptic, P its pole, PE a secondary to it, PC perpendicular to EC, C the place of the eye, and let ab be parallel to CE, then it will be that diameter of the circle $anbm$ of aberration which is seen most obliquely, and consequently that diameter which is projected into the minor axis of the ellipse; let mn be perpendicular to ab, and it will be seen directly, being perpendicular to a line drawn from it to the eye, and therefore will be the major axis; draw Ca, Cbd, and ad is the projection of ab; and as ad may be considered as a straight line, we have mn or ab, the major axis : ad the minor :: rad. : sin. abd, or ECd the star's latitude. As the angle bad is the complement of abd, or of the star's latitude, the circle is projected upon a plane making an angle with it equal to the complement of the star's latitude.

512. As the minor axis da coincides with a secondary to the ecliptic, it must be perpendicular to it, and the major axis mn is parallel to it, its position not being altered by projection. Hence, in the pole of the ecliptic, the sine of the star's latitude being radius, the ellipse becomes a circle; and in the plane of the ecliptic, the sine of the star's latitude being $=0$, the minor axis vanishes, and the ellipse becomes a straight line, or rather a very small part of a circular arc.

513. To find the aberration in *latitude* and *longitude*. Let $ABCD$ be the earth's orbit supposed to be a circle with the sun in the center at x, and conceive P to be in a line drawn from x perpendicular to $ABCD$, and to represent the pole of the ecliptic; let S be the true place of the star, and conceive $apcq$ to be the circle of aberration parallel to the ecliptic, and $abcd$ the ellipse into which it is projected; let ♈ T be an arc of the ecliptic, and draw the secondary PSG to it, and (511) it will coincide with the minor axis bd into which the diameter pq is projected; draw $GCxA$, and it (511) is parallel to pq, and BxD perpendicular to it must be parallel to the major axis ac; then when the earth is at A, the star is in conjunction, and in opposition when the earth is at C. Now as the place of the star in the circle of aberration is (510) always 90°

before the earth in its orbit, when the earth is at A, B, C, D the apparent places of the star in the circle will be at a, p, c, q, or in the ellipse at a, b, c, d; and to find the place of the star in the circle when the earth is at any point E, take the angle $pSs = ExB$, and s will be the corresponding place of the star in the circle; and to find the projected place in the ellipse, draw sv perpendicular to Sc, and vt perpendicular to Sc in the plane of the ellipse, and t will be the apparent place of the star in the ellipse; join st and it will be perpendicular to vt, because the projection of the circle into the ellipse is in lines perpendicular to the ellipse; draw the secondary $PvtK$, which will, as to sense, coincide with vt, unless the star be very near to the pole of the ecliptic; therefore the rules here given will be sufficiently accurate, except in that case. Now as cvS is parallel to the ecliptic, S and v must have the same latitude, hence vt is the aberration in latitude; and as G is the true, and K the apparent place of the star in the ecliptic, GK is the aberration in longitude. To find these quantities, put m and n for the sine and cosine of the angle sSc, or CxE the earth's distance from syzygies, radius being unity; and as (511) the angle svt = the complement of the star's latitude, the angle vst = the star's latitude, for the sine and cosine of which put v and w, and put $r = Sa$ or Ss; then in the right angled triangle Ssv, $1 : m :: r : sv = rm$; hence, in the triangle vts, $1 : v :: rm : tv = rvm$ the aberration in *latitude*. Also, in the triangle Ssv, $1 : n :: r : vS = rn$; hence, w (13) : $1 :: rn : GK = \frac{rn}{w}$ the aberration in *longitude*. When the earth is in syzygies $m = 0$, therefore there is no aberration in latitude; and, as n is then greatest, there is the greatest aberration in longitude; if the earth be at A, or the star in conjunction, the apparent place of the star is at a, and reduced to the ecliptic at H, therefore GH is the aberration, which diminishes the longitude of the star, the order of the signs being ♈ GT; but when the earth is at C, or the star in opposition, the apparent place c reduced to the ecliptic is at F, and the aberration GF increases the longitude; hence the longitude is the greatest when the star is in opposition, and least when in conjunction. When the earth is in quadratures at D or B, then $n = 0$, and m is greatest, therefore there is no aberration in longitude, but the greatest in latitude; when the earth is at D, the apparent place of the star is at d and the latitude is there increased; but when the earth is at B, the apparent place of the star is at b and the latitude is diminished; hence, the latitude is least in quadratures before opposition, and greatest in quadratures after. From the mean of a great number of observations, Dr. BRADLEY determined the value of r to be $20''$.

Ex. 1. What is the greatest aberration in latitude and longitude of γ *Ursæ minoris*, whose latitude is 75°. 13′? Here $m = 1$, $v = ,9669$ the sine of 75°. 13′; hence, $20'' \times ,9669 = 19'',34$ the greatest aberration in *latitude*. For the great-

est aberration in longitude, $n=1$, $w=,2551$; hence, $\frac{20''}{,2551}=78'',4$ the greatest aberration in *longitude*.

Ex. 2. What is the aberration in latitude and longitude of the same star, when the earth is 30° from syzygies? Here $m=,5$; hence, $19'',34\times,5=9'',67$ the aberration in *latitude*. If the earth be 30° beyond conjunction or before opposition, the latitude is diminished; but if it be 30° after opposition or before conjunction the latitude is increased. Also, $n=,866$; hence, $78'',4\times,866=67'',89$ the aberration in *longitude*. If the earth be 30° from conjunction, the longitude is diminished; but if it be 30° from opposition, it is increased.

Ex. 3. For the *Sun*, $m=0$ and $n=1$, $w=1$; hence it has no aberration in latitude, and the aberration in longitude $=r=20''$ constantly. This quantity 20″ of aberration of the sun answers to its mean motion in 8′. 7″. 30‴, which is therefore the time the light is moving from the sun to us at its mean distance. Hence, the sun always appears 20″ backwarder than its true place.

The following Table will render the calculation shorter.

The Argument for the Longitude is, *Long. Sun — Long. Star.* The Argument for the Latitude is, *Long. Sun — Long. Star — 3 Signs.*

Deg.	O. VI. − +	I. VII. − +	II. VIII. − +	Deg.
0	20″, 0	17″, 32	10″, 0	30
1	20, 0	17, 14	9, 70	29
2	19, 99	16, 96	9, 39	28
3	19, 97	16, 77	9, 8	27
4	19, 95	16, 58	8, 77	26
5	19, 92	16, 38	8, 45	25
6	19, 89	16, 18	8, 13	24
7	19, 85	15, 97	7, 81	23
8	19, 81	15, 76	7, 49	22
9	19, 75	15, 54	7, 17	21
10	19, 70	15, 32	6, 84	20
11	19, 63	15, 9	6, 51	19
12	19, 56	14, 86	6, 18	18
13	19, 49	14, 63	5, 85	17
14	19, 41	14, 39	5, 51	16
15	19, 32	14, 14	5, 18	15
16	19, 23	13, 89	4, 84	14
17	19, 13	13, 64	4, 50	13
18	19, 2	13, 38	4, 16	12
19	18, 91	13, 12	3, 81	11
20	18, 80	12, 86	3, 47	10
21	18, 67	12, 59	3, 12	9
22	18, 54	12, 21	2, 78	8
23	18, 41	12, 4	2, 44	7
24	18, 27	11, 76	2, 9	6
25	18, 13	11, 47	1, 74	5
26	17, 98	11, 18	1, 40	4
27	17, 82	10, 89	1, 5	3
28	17, 66	10, 60	0, 70	2
29	17, 49	10, 30	0, 35	1
30	17, 32	10, 0	0, 0	0
	− + XI. V.	− + X. IV.	− + IX. III.	

To find the aberration in $\begin{Bmatrix}\text{longitude}\\ \text{latitude}\end{Bmatrix}$, multiply the quantities taken from this Table by $\begin{Bmatrix}\text{secant}\\ \text{sine}\end{Bmatrix}$ of the star's latitude.

Ex. Let the longitude of the sun be $7^s.\ 5^\circ.\ 18'$, the longitude of a star $5^s.\ 18^\circ.\ 14'$, and its latitude $31^\circ.\ 10'$.

$7^s.\ 5^\circ.\ 18'$		
$5.\ 18.\ 14$		
$1.\ 17.\ 4$ - - - - - - - -	$-13'',62$	
$31.\ 10$ sec. - - - -	$1,169$	
Aberration in *Longitude* -	$-15,92$	Product.
$1^s.17^\circ.\ 4' - 3^s = 10^s.\ 17^\circ.\ 4'$	$-14'',65$	
$31.\ 10$ sine - - - -	$0,5175$	
Aberration in *Latitude* - -	$-7,58$	Product.

514. When the earth is at A, the star is in conjunction, and its apparent place at a; therefore the angle AxE described by the earth from conjunction, or the angle sSa, shows the elongation of the star from the sun.

To find the aberration in *Right Ascension* and *Declination*, we shall, in part, follow the method given by M. CAGNOLI in his Trigonometry, as being the most convenient for practice, and from which M. de LAMBRE has computed a set of Tables, by which the aberration may, at any time, be very readily found.

515. To find the aberration in *Declination*. Let $abcd$ be the ellipse of aberration, and P the pole of the ecliptic, as described in the last figure; on the major axis ac describe the circle $apcq$, which we will now suppose to lie in the plane of the ellipse, and then every point of this circle will be projected into the same point of the ellipse as before; let R be the pole of the equator, and perpendicular to RS draw the diameter MN of the ellipse; also draw BMC, YNW perpendicular to ac, and YB, will be the corresponding diameter of the circle; draw FS perpendicular to BY, FQD perpendicular to ac, and QH perpendicular to MS; from any point X let fall XsE perpendicular to ac; draw

Xz, st, perpendicular to BY and MN respectively, and sv an ordinate to the diameter MN.

516. As the point F of the circle lies at the distance of 90° from the diameter BY, the diameter FSf will be projected into a diameter QSr which will be conjugate to MSN, and therefore a tangent at Q is parallel to MN, hence, QH is the greatest perpendicular on MN, and consequently it is the greatest aberration in declination; for as MN is the projection of BY, which is perpendicular to the circle of declination RS, there can be no aberration in MN; also st is the aberration in declination at any point s. Now when the apparent place of the star is at a, the star is (513) then in conjunction ;and as the motion of the sun is (510) equal to the motion of the star in the circle $apcq$, whilst the star moves from s to Q in the ellipse, its motion in the circle would be XF, which therefore represents the sun's motion in the same time, or the motion from the time when the star is at s to the time when the aberration in declination is the greatest. Also (514), the arc Fa shows the elongation of the star from the sun when the star appears at Q, and Xa the elongation when at s.

517. When the star is at s, st is the aberration in declination; and as the position of st to sv is constant, st varies as sv; but sv is the projection of Xz, and therefore in a given ratio to it; hence, st varies as Xz the sine of XY, or cosine of XF; that is, the aberration in declination at any time is as the cosine of the sun's distance from the point where it was when the aberration in declination was the greatest.

To find QH, we have, by the property of the ellipse, $QH \times SM = Sd \times Sc$; hence, $\frac{QH}{Sc} = \frac{Sd}{SM} = \frac{Sd \times BS}{SM \times BS} =$ (because $\left.\frac{CM}{CB} = \frac{Sd}{Sq} = \frac{Sd}{SB}\right)$ $\frac{CM \times BS}{SM \times CB} = \frac{CM}{SM}$ divided by $\frac{CB}{BS} = \frac{\text{sin. } MSa}{\text{sin. } BSa}$; consequently $QH : cS :: \text{sin. } MSa : \text{sin. } BSa :: \text{sin. } PSR : \text{cos. } FSa$; hence, $QH = \frac{20'' \times \text{sin. } PSR}{\text{cos. } FSa}$, the greatest aberration in declination.

518. Let P be the pole of the equator QEW, O the pole of the ecliptic CEV, S the place of the star, $PSAM$ a circle of declination, OSL a circle of latitude; then L has the same longitude as the star, and therefore (513) it marks the place of the sun when the aberration in latitude is nothing. Draw the circle STR perpendicular to PSA, and T will be the place of the sun when the aberration in declination is the greatest; for by *Conics*, $WN : WY :: Sb : Sp$;: (511) sin. star's lat. : rad. also, $WN : WY ::$ tan. NSW, or PSR, : tan.

YSW, or cot FSa; hence, sin star's lat : rad :: tan PSR : cot FSa. But sin SL (the star's lat.) : rad :: cot TSL, or tan LSM, : cot TL; hence, the three first terms in these two last proportions being respectively equal, the arc aF (Fig. 112) $= TL$ (Fig. 113); and as aF represents the motion of the sun from the time when the aberration in declination is the greatest to conjunction, and L represents the place of the sun at conjunction, T must be the place at the greatest aberration. Hence, by the last Article, the greatest aberration in declination $= \dfrac{20'' \times \sin MSL}{\cos LT}$. But in the triangle STL, cos TSL, or sin MSL, $= \sin LTS \times \cos LT$; hence, the greatest aberration in declination becomes $20'' \times \sin LTS$. Also, in the triangle ETR, sin ETR, or sin LTS, $= \dfrac{\sin ER \times \sin ERT}{\sin ET} =$ (because the measure of ERT is AS, and AE is the complement of ER) $\dfrac{\cos AE \times \sin SA}{\sin ET}$. Hence we get the greatest aberration in declination $= 20'' \times$ cos right ascension $\times$ sin dec. divided by the sin of the sun's longitude at the time when the aberration is greatest subtractive. Therefore, as the aberration at any other time is (517) as the cosine of the sun's distance from that place where it was when the aberration was the greatest, if L be the sun's longitude at the time of the greatest aberration in declination subtractive, S its longitude at any other time, A the star's right ascension, D its declination, O the obliquity of the ecliptic, the aberration at that time $= \dfrac{-20'' \times \cos A \times \sin D \times \cos \overline{L-S}}{\sin L} =$ $\dfrac{-20'' \times \cos A \times \sin D \times \overline{\cos L \times \cos S + \sin L \times \sin S}}{\sin L} =$ (because $\dfrac{\cos L}{\sin L} = \cot L$) $-20'' \times \cos A \times \sin D \times \cot L \times \cos S - 20'' \times \cos A \times \sin D \times \sin S$. But (Trig. Prop. XLV.) cot ET, or cot $L = \dfrac{\cot ERT \times \sin E}{\sin ER} + \cos E \times \cot ER =$ (because $\cot ER = -\tan A$) $\dfrac{\cot D \times \sin O}{\cos A} - \cos O \times \tan A$; hence, the aberration in declination becomes $-20'' \times \sin D \times \cos S \times \cot D \times \sin O + 20'' \times \cos A \times \sin D \times \cos S \times \cos O \times \tan A - 20'' \times \cos A \times \sin D \times \sin S =$ (because $\sin D \times \cot D = \cos D$, and $\cos A \times \tan A = \sin A$) $-20'' \times \sin O \times \cos D \times \cos S + 20'' \times \cos O \times \sin A \times \sin D \times \cos S - 20'' \times \cos A \times \sin D \times \sin S = -20'' \times \sin O \times \cos D \times \cos S - 20'' \times \sin D \times \overline{\cos A \times \sin S - \cos O \times \sin A \times \cos S}$. For south declination we must change the signs. But by Trigonometry, $\cos D \times \cos S = \frac{1}{2} \cos \overline{S+D} + \frac{1}{2} \cos \overline{S-D}$, and $\cos A \times \sin$

$S = \frac{1}{2}$ sin. $\overline{A+S} - \frac{1}{2}$ sin. $\overline{A-S}$, also sin. $A \times$ cos. $S = \frac{1}{2}$ sin. $\overline{A+S} + \frac{1}{2}$ sin. $\overline{A-S}$; hence, the aberration in *Declination* =

$$\left.\begin{array}{l} +10'' \times \overline{1+\text{cos. } O} \times \text{sin. } \overline{A-S} \times \text{sin.} D \\ -10'' \times \overline{1-\text{cos. } O} \times \text{sin. } \overline{A+S} \times \text{sin.} D \\ -10'' \times \text{sin. } O \times \text{cos. } \overline{S-D} \\ -10'' \times \text{sin. } O \times \text{cos. } \overline{S+D}. \end{array}\right\} = \left\{\begin{array}{l} +19'',17 \times \text{sin. } \overline{A-S} \times \text{sin. } D \\ -\ 0,83 \times \text{sin. } \overline{A+S} \times \text{sin. } D \\ -\ 3,98 \times \text{cos. } \overline{S-D} \\ -\ 3,98 \times \text{cos. } \overline{S+D}. \end{array}\right.$$

The two last terms must have their signs changed, when the declination is south.

519. To find the sun's place when the aberration is the greatest, we have in the triangle LST, sin. SL : rad. :: cot. TSL : cot. TL; therefore knowing the longitude of the star, or of the point L, the longitude of T the place of the sun is known. Hence, we find the sun's longitude when the aberration is greatest subtractive.

520. To find the aberration in *Right Ascension*. Let S be the true place of the star, $abcd$ the ellipse of aberration, $apcq$ the circumscribing circle, P the pole of the ecliptic and R that of the equator, and let MSN be a conjugate diameter to ASB; draw FNC, DAV perpendicular to ca, join VS, draw CSK which must be perpendicular to VS, and draw MG perpendicular to AB, also, from any point Q draw QsW perpendicular to ca, and QT, sv perpendicular to SV, SA respectively, and sr an ordinate to the diameter AB. Now it is manifest, that A is the apparent place of the star when the aberration in right ascension is nothing, and M when it is greatest, because a tangent at M is parallel to AB. By the property of the ellipse, $MG \times AS = dS \times cS$, therefore $AS : cS$ or $SV :: dS : MG$; hence, $\frac{AD}{SV} : \frac{AD}{AS} :: dS : MG$, but $AD : dS ::$ $VD : Sq$, therefore $\frac{VD}{SV} : \frac{AD}{AS} :: Sq : MG$, that is, the sine of Va : the sine of ASa, or cos. PSR, :: $20'' : MG = \frac{20'' \times \text{cos. } PSR}{\text{sin. } Va}$, the greatest aberration in right ascension. If the star be at any other point s, then sv is the aberration in right ascension; but sv is in a given ratio to sr, and sr is in a given ratio to QT, because QT is projected into sr; hence, sv varies as QT the sine of QV, or cosine of KQ the distance of the sun from that point where it was when the aberration was greatest. Now tan. ASD, or cot. PSR : tan. VSa :: ($AD : VD$ ::) sin. star's lat. : rad. (511); but tan. ML : tang. MSL :: sin. star's lat. : rad. hence, as $PSR = MSL$, the tang. $VSa \times$ tan. LM is constant, therefore LM is the complement of VSa; hence, $LM = Ka$ the elongation of the sun from the star when the aberration is greatest; therefore M is

the place of the sun at that time, the longitude of which put $=L$ at the time when the aberration is greatest subtractive. Hence, the greatest aberration in right ascension $=\frac{20''\times\cos MSL}{\cos LM}$. This is the aberration at the star, and therefore reduced to the equator it (13) becomes $\frac{20''\times\cos MSL}{\cos ML\times\cos SA}$. But $\frac{\cos MSL}{\cos ML}=\sin M=\frac{\sin AE}{\sin MF}=\frac{\sin A}{\sin L}$, therefore the greatest aberration subtractive becomes $\frac{-20''\times\sin A}{\cos D\times\sin L}$, hence, the aberration in right ascension at any other time $=\frac{-20''\times\sin A\times\cos\overline{L-S}}{\cos D\times\sin L}=\frac{-20''\times\sin A}{\cos D\times\sin L}\times\overline{\cos L\times\cos S+\sin L\times\sin S}=$
$\frac{-20''\times\sin A\times\cot L\times\cos S-20''\times\sin A\times\sin S}{\cos D}=$ (because $\cot L=\cos O\times\cot A$) $\frac{-20''\times\sin A\times\cos S\times\cos O\times\cot A-20''\times\sin A\times\sin S}{\cos D}$
$=\frac{-20''\times\cos O\times\cos S\times\cos A-20''\times\sin A\times\sin S}{\cos D}$. Now if we augment A by 90°, or 3 signs, the numerator of this fraction becomes the same as the coefficient of $\sin D$ in the aberration of declination, because the $\sin A=\cos\overline{A+3s}$, and $\cos A=\sin\overline{A+3s}$. But to reduce this further, we have $\cos A\times\cos S=\frac{1}{2}\cos\overline{A+B}+\frac{1}{2}\cos\overline{A-B}$, and $\sin A\times\sin S=\frac{1}{2}\cos\overline{A-B}-\frac{1}{2}\cos\overline{A+B}$, hence, the aberration in *Right Ascension* $=-\frac{10''\times\overline{1+\cos O}\times\cos\overline{A-S}-10''\times\overline{1-\cos O}\times\cos\overline{A+S}}{\cos D}=-$
$\overline{19'',17\times\cos\overline{A-S}-0'',83\times\cos.\overline{A+S}}\times\sec D$.

521. As M (520) is the place of the sun when the aberration in right ascension is the greatest, we have, $\cos AEM$ $\tan. AE$ the star's right ascension rad $\tan EM$ the sun's longitude. Hence we can find the sun's longitude when the aberration is greatest subtractive.

522. From these expressions for the aberration in right ascension and declination, M. de LAMBRE has computed the following TABLES, by which the aberration of a star at any time may be very readily found.

TABLE I. Arg. $A-S$.

S	O. VI	I. VII.	II. VIII	S
D	− +	− +	− +	D
0	19″,17	16″,60	9″,59	30
1	19,17	16,43	9,30	29
2	19,16	16,26	9,00	28
3	19,15	16,08	8,70	27
4	19,13	15,89	8,40	26
5	19,10	15,71	8,10	25
6	19,07	15,51	7,80	24
7	19,03	15,31	7,49	23
8	18,99	15,11	7,19	22
9	18,94	14,90	6,87	21
10	18,88	14,69	6,56	20
11	18,82	14,47	6,24	19
12	18,75	14,25	5,93	18
13	18,68	14,02	5,61	17
14	18,60	13,79	5,28	16
15	18,52	13,56	4,96	15
16	18,43	13,32	4,64	14
17	18,33	13,08	4,31	13
18	18,23	12,83	3,99	12
19	18,13	12,58	3,66	11
20	18,02	12,32	3,33	10
21	17,90	12,07	3,00	9
22	17,78	11,80	2,67	8
23	17,65	11,54	2,34	7
24	17,52	11,27	2,00	6
25	17,38	11,00	1,67	5
26	17,23	10,72	1,34	4
27	17,08	10,44	1,00	3
28	16,93	10,16	0,67	2
29	16,77	9,87	0,33	1
30	16,60	9,59	0,00	0
D	− +	− +	− +	D
S	XI. V.	X. IV.	IX. III	S

TABLE II. Arg. $A+S$

S	O. VI.	I. VII	II. VIII	S
D	+ −	+ −	+ −	D
0	0″,83	0″,72	0″,41	30
1	0,83	0,71	0,40	29
2	0,82	0,70	0,39	28
3	0,82	0,69	0,38	27
4	0,82	0,68	0,37	26
5	0,82	0,67	0,35	25
6	0,82	0,67	0,33	24
7	0,82	0,66	0,32	23
8	0,82	0,65	0,30	22
9	0,82	0,64	0,29	21
10	0,82	0,63	0,28	20
11	0,82	0,62	0,27	19
12	0,82	0,61	0,25	18
13	0,81	0,61	0,24	17
14	0,81	0,60	0,23	16
15	0,80	0,58	0,22	15
16	0,80	0,57	0,20	14
17	0,80	0,56	0,19	13
18	0,79	0,55	0,17	12
19	0,78	0,54	0,15	11
20	0,78	0,53	0,14	10
21	0,77	0,52	0,12	9
22	0,76	0,51	0,11	8
23	0,76	0,50	0,10	7
24	0,75	0,49	0,09	6
25	0,75	0,47	0,07	5
26	0,75	0,46	0,06	4
27	0,74	0,45	0,05	3
28	0,73	0,44	0,03	2
29	0,72	0,43	0,02	1
30	0,72	0,41	0,00	0
D	+ −	+ −	+ −	D
S	XI. V.	X. IV	IX. III	S

TAB. III. Arg. $S+D$ & $S-D$

S	O. VI	I. VII.	II. VIII	S
D	− +	− +	− +	D
0	3″,98	3″,45	1″,99	30
1	3,98	3,42	1,93	29
2	3,98	3,38	1,87	28
3	3,98	3,34	1,81	27
4	3,97	3,30	1,75	26
5	3,97	3,26	1,68	25
6	3,96	3,22	1,62	24
7	3,95	3,18	1,56	23
8	3,94	3,14	1,49	22
9	3,93	3,10	1,43	21
10	3,92	3,05	1,36	20
11	3,91	3,01	1,30	19
12	3,90	2,97	1,23	18
13	3,89	2,92	1,17	17
14	3,87	2,87	1,10	16
15	3,85	2,82	1,03	15
16	3,83	2,77	0,97	14
17	3,81	2,72	0,90	13
18	3,79	2,67	0,83	12
19	3,77	2,62	0,76	11
20	3,74	2,56	0,69	10
21	3,72	2,51	0,63	9
22	3,70	2,46	0,56	8
23	3,67	2,40	0,49	7
24	3,64	2,34	0,42	6
25	3,61	2,28	0,35	5
26	3,58	2,23	0,28	4
27	3,55	2,17	0,21	3
28	3,52	2,11	0,14	2
29	3,49	2,05	0,07	1
30	3,45	1,99	0,00	0
D	− +	− +	− +	D
S	XI. V.	X. IV	IX. III	S

USE OF THE TABLES

A = the right ascension } of the star
D = the declination }
S = the longitude of the sun

Enter Table I with the argument $A-S$, and Table II with $A+S$, and the sum of the two corresponding numbers multiplied by the secant of D will be the aberration in *Right Ascension*

Enter Table I with the argument $A-S+3$ signs, and Table II with $A+S+3$ signs, and the sum of the two corresponding numbers multiplied by the sine of D will be the first part of the aberration in declination

Enter Table III with the arguments $S+D$ and $S-D$, and you will have two other parts of the aberration in declination, and the sum of these three parts will give the whole aberration in *Declination*

If the declination of the star be south, add six signs to $S+D$ and $S-D$

Ex To find the aberration of *α Aquilæ*, on May 10, 1795, at 12 o'clock in the evening

$A=$	9^s	25°	$12'$		
$S=$	1	20	12		
$A-S=$	8	5	0	Table I - - - - - -	$+8'',1$
$A+S=$	11	15	24	Table II - - - -	$+0,8$
					$+8,9$
$D=8^\circ\ 20'$ secant				- - - - - - -	1,011
Aberration in *Right Ascension*				- - - -	$+8,998$ Product

$A-S+3$ signs	$11^s.\ 5^\circ.\ 0'$	Table I.	$-17'',38$
$A+S+3$ signs	$2.\ 15.\ 24$	Table II.	$+\ 0,21$
			$-17,17$
$D=0^s.\ 8^\circ.\ 20'$ sine			$0,145$
$S=1.\ 20.\ 12$			
			$-\ 2,49$ Product.
$S+D=1^s.\ 28^\circ.\ 32'$		Table III.	$-\ 2,08$
$S-D=1.\ 11.\ 52$		Table III.	$-\ 2,97$
Aberration in *Declination*			$-\ 7,54$

If the star's declination had been *south*, then

$S+D+6$ signs $=7^s.\ 28^\circ.\ 32'$	Table III.		$+\ 2'',08$
$S-D+6$ signs $=7.\ 11.\ 52$	Table III.		$+\ 2,97$
First Part			$-\ 2,49$
Aberration in *Declination*			$+\ 2,56$

The aberration of a star applied to its apparent place gives the true place.

523. Or the aberration of a star may be thus found:

For the Aberration in Longitude.

Cosin. lat. : rad. :: $20''$: M, or maximum.

Aberration is 0 tending to excess when the sun's longitude is 3^s greater than that of the star; or the argument of aberration is = the sun's long. − the star's long. − 3^s.

For the Aberration in Latitude.

Rad. : sin. lat. :: $20''$: M, or maximum.

Aberration is 0 tending to excess when the sun's longitude is opposite that of the star; or the argument is always the sun's long. − the star's long. $\pm 6^s$.

For the Aberration in Right Ascension

Sin lat rad cot P, or $\angle$ position, , tan Z
Cos declin $\times$ sin Z cos $P \times$ rad $20''$ M.

$$\begin{array}{l} \text{Star in } \textit{first} \text{ or } \textit{last} \text{ quad of long} \\ \\ \text{Star in } \textit{second} \text{ or } \textit{third} \text{ quad of long} \end{array} \left\{\begin{array}{l} \text{with N lat} \\ \text{with S lat} \\ \text{with N lat,} \\ \text{with S lat} \end{array}\right\} \text{star's long} \left\{\begin{array}{l} +Z-6^s \\ +3^s-Z \\ +3^s-Z \\ +Z-3^s \end{array}\right\} = X$$

Take $X \sim S$ so as to be *less* than 6^s Then the aberration in right ascension $=$ $-M \times \cos\ (X \sim S)$, and if $X \sim S$ be *less* than 3^s, the aberration is $-$, if greater than 3^s it is $+$, where $S =$ sun's longitude

For the Aberration in North polar distance

Sin star's lat rad tan P tan Z
Sin Z sin P $20''$ M

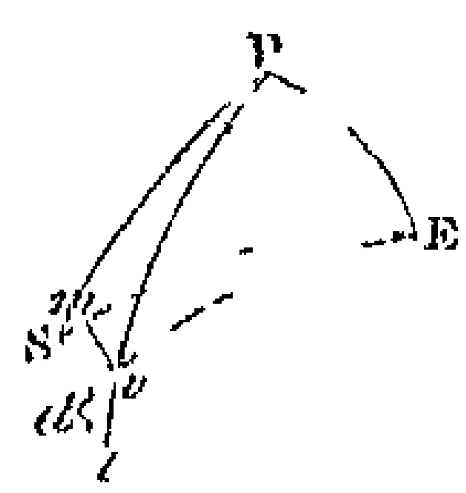
$$\begin{array}{l} \text{Star in } \textit{first} \text{ or } \textit{last} \text{ quad of long} \\ \\ \text{Star in } \textit{second} \text{ or } \textit{third} \text{ quad of long.} \end{array} \left\{\begin{array}{l} \text{with N lat} \\ \text{with S lat} \\ \text{with N lat} \\ \text{with S lat.} \end{array}\right\} \text{star's long} \left\{\begin{array}{l} -Z-3^s \\ +Z+3 \\ +Z-3 \\ +3^s-Z \end{array}\right\} = X$$

Let $S =$ sun's longitude, and take $X \sim S$ so as to be *less* than 6^s. Then the aberration in N P D $= -M \times \cos\ (X \sim S)$, and if $X \sim S$ be *less* than 3 signs, the aberration is $-$, if *greater* than 3 signs, it is $+$

The rule Dr MASKELYNE thus investigated Let E be the pole of the equator, P that of the ecliptic, S the star, t the place by aberration, Sv the aberration parallel to the ecliptic, vt that in north polar distance, draw vm perpendicular to SE, and td to Evd Then 1 (rad) cos PSE Sv $vm = Sv \times \cos$ PSE, 1 sin PSE Sv $Sm = Sv \times \sin$ PSE, 1 cos PSE tv $vd = tv \times \cos$ PSE, 1 sin PSE tv $td = tv \times \sin$ PSE, hence, the aberration parallel to the equator $= vm + td =$ (calling S the angle PSE) $Sv \times \cos\ S + tv \times \sin\ S$, and this divided by cos *Dec* gives the aberration in AR, also, the aberration in N P D $= Sm - vd = Sv \times \sin\ S - tv \times \cos\ S$ But by Art 513, $Sv = 20'' \times \cos$ (☉long $-$ ✱lon), and $tv = 20'' \times \sin$ lat $\times \sin$ (☉lon $-$ ✱lon) Now let sin S sin lat $\times \cos\ S$: sin. Z . cos tan. Z 1, then by substitution, the aberration in N. P. D. $= 20'' \times \dfrac{\sin\ S}{\sin\ Z} \times$ (sin. Z $\times$ cos (☉l.

$-\ast\text{l.}) - \cos. Z \times \sin. (\odot\text{l.} - \ast\text{l.}) = 20'' \times \frac{\sin. S}{\sin. Z} \times \sin. (\odot\text{l.} - \ast\text{l.} - Z)$.

But when $(\odot\text{l.} - \ast\text{l.} - Z) = 90°$, the aberration $=$ max. (M); hence, sin. Z : sin. S :: $20''$: M, and in any other case, the aberration $= M \times \sin. (\odot\text{l.} - \ast\text{l.} - Z) = M \times \cos. (\odot\text{l.} - \ast\text{l.} - Z + 3\text{ sig.})$. Also, let sin. $\ast$ lat. : 1 :: cot. S : tan. Z; then by substitution, the aberration parallel to the equator $= 20'' \times \frac{\cos. S}{\sin. Z} \times (\sin. Z \times \cos. (\odot\text{l.} - \ast\text{l.}) + \cos. Z \times \sin. (\odot\text{l.} - \ast\text{l.})) = 20'' \times$



$\frac{\cos. S}{\sin. Z} \times \sin. (\odot\text{l.} - \ast\text{l.} + Z)$; hence, the aberration in $AR = \frac{20''}{\cos. \text{dec.}} \times$

$\frac{\cos. S}{\sin. Z} \times \sin. (\odot\text{l.} - \ast\text{l.} + Z)$. But when $(\odot\text{l.} - \ast\text{l.} + Z) = 90°$, the aberration $=$max. (M); hence, cos. dec. $\times$ sin. Z : cos. S :: $20''$: M, and in any other case, the aberration in $AR = M \times \sin. (\odot\text{l.} - \ast\text{l.} + Z) = M \times \cos. (\odot\text{l.} - \ast\text{l.} + Z + 3\text{ sig.})$. Putting the star therefore into all the quadrants, and varying the signs of the quantities accordingly, we get the different cases specified above, as given by Dr. MASKELYNE.

524. Dr. BRADLEY has shown the agreement of his theory with observation, which we shall here put down for the satisfaction of the reader.

γ Draconis 1727	D	Aber. in dec. by Observ.	Aber. in dec. by Theory	*γ Draconis* 1728	D	Aber. in dec. by Observ.	Aber. in dec. by Theory
Oct	20	4,5	4',5	March	24	37'	38'
Nov	17	11,5	12	April	6	36	36,5
Dec	6	17,5	18,5	May	6	28,5	29,5
——	28	25	26	June	5	18,5	20
1728				——	15	17,5	17
Jan	21	31	34	July	3	11,5	11,5
Feb	10	38	37	August	2	1	1
March	7	39	39	Sep	6	0	0

η Ursæ maj. 1727	D	Aber. in dec. by Observ.	Aber. in dec. by Theory	*η Ursæ maj.* 1728	D	Aber. in dec. by Observ.	Aber. in dec. by Theory
Sep	14	29',5	28',5	April	16	18',5	18'',
——	21	21,5	25,5	May	5	24,5	23,5
Oct	16	19,5	19,5	June	5	32	31,5
Nov	11	11,5	10,5	——	25	35	34,5
Dec	11	4	3	July	17	36	36
1728				August	2	35	35,5
Feb	17	2	3	Sep	20	26,5	26,5
March	21	11,5	10,5				

Dr BRADLEY further observes, that in above 70 observations made in a year on *γ Draconis*, there was but one (and that is noted very dubious on account of clouds) which differed more than 2" from the theory, and that did not differ 3". And in about 50 observations made in a year on *η Ursæ majoris*, he did not find a difference of 2', except in one marked doubtful on account of the undulation of the air, &c. and that did not differ 3' This agreement between the theory and observation leaves no room to doubt but that the cause is rightly assigned And if this be the case, the annual parallax of the fixed stars must be extremely small "I believe," says the Dr "that I may venture to say that in either of the above mentioned stars, it does not amount to 2" I believe if it were 1" I should have perceived it in the great number of observations that I made, especially on γ Draconis, which agreeing with the theory (without allowing any thing for parallax) nearly as well in conjunction with, as in opposition to this star, it seems very probable that the parallax is not so great as one

single second; and consequently that it is above 400000 times further from us than the sun." The observations of Mr. FLAMSTEAD of the different distances of the pole star from the pole at different times of the year, and which was looked upon as a proof of its annual parallax, was undoubtedly owing to this cause. For he concluded that the star was 35″, 40″ or 45″ nearer the pole in December than in May or July; and according to this hypothesis, it ought to appear 40″ nearer in December than in June. This agreement is greater than could have been expected from observations made with his instrument.

525. Hence Dr. BRADLEY deduced the following conclusions. 1. That the light of all the fixed stars arrives at the earth with equal velocities; for the major axis of the ellipse is the same in all the stars, that is 40″ according to his last determination. 2. That unless their distances from us are all equal, which is very improbable, their lights are propagated *uniformly* to all distances from them. 3. That light moves from the sun to the earth in 8′. 7″,5, and its velocity is to the velocity of the earth in its orbit as 10314 : 1. 4. That the time thus determined can scarce err from the truth by above 5″ or 10″ at most, which is such a degree of exactness as can never be expected from the eclipses of *Jupiter's* satellites. 5. That as this velocity of star light comes out about a mean of the several velocities found from the eclipses of Jupiter's satellites, we may reasonably conclude that the velocities of these reflected lights are equal to the velocity of direct light. 6. And as it is highly probable that the velocity of the sun's emitted light is equal to that of star light, it follows that its velocity is not altered by reflection into the same medium.

On the Aberration of Light in the Planets.

526. Let S be the sun, T the earth, P the corresponding place of the planet, and let us suppose Tt to be the direction in which the earth is moving, parallel to which draw PQ, and whilst light moves from P to T let PQ be equal to the space through which the earth has moved, and (504) Q is the apparent place of the planet. Now let Pp be the motion of the planet in the same time, then Q being the apparent and p the corresponding true place, the angle QTp is the aberration arising from the progressive motion of light and the motion of the planet. As PQ, Pp represent the motions of the earth and planet, Qp represents their relative motion; hence, the motion of the planet about the earth in the time in which light comes from the planet to the earth, is the aberration. Let $ST=1$, $PT=d$, $m=$ the angle described by the planet about the earth, or its geocentric motion, either in latitude, longitude, right ascension, or declination, in 24 hours; then $1 : d :: 8'.\ 7'',5 : 8'.\ 7'',5\ d$ the time light is moving from P to T; consequently $24h. : 8'.\ 7'',5\ d :: m$: the aberration $= \frac{8'.\ 7'',5\ dm}{24h.} =$

$0{,}00564\ dm$ Thus we find the aberration of a planet either in latitude, longitude, right ascension or declination The geocentric motion may be taken from the *Nautical Almanac*, and the distance need not be calculated to any very great degree of accuracy We may also further observe, that when $m=0$, or the planet is stationary, the aberration becomes equal to nothing

Ex 1 On May 1, 1791, at noon, what is the aberration in longitude of *Mars*?

Here $SP=1{,}5237$ the mean distance, the longitude of the sun is $1^s\ 11°$, and the geocentric longitude of Mars is $0^s\ 29°\ 19'$, hence, the angle $PTS=11°\ 11'$, and consequently $PT=2{,}489=d$, also, $m=44'\ 50''=2690''$ from the *Nautical Almanac*, hence $0{,}00564\ dm=37''$ is the aberration in longitude

Ex 2 For the *Moon*, $d=0{,}00253$ the mean distance, $m=13°\ 10'\ 35''=17435''$ the mean diurnal motion, hence, $0{,}00564\ dm=0'',67$ the aberration, which is so small that it may be neglected

527 Dr MASKELYNE observes, that since a planet, as affected by aberration, appears in the place where it should have appeared at the instant of the emission of its light, exclusive of this cause of error, it follows that the most simple as well as the most elegant method of computing the apparent geocentric place of a planet, is to compute its geocentric place by the common rules for that instant which precedes the given time by the interval of time taken up by light to move from the planet to the earth For this purpose the distance of the planet need not be computed very accurately, and then the time may be found by Table XXII at the end of Volume II The sun's longitude must be computed with the epoch of its mean longitude advanced by 20″, because it always appears so much too backward in the ecliptic by aberration, and the Tables have been constructed without making any correction on this account, and consequently they show the epoch of the mean longitude 20″ too little

528 If we suppose the planet and earth to describe circles which lie in the same plane, which will make no sensible difference, then if in Tt produced we take $ET=PQ$, and draw Pa parallel to pT, the angle $EPa=pTQ$ the aberration, draw also Tu parallel to Pp By Art 506 the angle $TPE=\sin E\times 20''$, also, $\sin E : \sin ErT :: Tr : TE :: Tu : Ta$* $=\dfrac{\sin ErT}{\sin E}\times Tu$, hence, $Ea=TE\mp\dfrac{\sin ErT}{\sin E}\times Tu$, consequently $TE : Ea = TE\mp\dfrac{\sin ErT}{\sin. E}$

* This is not accurately true, because ua is not strictly parallel to rE, but sufficiently so for all practical purposes as the angle TPE is very small

$\times Tu$ ∴ angle TPE : angle EPa :: sin. $E \times 20''$ ∴ angle $EPa =$ sin. $E \times 20'' \mp \frac{\text{sin. } ErT \times Tu}{TE} \times 20''$, but TE : Tu :: velocity of the earth : velocity of the planet :: $\sqrt{SP}$: $\sqrt{1}$;* also, sin. E, or PTt, $=$ cos. STP, and sin. ErT, or EPp $=$ cos. SPE, or SPT; hence by substitution, the angle $EPa =$ cos. $STP \times 20'' \mp \frac{\text{cos. } SPT}{\sqrt{SP}} \times 20''$ the aberration in longitude. The first term cos. $STP \times 20''$, or sin. $E \times 20''$, is (506) the aberration for a fixed star, hence the other term, or the aberration which arises from the motion of the body, varies as the cosine of the angle at the body between the sun and earth directly, and as the square root of the distance of the body from the sun inversely. The first part is common to all the planets, the elongation being given. If we take the sum of the aberrations when the planet is in conjunction and opposition, the last part will be destroyed by the opposition of its signs, and as the cos. STP in each case $= 1$, the sum of the two aberrations is always $= 40''$.

529. When p coincides with Q, or when a line joining the earth and planet continues parallel to itself, there is no aberration; this therefore happens when the planet is stationary. In this case (putting $a = SP$), cos. $STP \times 20'' - \frac{\text{cos. } SPT}{\sqrt{a}} \times 20'' = 0$, or $a \times \text{cos. } STP^2 = \text{cos. } SPT^2$, or $a \times \overline{1 - \text{sin. } STP^2} = 1 - \text{sin. } SPT^2$; but $a : 1 :: \text{sin. } STP : \text{sin. } SPT = \frac{\text{sin. } STP}{a}$, which substituted for sin. SPT in the last equation, by reduction gives sin. $STP = \sqrt{\frac{a^3 - a^2}{a^3 - 1^2}}$, the same as in Art. 313.

* This will appear when we treat on the Physical Principles of Astronomy

530 M de la LANDE has calculated the following Table, showing what we must apply to the true place of a planet in longitude to find the apparent place, in which the quantities are to be applied according to their signs All the orbits are supposed to be circular, except that of *Mercury* When the aberration is *negative*, the planet's motion is *direct*, when *positive*, it is *retrograde*

Elong from Sun	*Mars*	*Jupiter*	*Saturn*	*Georgian*	Elong	*Venus*
O conjun 0°	−36″	−29″	−27″	−25″	Conj sup	−43″,5
15	−35	−28	−26	−24	15°	−41
I. XI 0	−32	−26	−24	−22	30	−34
15	−28	−23	−21	−19	45	−19
II X 0	−23	−19	−16	−15	Gr elong	−14
15	−18	−14	−12	−10	45°	− 9
III IX 0	−12	− 9	− 6	− 5	30	0
15	− 7	− 1	− 1	0	15	+ 3
IV VIII 0	− 3	+ 1	+ 4	+ 5	Conj inf	+ 3,5
15	0	+ 5	+ 8	+ 9		
V VII 0	+ 2	+ 9	+11	+13		
15	+ 3	+10	+12	+15		
VI oppos 0	+ 1	+11	+13	+15		

	MERCURY		
	Aphelion	Mean Distance	Perihelion
Conjunc super	−46″	−51″,5	−59″,5
5°	−15	−51	−58
10	−44	−48	−52
15	−41	−43	−41
20	−36	−33	
25	−29		
Greatest Elong	−18	−18	−19
25	− 7		
20	− 1	− 4	
15	+ 2	+ 4	+ 2
10	+ 4	+ 8	+13
5	+ 6	+11	+18
Conjunc. infer	+ 6	+11,5	+19,5

531. M. de la LANDE observes, that in the passage of *Mercury* over the sun in 1782, the aberration retarded the phases by computation, 6′. 34″, as will appear by augmenting its longitude by 18″,8, the aberration at that time, and diminishing that of the sun 20″, which is always its aberration. Compute the phases by supposing each body to be at its true place, and at its apparent place at the same time, and the difference shows how much the aberration affects the time. Moreover, when we calculate the true geocentric place of a planet, we must add 20″ to the place of the sun in the Tables of its motion, the place of the sun being put down as affected by aberration.

532. By Article 526, the aberration $=0,00564\ dm$, if the earth's distance from the sun be unity; if therefore that distance be represented by 10, the aberration $=0,000564\ dm$, from which the following Table was constructed, to be entered with the distance of the planet from the earth, and the angle described by the planet about the earth in 24 hours, in latitude, longitude, right ascension or declination.

If the distance of the body from the earth be greater than 10, as 37 for instance, find the value for 10 and then multiply it by 3, and to it add the value for 7.

A TABLE

To find the Aberration of a Planet *or* Comet, *in Latitude, Longitude, Right Ascension or Declination*

Diurnal Motion	Distance from the Earth, that of the Sun being 10								
	2	3	4	5	6	7	8	9	10
D M	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec	Sec
0 8	0,5	0,8	1,1	1,1	1,6	1,9	2,2	2,4	2,71
0 16	1,1	1,6	2,2	2,7	3,3	3,8	1,3	4,9	5,41
0 21	1,6	2,4	3,3	4,1	4,9	5,7	6,5	7,3	8,12
0 32	2,2	3,3	4,3	5,4	6,5	7,6	8,7	9,7	10,83
0 40	2,7	4,1	5,4	6,8	8,1	9,5	10,8	12,2	13,53
0 48	3,3	4,9	6,5	8,1	9,8	11,4	13,0	14,6	16,24
0 56	3,8	5,7	7,6	9,5	11,4	13,0	15,2	17,1	18,95
1 4	4,3	6,5	8,7	10,8	13,0	15,2	17,3	19,5	21,66
1 12	4,9	7,3	9,8	12,2	14,6	17,1	19,5	21,9	24,36
1 20	5,4	8,1	10,8	13,5	16,2	19,0	21,7	24,4	27,07
1 28	6,0	8,9	11,9	14,9	17,9	20,8	23,8	26,8	29,78
1 36	6,5	9,8	13,0	16,2	19,5	22,7	26,0	29,2	32,48
1 44	7,0	10,6	11,1	17,6	21,1	24,6	28,2	31,7	35,19
1 52	7,6	11,4	15,2	19,0	22,7	26,5	30,3	34,1	37,90
2 0	8,1	12,2	16,2	20,3	24,4	28,4	32,5	36,6	10,61
2 8	8,7	13,0	17,3	21,7	26,0	30,3	34,7	39,0	43,31
2 16	9,2	13,8	18,1	23,0	27,6	32,2	36,8	41,4	46,02
2 24	9,8	14,6	19,5	24,1	29,2	31,1	39,0	43,9	48,73
2 32	10,3	15,4	20,6	25,7	30,9	36,0	41,2	46,3	51,43
2 40	10,8	16,3	21,7	27,1	32,5	37,9	43,3	48,7	54,14
2 48	11,4	17,1	22,8	28,4	34,1	39,8	45,5	51,2	56,85
2 56	11,9	17,9	23,8	29,8	35,7	41,7	47,6	53,6	59,55
3 0	12,2	18,3	24,1	30,5	36,5	42,6	48,7	54,8	60,91

Ex. Suppose the distance of a comet from the earth to be 43, and its apparent motion in 24 hours to be 2°. 15′ in longitude; to find the aberration in longitude.

Enter with the distance 10 and daily motion 2°. 15′, and we get 45″,68, which multiplied by 4 gives 182″,7, and by entering with the distance 3 we get 13″,7; hence the aberration is 196″,4.

To reduce the place of the body computed from the Tables to the apparent place, *add* the aberration, if the latitude, longitude, right ascension or declination of the body *decrease*, but *subtract*, if it *increase*; and the contrary, to reduce the apparent to the computed place.

CHAP. XXIII.

ON THE PROJECTION FOR THE CONSTRUCTION OF SOLAR ECLIPSES.

Art. 533. AS the ecliptic is inclined to the equator and cuts it in two opposite points, the sun keeps continually approaching to one pole and receding from the other by turns, and therefore to a spectator at the sun, the poles must appear and disappear by turns. When the sun is on the north side of the equator, the north pole must appear; and when on the south side, the south pole. When the sun is in the equator, the plane of illumination is perpendicular to the equator, and consequently the poles will lie in the circumference of the circle of illumination; when the sun comes to the tropic, the pole will appear in the middle of its path over the circle of illumination; and when the sun comes to the next equinox, the pole will appear on the other side of the circle of illumination. When the sun gets on the other side of the equator, this pole will disappear, and the other will appear in like manner. Hence, to a spectator at the sun, the apparent motion of the pole P is the same as if the axis Pp of the earth had an annual conical motion $PrQs$, $pnqm$ about an axis GOF perpendicular to the ecliptic EOC, the angle POG being equal to the greatest declination of the sun. As these circles $PrQs$, $pnqm$ are parallel to the ecliptic, their planes will pass through the sun, and therefore to a spectator at the sun the apparent motion of the poles will be in the straight lines PQ, pq; and as P moves as fast in the circle $PrQs$ as the sun does in the ecliptic, if P be the place of the pole at the equinox, and we take the arc Pv equal to the sun's distance from that equinox, and draw vo perpendicular to PQ, o will be the apparent place of the pole at that time. It is manifest that Pv may be set off upon any circle described on PQ. Hence also, the angle which the axis vOw makes with the plane of illumination must be equal to the declination of the sun. As this apparent motion of the pole over the enlightened disc of the earth is caused by the motion of the earth in its orbit, the motion of the pole over the disc will be in a direction contrary to the diurnal motion of the disc; if therefore P be the position of the pole at the vernal equinox, and PrQ be its motion over the disc of the earth to the next equinox, the diurnal motion of the disc will be made in the contrary direction.

534. When the sun, and consequently the spectator who is supposed to be at the sun, is in the equator, the spectator being in the plane of the equator, and, as to sense, in the plane of all the circles parallel to it, they will all appear to be

projected upon the circle of illumination into right lines parallel to each other. But when the sun, and consequently the spectator, is out of the equator, the equator, and all the circles parallel to it, being seen obliquely, will appear to be projected into ellipses upon the plane of illumination, as the eye may be considered at an infinite distance; and as the eye has the same relative situation to all these circles, the ellipses must be all similar. When the sun is on the north side of the equator, that part of the ellipse which is the projection of that part of the circle which lies between the north pole and equator on the enlightened hemisphere will be concave to the pole, but when the sun is on the other side of the equator, that part will be convex. That is, let P be the north pole on the enlightened hemisphere, the sun being on the north side of the equator, and *vxyz*, *ambn*, the ellipses into which the equator and any parallel to it are projected; then *amb* is that part of the ellipse which the place on this parallel describes in the *day*, and the other part *bna* is that which is described in the *night;* and the place is at m at 12 at noon, and at n at 12 at midnight. In this case, the other pole p must be considered as being on the other, or dark side of the earth. But if P be supposed on the dark side, and consequently p on the light side, or if the sun be on the south side of the equator, n will be 12 at noon, and m will be 12 at midnight. For if Pp be the axis, LN the plane upon which the circle ab is to be projected, E the sun on that side next to the north pole; then drawing Eam, Enb, the point a, answering to noon, the sun being on the meridian, is projected at m, and the point b, answering to midnight, is projected at n, but when the sun is on the other side of ab, as at e, a is projected to n' and b to m', therefore n' represents noon and m' midnight. On account of the great distance of the sun compared with the radius of the earth, the lines Ea, Eb, and ea, eb may be considered as parallel, and therefore the circle ab is orthographically projected upon the plane LN into an ellipse, whose minor axis is mn, or $m'n'$.

535. The next thing to be done is to determine the magnitude of the ellipse into which the circle ab is projected, and its position upon the plane of illumination. Let Pp represent the axis of the earth, *asbt* a circle of latitude to any place, $IPNp$ the meridian passing through the sun, and LON the plane upon which the projection is made; then (*533*) the angle LOP is equal to the sun's declination; draw am, bn, vr perpendicular to LO, and (*534*) mn is the minor axis of the ellipse; let vs be that radius of the circle ab which is parallel to the plane of projection, and it will be projected into a line equal to itself, and consequently it will be the major axis; hence, $2vs$, or $2va$, or 2 cos. lat. is the major axis of the ellipse; but mn (the projection of ab upon LN) : ab :: sin. mab, or POL the dec. : radius; that is, the axis major : axis minor :: rad. : sin. declination. And to find the distance Or from the center of projection to the center of the ellipse, we have, rad. $=1$: cos. vOr the dec. :: vO : Or

$=vO\times\cos$ dec. $=\sin$ lat. $\times\cos$ dec. But (541) the radius of the projection is the horizontal parallax of the moon diminished by the horizontal parallax of the sun; the radius therefore thus expressed being multiplied by the quantities whose values are expressed when radius is supposed to be unity, give the value in terms of that radius; hence, if hor. par. ☾ − hor. par. ☉ $=h$, then $h\times\cos$ lat. $=$ the semi-axis major of the ellipse, $h\times\cos$ lat. $\times\sin$ dec. $=$ the semi-axis minor, and $Or=h\times\sin$ lat. $\times\cos$ dec. Thus we have gotten the dimensions and position of the ellipse in terms of the radius of projection. Hence we have the following construction for the apparent ellipse described by any place on the earth's surface to a spectator at the sun.

536. Let $GCFE$ be that half of the earth which is illuminated, EC the plane of the ecliptic, GOF perpendicular to it; take $GQ=GV$ equal to the sun's greatest declination, join QV, and on it describe the semicircle VKQ, and take Vh equal to the sun's distance from the vernal equinox corresponding to the pole at V, and draw hP perpendicular to VQ, and P (533) is the place of the pole, which we will suppose to be on the enlightened disc of the earth. Put $c=\cos.\ Vh$, $n=\sin.\ VG$, $m=$ its cosine, to radius unity; then

$$\begin{aligned} Pc &: cV :: c : 1 \\ cV &: OG :: n : 1 \\ \hline Pc &: OG\,(=h) :: c\times n : 1, \text{ hence, } Pc=h\times c\times n. \end{aligned}$$

Also, $h\times m=Oc$; hence, $h\times m : h\times c\times n ::$ rad. : tan. $POc=\frac{c\times n}{m}=c\times$ tan. $23^\circ\ 28'=0{,}4341208\times c$. Draw POp, and upon OP take $Or=h\times\sin$ lat. $\times\cos$ dec., draw bra perpendicular to OP, and take $ra=rb=h\times\cos$ lat. and $rm=rn=h\times\cos$ lat. $\times\sin$ dec. and describe an ellipse $ambn$, and (535) it will represent the apparent diurnal path of the place to a spectator at the sun, for the given declination of the sun. If z and s be the points where the ellipse touches the circle $GCFE$, the part $zambs$ will (531) be on the illuminated part of the earth, and therefore visible to a spectator at the sun, and the part snz on the dark part, P being the north pole, and the sun's declination north; but if the declination be south, zns will be the part on the illuminated side of the earth, and $sbmaz$ on the dark part. Let the declination be north, and a the west side of the disc; then to find where the given place on the earth's surface, is at any time, we may observe that the place describing the circle which is projected into the ellipse $ambn$ moves uniformly in that circle, from the uniform motion of the earth about its axis; let therefore ayb be a semicircle; then if every ordinate be diminished in the ratio of $yr : mr$, the circle will be projected into the ellipse amb; this semicircle may therefore represent the half of the diurnal

motion of the given place, so far as it is necessary to obtain the corresponding positions of the place in the ellipse. For divide the semicircle ayb into 12 equal parts from a, at 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, representing the positions of the given place from a at six o'clock in the morning to b at six in the evening, and these figures will represent the positions of the given place at the respective hours denoted by the figures, and if the dotted lines be drawn perpendicular to ab, the corresponding points denoted by the same figures will represent the positions of the place in the ellipse. This ellipse may be very accurately described by diminishing each ordinate of the circle perpendicular to ab in the ratio of yr to mr, by taking a proper number of ordinates and then describing a curve through all the points; and if these lines be continued to the other half of the ellipse, the hours, as there marked, will correspond to the positions of the given place. If each division of the semicircle be divided into 10 equal parts, and ordinates be drawn to ab, the ellipse will be divided into every six minutes; and if the scale be large enough, and these divisions on the ellipse be subdivided into six equal parts, the ellipse will be divided into minutes, for there will be no occasion to use the circle for this last subdivision. Thus we can always find the apparent position of any place on the earth's surface to a spectator at the sun.

537. Draw $11w$ and $11d$ perpendicular to ry; then $11w=11d$ is the sine of $15°$ to the radius ra; and by the principles of projection, $yr : mr :: yd : mw$, therefore as yd is the versed sine of $15°$ to the radius yr, mw must be the versed sine of $15°$ to the radius mr; hence, if we take the sine and versed sine of $15°$ to radius unity, and multiply them into ra and rm respectively, they will give the values of $11w$ and mw; and if rm be multiplied into the cosine of $15°$, it gives rw. The same for any other angle.

538. By Art. 535. $Or = h \times$ sin. lat. $\times$ cos. dec. and $ra = h \times$ cos. lat. hence, $Or : ra ::$ sin. lat. $\times$ cos. dec. : cos. lat. consequently $Or = ra \times \frac{\text{sin. lat.} \times \text{cos. dec.}}{\text{cos. lat.}} = ra \times$ tan. lat. $\times$ cos. dec. therefore if ra and cos. dec. be constant, Or varies as tan. lat. Also (535), the radius of projection must vary inversely as the cosine of the latitude.

539. Having determined the situation of the ellipse for any one latitude in respect to the center of projection, as the ellipses for all latitudes are similar if the declination be given, we may make use of the same ellipse for all latitudes, only by altering Or in a proper ratio; for (538) if ra and the declination remain constant, rO varies as the tangent of latitude. Hence, take $rO : rO'$ as the tangent of the latitude for which the projection was made : tangent of any other latitude, and O' will be the center of projection, whose radius is (538) also known, and amn is the ellipse for that latitude.

540. Let e be any position of the given place, and join eO; then the angle under which EO appears at the sun is the sun's horizontal parallax, also, the

angle under which eO appears at the sun must be the parallax in altitude at the point e, for the sun being vertical to O, the arc corresponding to eO is the zenith distance of the sun at the given place, and eO is the sine of that arc from the nature of the projection, but (154) the hor parallax parallax at any altitude rad sine of the zenith distance OE Oe, hence, if OE represent the horizontal parallax, Oe will represent the parallax in altitude at e Also, as e represents the zenith of the given place, eO represents the vertical circle passing through the sun The use of this projection is to construct the phases and times of a solar eclipse, as we shall now proceed to explain

541 Let S be the center of the sun, aw the enlightened hemisphere of the earth, which we must conceive to be perpendicular to SC, draw SD, SV tangents to two opposite points of the earth, and let $ambn$ be the apparent ellipse (536) described by any point m on the earth's surface, let OC be the distance of the moon from the earth, and vd, $a'm'b'n'$ be the projection of VD, $ambn$ upon a plane at the moon perpendicular to SO, to an eye at S, and $a'm'b'n'$ will be the apparent motion of the center of the sun at S to the spectator describing $ambn$ The curve $a'm'b'n'$ may be considered as an ellipse, for the angle DSC being only $8\frac{1}{4}''$, DS, CS may be reckoned as parallel, and therefore the projection of DV upon a plane parallel to it may be considered as an orthographic projection, and consequently the two figures may in all respects be considered as similar Let LM be the orbit of the moon, then if we know at any time the point of the ellipse $ambn$ where the spectator is, we know the corresponding point where the center of the sun is in the ellipse $a'm'b'n'$, if therefore we determine at the same time the point where the moon is in its orbit LM, we shall know the apparent situation of the moon in respect to the sun Hence, if we find two points, one in the ellipse $a'm'b'n'$ where the center of the sun is, and another in LM where the center of the moon is at the same time, and about these centers, with radii equal to the apparent semidiameters of the sun and moon, we describe two circles, they will represent the apparent situations of the two discs If that of the moon fall upon the sun, it shows how much the sun is eclipsed at that instant Now the angle $OVv = COV - OSV$, that is, the radius of projection is equal to the difference of the horizontal parallaxes of the moon and sun The projection Oc' of Ce is the parallax in altitude of the moon from the sun, supposing the moon to be at the same altitude as the sun, for the radius Ov represents the difference of the horizontal parallaxes of the sun and moon, or the horizontal parallax of the moon from the sun, and as the parallax of each varies (154) as the sine of the apparent zenith distance, the difference of the parallaxes must vary as the sine of their common apparent zenith distance, hence, Ov Oe' difference of the horizontal parallaxes difference of the parallaxes at their common apparent altitude, therefore if Ov represent the third term, Oe' will represent the fourth In an eclipse of the sun therefore this will be nearly true, but

not accurately so, except when the sun and moon are at the same altitude. The place of the pole of the earth is here supposed to be fixed during the time of the eclipse, and consequently the earth is supposed to be immoveable for that time; the sun's declination is also supposed to be constant for the same time; but as these circumstances do not take place, the projected path of the spectator will not be accurately an ellipse. M. de la CAILLE observes, that in this projection, all the errors arising from the finite distances of the sun and moon are supposed to be compensated, by making the semidiameter of the projection equal to the difference of their horizontal parallaxes; whereas only a part of the lines should be diminished in that ratio. The sun also not being at an infinite distance, the projection will not be an accurate ellipse. The spheroidical figure of the earth is also here not considered. All these circumstances tend to render the method of determining the phases of an eclipse by this construction subject to a certain degree of inaccuracy; but if the construction be made upon a large scale, it will be sufficiently accurate, when we only want to predict an eclipse. If S be a fixed star, the same construction will give the time of its occultation by the moon. In this case, as the fixed star has no parallax, the radius of projection is equal to the horizontal parallax of the moon. This projection was first given by Mr. FLAMSTEAD.

542. If we make this projection upon a plane at the orbit of *Venus* or *Mercury*, the radius of projection must be taken equal to the difference of the horizontal parallaxes of Venus or Mercury and the sun; and by proceeding as for the moon, we may determine the times of the phases of the transits of Venus and Mercury over the sun's disc; but this is best done by calculation, as will be afterwards explained.

CHAP. XXIV.

ON ECLIPSES OF THE SUN AND MOON, AND OCCULTATIONS OF FIXED STARS BY THE MOON.

Art. 513. AN eclipse of the *Moon* is caused by its entering into the earth's shadow, and consequently it must happen when the moon is in opposition to the sun, or at the full moon. An eclipse of the *Sun* is caused by the interposition of the moon between the earth and sun, and therefore it must happen when the moon is in conjunction with the sun, or at the new moon. If the plane of the moon's orbit coincided with the plane of the ecliptic, there would be an eclipse at every opposition and conjunction; but the plane of the moon's orbit being inclined to the ecliptic, there can be no eclipse at opposition or conjunction, unless at that time the moon be at, or near to the node. For let *Mamb* be the orbit of the moon, *Mcmd* the plane of the earth's orbit, or that plane in which the sun *S*, *S'* appears as seen from the earth, and let these two planes be inclined to each other, so that we may conceive the part *Mam* to lie *above Mcm*, and the part *mbM below mdM*, and *M*, *m* are the nodes. Now if when the moon is at *M* the sun be in conjunction at *S*, the three bodies are then in the same plane, and therefore the moon must interpose between the earth and sun, and cause an eclipse of the sun. But if the moon be at *M'* when the sun comes into conjunction at *S'*, *M'* is now elevated above the line joining *E* and *S'*, and the farther *M'* is from *M* the more elevated will *M'* appear above *S'*, so that *M'* may be so far from *M*, that the moon may not at all interpose between *E* and *S'*, in which case there will be no eclipse of the sun. Whether therefore there will, or will not be an eclipse of the sun at the conjunction, depends upon the distance of the moon from the node at that time. If the moon be at *m* at the time of opposition, then the three bodies being in the same plane, the shadow *EV* of the earth must fall upon the moon, and the moon must suffer an eclipse. But if the moon be at *m'* at the time of opposition, *m'* may be so far below the shadow *Ev* of the earth, that the moon may not pass through it, in which case there will be no eclipse. Whether therefore there will be a lunar eclipse at the time of opposition, depends upon the distance of the moon from the node at that time. If the two planes coincided, there would evidently be a central interposition every conjunction and opposition, and consequently a total eclipse. METON, who lived about 430 years before CHRIST, observed, that after 19 years the new and full moons returned again on the same day of the month. The ancient Astronomers also observed, that at the end of 18 years 10 days, a period of 223 lunations, there was a return of the same

eclipses; and hence they were enabled to foretel when they would happen. This is mentioned by PLINY the naturalist, Lib. II. Ch. 13. and by PTOLEMY, Lib. IV. Ch. 2. This restitution of eclipses depends upon the return of the following elements to the same state.—1. The sun's place. 2. The moon's place. 3. The place of the moon's apogee. 4. The place of the ascending node of the moon. The exact restitution of these can never take place, but it so nearly happens in the above time, as to produce eclipses remarkably corresponding. In this manner Dr. HALLEY predicted and published a return of eclipses from 1700 to 1718, many of them corrected from observations; together with the following elements.—1. The apparent time of the middle. 2. The sun's anomaly. 3. The annual argument. 4. The moon's latitude. He says, that in this period of 223 lunations there are 18 years 10 or 11 days (according as there are five or four leap-years) $7h.\ 43'\frac{1}{4}$; that if we add this time to the middle of any eclipse observed, we shall have the return of a corresponding one, certainly within $1h.\ 30'$; and that by the help of a few equations, we may find the like series of eclipses for several periods.

To calculate an Eclipse of the Moon.

544. The first thing to be done, is to find the time of the *mean* opposition. To get which, from the Tables of Epacts*, amongst the Tables of the moon's motion, take out the epact for the year and month, and subtract the sum from $29d.\ 12h.\ 44'.\ 3''$ one synodic revolution of the moon, or two if necessary, so that the remainder may be less than a revolution, and that remainder gives the time of the mean conjunction. If to this we add $14d.\ 18h.\ 22'.\ 1'',4$ half a revolution, it gives the time of the next mean opposition; or if we subtract, it gives the time of the preceding mean opposition. If it be leap-year, in January and February, subtract a day from the sum of the epacts, before you make the subtraction. When the day of the mean conjunction is 0, it denotes the last day of the preceding month.

* The epact for any year is the age of the moon at the beginning of the year from the last *mean* conjunction, that is, from the time when the mean longitudes of the sun and moon were last equal. The epact for any month is the age which the moon would have had at the beginning of the month, if its age had been nothing at the beginning of the year.

Ex. To find the time of the *mean* new and full moons in February, 1795

Epact 1795 - - -	9^d	11^h	$6'$	$17''$
February - - -	1	11	15	57
	10	22	22	14
	29	12	44	3
Mean new moon - -	18.	14	21	49
	14	18	22	1,4
Mean full moon - -	3	19	59	47,6

545 To determine whether an eclipse may happen at opposition, find the *mean* longitude of the earth at the time of *mean* opposition, and also the longitude of the moon's node, then, according to M CASSINI, if the difference between the *mean* longitudes of the earth and the moon's node be less than 7° 30′, there *must* be an eclipse, if it be greater than 14° 30′, there *cannot* be an eclipse, but between 7° 30′ and 14° 30′ there *may*, or may *not* be an eclipse M de LAMBERT makes these limits 7° 47′ and 13° 21′

Ex To find whether there will be an eclipse at the full moon on February 3, 1795

Sun's mean long at 3^d. 19^h $59'$. $47''$,6 (543)	10^s	13°	$27'$	$20''$,8
Mean long of the earth - - -	4	13	27	20,8
Long of the moon's node - - -	4	8	1	48,5
Difference - - - - - -	0	5	25	32,3

Hence there must be an eclipse

Examine thus all the new and full moons for a month before and a month after the time at which the sun comes to the place of the nodes of the lunar orbit, and you will be sure not to miss any eclipses Or having the eclipses for the last 18 years, if you add to the times of the middle of these eclipses, 18*y* 10*d* or 11*d* 7*h* 43′¼, it will give the times when you may expect the eclipses will return

546. To the time of *mean* opposition, compute * the true longitudes of the sun and moon, and the moon's true latitude; and find, from the Tables of their motions, the horary motions of the sun and moon in longitude, and the difference (d) of their horary motions is the relative horary motion of the moon in respect to the sun, or the motion with which the moon approaches to, or recedes from the sun; find also the moon's horary motion in latitude, and suppose at the time (t) of *mean* opposition, the moon is at the distance (m) from opposition; then $d : m :: 1$ hour : the time (w) between t and the opposition, which added to, or subtracted from the time t, according as the moon is not yet got into opposition, or is beyond it, gives the time of the ecliptic opposition.

547. To find the place of the moon in opposition, let n be the moon's horary motion in longitude; then, 1 hour : $w :: n$: the increase of the moon's longitude in the time w, which applied to the moon's longitude at the time of the mean opposition, gives the true longitude of the moon at the time of the ecliptic opposition. The opposite to that must be the true longitude of the sun. Find also the moon's true latitude at the time of opposition, by saying, 1 hour : $w ::$ the horary motion in latitude : the motion in latitude in the time w, which applied to the moon's latitude at the time of the mean opposition, gives the true latitude at the time of the true opposition †. In like manner you may compute the true time of the ecliptic conjunction, and the places of the sun and moon for that time, when you calculate a solar eclipse.

548. With the sun's horary motion in longitude, and the moon's in longitude and latitude, find the inclination of the *relative* orbit, and the horary motion upon it. To do this, let LM be the horary motion of the moon in longitude, SM that of the sun; draw Ma perpendicular to LM and equal to the moon's horary motion in latitude; take $Sb = Ma$ and parallel to it, and join La, Lb; then La is the moon's *true* orbit, and Lb its *relative* orbit in respect to the sun. Hence, LS (the difference of the horary motions in longitude) : Sb (the moon's horary motion in latitude) :: radius : tan. bLS the inclination of the relative orbit; and cos. bLS : radius :: LS : Lb the horary motion in the relative orbit. By Logarithms the calculations are thus.

* The method of doing this will be explained in the Introduction to the Tables, in the third Volume.

† For greater certainty you may compute again from the Tables the places of the sun and moon, and if they be not exactly in opposition, which probably may not be the case, as the moon's longitude does not increase uniformly, repeat the operation. This accuracy however in eclipses is generally unnecessary, for the best lunar Tables cannot be depended upon to give the moon's longitude nearer than 10″, therefore the probable error from the Tables is vastly greater than that which arises from the motion in longitude not being uniform. Unless therefore very great accuracy be required, this operation is unnecessary.

$$\text{Log } Sb + 10, -\log LS = \log \tan bLS$$
$$\text{Log } LS + 10, -\log \cos bLS = \log Lb$$

M. de la LANDE observes, that if we add 8″ to the difference of the horary motions in longitude it will give the horary motion in the *relative* orbit, for in a right angled triangle, of which the base is the difference of the horary motions in longitude, which is about half a degree, and the angle at the base about 5⅔°, the difference between the base and hypothenuse will always be about 8″.

549. At the time of opposition, find, from the Tables, the moon's horizontal parallax, its semidiameter, and the semidiameter of the sun, the horizontal parallax of which we may here take = 9″.

550. To find the semidiameter of the earth's shadow at the moon, seen from the earth. Let AB be the diameter of the sun, TR the diameter of the earth, O and C their centers, draw AT, BR to meet at I, and join OCI, let FGH be the diameter of the earth's shadow at the distance of the moon, and join OT, CF. Now the angle $FCG = CFA - CIA$, but $CIA = OTA - TOC$, therefore $FCG = CFA - OTA + TOC$, that is, *the angle under which the semidiameter of the earth's shadow at the moon appears, is equal to the sum of the horizontal parallaxes of the sun and moon diminished by the apparent semidiameter of the sun.* In eclipses of the moon, the shadow is found to be a little greater than this Rule gives it, owing to the atmosphere of the earth. This augmentation of the semidiameter is, according to M. CASSINI, 20″, according to M. MONNIER, 30′, and according to M. de la HIRE, 60″. MAYER thinks the correction is about $\frac{1}{60}$ of the semidiameter of the shadow, or that you may add as many seconds as the semidiameter contains minutes. Some Computers always add 50″, but this must be subject to some uncertainty.

551. As the angle CIT $(= OTA - TOC)$ is known, we have sin TIC : cos. TIC :: TC : CI the length of the earth's shadow. If we take the angle ATO = 16′ 3″ the mean semidiameter of the sun, TOC = 9″ the horizontal parallax of the sun, we have CIT = 15′ 54″; hence, sin. 15′. 54″ : cos. 15′. 54″, or 1 : 216,2 :: TC : CI = 216,2 TC.

552. Let PQ represent the section of the earth's shadow at the moon, CN the ecliptic, NL the moon's orbit; draw Cn perpendicular to CN, and Cm perpendicular to NL, and let the moon at m just touch the earth's shadow at r externally, so that Cm may be the sum of the radii of the moon and earth's shadow; then to determine when this happens, we may take the angle at N = 5°. 17′, which is very nearly its value in all eclipses, the inclination of the lunar orbit being at that time always greatest, as will afterwards be shown;* hence,

sine $5°. 17'$: rad. :: sin. Cm : sin. CN; now the greatest value of Cm is about $1°. 3'. 30''$; hence, the corresponding value of $CN = 11°. 34'$; when therefore CN is greater than that quantity, there can be no eclipse. According to M. CASSINI, if the latitude Cn of the moon at the time of the ecliptic conjunction exceed the sum of the semidiameters of the earth's shadow and moon by $18''$, there will be no eclipse; but if it do not exceed that sum by $16''$, there will be an eclipse. If $Cm = Cr - rm$, or the limbs touch internally, the eclipse will be just total; hence, if the distance of the moon's node from the place of the earth be less than the computed value of CN in this case, there must be a total eclipse of some duration. If therefore it was before doubtful, and it now appears that there will be an eclipse, proceed as follows to compute it.

553. Let APB be that half of the earth's shadow where the moon passes through, NL the relative orbit of the moon, one figure representing a partial eclipse, and the other a total one; draw Cmr perpendicular to NL, and let z be the center of the moon at the beginning of the eclipse, m at the middle, x at the end, v at the beginning of total darkness, w at the end; also, let AB be the ecliptic, and Cn perpendicular to it. Now in the right angled triangle Cnm, we know Cn the latitude of the moon at the time of the ecliptic conjunction, and (548) the angle Cnm* the complement of the angle which the relative orbit of the moon makes with the ecliptic; hence, radius : cos Cnm :: Cn : nm, which we call the *Reduction*; and radius : sine Cnm :: Cn : Cm. By Logarithms the calculations are thus.

$$\text{Log. cos. } Cnm + \text{log. } Cn - 10, = \text{log. } nm.$$
$$\text{Log. sin. } Cnm + \text{log. } Cn - 10, = \text{log. } Cm.$$

The horary motion (h) of the moon upon its relative orbit being known, we know the time of describing mn, by saying, h : mn :: 1 *hour* : *the time of describing* mn; the computation of this is most readily performed by logistic† Logarithms. Hence, knowing the time of the ecliptic conjunction at n, we know the time of the middle of the eclipse at m. Next, in the right angled triangle Cmz, we know Cm, and Cz the sum of the semidiameters of the earth's shadow and the moon, to find mz, which is done thus by Logarithms; as $mz = \sqrt{Cz^2 - Cm^2} = \sqrt{\overline{Cz + Cm} \times \overline{Cz - Cm}}$, the *log. of* $mz = \frac{1}{2} \times$ *log.* $\overline{Cz + Cm}$ + *log.* $\overline{Cz - Cm}$. Hence, the horary motion of the moon being known, we know the time of describing zm, which *subtracted* from the time at m gives the time of the beginning,

* If the moon at m have north or south latitude increasing, the angle Cnm is to be set off to the right, otherwise to the left of Cn.

† For the nature and use of these Logarithms, see Table XLIX. at the end of Volume II.

and *added*, gives the time of the end. In the same manner, in the right angled triangle *Cmv*, we know *Cm*, and *Cv* the difference of the semidiameters of the earth's shadow and moon; hence, by Logarithms, the *log. of* $mv = \frac{1}{2} \times log.\ \overline{Cv + Cm} + log.\ \overline{Cv - Cm}$, from whence, as before, we know the time of describing *mv*, which *subtracted* from the time at *m* gives the time of the beginning of total darkness, and *added*, gives the time of the end. The magnitude of the eclipse at the middle is represented by *tr*, which is the greatest distance of the moon within the earth's shadow, and this is measured in terms of the diameter of the moon, conceived to be divided into 12 equal parts, called *Digits*, or *Parts deficient*; to find which, we know *Cm*, the difference between which and *Cr* gives *mr*, which added to *mt*, or if *m fall out* of the shadow take the difference between *mr* and *mt*, and we get *tr*; hence, to find the number of digits eclipsed, say, *mt* : *tr* :: 6 *digits, or* 360′, (it being usual to divide a digit into 60 equal parts, and call them minutes,) : *the digits eclipsed*. If the latitude of the moon be north, we use the *upper* semicircle; if south, we take the *lower*.

554. If the earth had no atmosphere, when the moon was totally eclipsed it would be invisible; but we have shown (201) that by the refraction of the atmosphere, some rays will be brought to fall on the moon's surface, upon which account the moon will be visible at that time, and appear of a dusky red colour. M. MARALDI (*Mem. de l'Acad.* 1723) has observed, that, in general, the earth's umbra at a certain distance is divided by a kind of penumbra, from the refraction of its atmosphere. This will account for the circumstance of the moon being more visible in some total eclipses than in others. It is said that the moon, in the total eclipses in 1601, 1620 and 1642, entirely disappeared.

555. An eclipse of the moon arising from its real deprivation of light, it must appear to begin at the same instant of time to every place on that hemisphere of the earth which is next the moon. Hence it affords a very ready method of finding the difference of longitudes of places upon the earth, as will be afterwards explained. The moon enters the penumbra of the earth before it comes to the umbra, and therefore it gradually loses its light; and the penumbra is so dark just at the umbra, that it is difficult to ascertain the exact time when the moon's limb touches the umbra, or when the eclipse begins. When the moon has entered into the umbra, the shadow upon its disc is tolerably well defined, and you may determine, to a considerable degree of accuracy, the time when any spot enters into the umbra. Hence, the beginning and end of a lunar eclipse are not so proper to determine the longitude from, as the times at which the umbra touches any of the spots.

EXAMPLE I.

A Computation of a Partial Eclipse of the Moon, on February 3, 1793; for the Meridian of the Royal Observatory at Greenwich.

The time of the mean full moon is at 3*d*. 19*h*. 49'. 47",6 by Art. 544.

By Art. 545, it appears that there will be an eclipse.

By computation (546) the mean time of the ecliptic opposition is at 12*h*. 46'. 18", from which subtract the equation of time 14'. 20", and we have 12*h*. 31'. 58" the apparent time at Greenwich.

To this time compute (547) the moon's place in the ecliptic, and it will be found 4s. 15°. 15'. 57", the opposite point to which is 10s. 15°. 15'. 57" the place of the sun. Compute also the moon's latitude *Cn*, and it will be found 37'. 49" N. ascending.

By the Tables, the horary motion of the moon in latitude is 2'. 57"; the horary motion of the sun is 2'. 32", and of the moon 32'. 9" in longitude; hence, the horary motion of the moon from the sun in longitude is 29'. 37"; consequently (548) the horary motion of the moon from the sun on the relative orbit is 29'. 45"; also, the inclination of the relative orbit is 5°. 41'. 27". The reduction *nm* (558) is 3'. 44"; reduce this into time by logistic Logarithms, and the operation is thus;

3'. 44"	- - - - - - - - -	1,2061
29. 45	- - - - - - - - -	0,3047
7. 32	time of describing *nm* -	0,9014

The nearest approach *Cm* of the centers is 37'. 28".

From 12*h*. 31'. 58" subtract 7'. 32" and it leaves 12*h*. 24'. 26", the *Middle* of the eclipse.

By the Tables, the horizontal parallax of the sun is 0'. 9", and of the moon (178) 56'. 30"; also, the apparent semidiameter of the sun is 16'. 18", and of the moon 15'. 24". Hence, hor. par. ☉ + hor. par. ☾ − semidiam. ☉ + 50" = 41'. 19", the semidiameter (550) of the earth's shadow increased by 50" for refraction. Hence, by Article 553,

Semid ☾ + semid ⊖'s shad	56′ 37″ = 3397″		
Nearest app of centers	37 28 = 2248		
Sum - - -	5645 – log	3,751664	
Difference - - - - -	1149 – log	3,060320	
		2)6,811984	
Log of 2546″,8 = 42′ 26″,8 mot of half duration		3,405992	

Reduce this into time by the logistic Logarithms

29′ 45″ - -	0,3047
42 27 - - -	0,1503
1*h* 25 37 half duration -	0,8456

Subtract this from and add it to 12*h* 24′ 26″, and we get 10*h* 58′ 49″ for the *Beginning*, and 13*h* 50′ 3″ for the *End*

From $Cr = 41'\ 13''$ subtract $Cm = 37'\ 28''$, and we get $mr = 3'\ 45''$, hence, (553) $mr + mt = rt = 19'\ 9''$ the parts deficient, consequently 15′ 24″ 19′ 9″ 6*d* or 360′ 7*d* 27′ 36″ the digits eclipsed

By logistic Logarithms the computation is thus,

19′. 9′ log + 1, - -	1,4960
15 24 - - -	0,5906
7ᵈ 27 36 - -	0,9054

Hence, the times of this eclipse are February 3, 1795, the

Beginning at	10ʰ 58′ 49″	apparent time at Greenwich
Middle -	12 24 26	
End -	13 50 3	
Duration -	2 51 12	

Digits eclipsed - 7ᵈ 27 36 on the moon's south limb, as represented in Fig 126 which was constructed for this eclipse.

EXAMPLE II.

A Computation of a Total Eclipse of the Moon, on December 3, 1797; for the Meridian of the Royal Observatory at Greenwich.

By Art 545, it appears that there will be an eclipse at this full moon.

By computation (546) the mean time of the ecliptic opposition is 3*d.* 16*h.* 16′. 46″; to which add 9′. 18″ the equation of time, and you get 3*d.* 16*h.* 26′. 4″ for the apparent time.

To this time compute (547) the moon's place in the ecliptic, and it will be found to be 2ˢ. 12°. 35′. 19″; consequently the sun's place is 8ˢ. 12°. 35′. 19″. Compute also the moon's latitude *Cn*, and it will be found 4′. 55″ S. decreasing.

By the Tables, the horary motion of the moon in latitude is 3′. 15″; the horary motion of the sun is 2′. 32″, and of the moon 35′. 14″ in longitude; hence, the horary motion of the moon from the sun in longitude is 32′. 42″; consequently the horary motion of the moon from the sun on the relative orbit (548) is 32′. 50″; also, the inclination of the relative orbit is 5°. 40′. 34″.

The reduction *mn* (553) is 0′. 29″; reduce this into time by the logistic Logarithms, and the operation is thus.

32′. 50″	- - - - -	0,2618
0. 29	- - - - -	2,0939
0. 53 time of describing *mn*	-	1,8321

The nearest approach *Cm* of the centers is 4′. 54″.

To 16*h.* 26′. 4″ add 53″ and it gives 16*h.* 26′. 57″ for the *Middle* of the eclipse.

By the Tables, the horizontal parallax of the sun is 0′. 9″, and of the moon (173) 59′. 9″; also, the apparent semidiameter of the sun is 16′. 17″, and of the moon 16′. 6″. Hence, hor. par. ☉ + hor. par. ☾ − semid. ☉ + 50″ = 43′. 51″, the semidiameter (550) of the earth's shadow increased by 50″ for refraction. And as *Cr* (= 43′. 51″) is greater than *Cm* + *ms* (= 21′), the eclipse must be total.

Hence by Article 553,

Semid ☾ + semid ⊖'s shad 59′ 57″=3597″
Nearest app. of the centers 4 54 = 294

Sum - - - - -	3891 — log	3,5900612
Difference - - -	3303 — log	3,5189086
		2)7,1089698
Log of 3585″=59′ 45″ mot of half duration -		3,5544849

Reduce this into time by the logistic Logarithms, but because the fourth term, in this case, would come out a greater quantity than that to which the Table extends, we will take the half of 59′ 45″, and then double the conclusion,

32′ 50″ - - - -	0,2618
29 52,5 - - - - -	0,30285
54 35,5 - - - -	0,04105

Hence, 1*h* 49′ 11″ is the half duration, which subtracted from and added to 16*h* 26′ 57″, gives 14*h* 37′ 46″ for the *Beginning*, and 18*h* 16′ 8″ for the *End*

By the same Article, we find the time of half the duration of total darkness thus

Semid ⊖'s shad — semid ☾ 27′ 45″=1665″
Nearest app of the centers 4 54 = 294

Sum - - - - -	1959 — log	3,2920344
Difference - - - -	1371 — log	3,1370375
		2)6,4290719
Log. of 1639″=27′ 19″ mot. of $\frac{1}{2}$ dur. of tot dark.		3,2145359

Reduce this into time by the logistic Logarithms.

52′. 50″	- - - -	0,2618
27. 19	- - - -	0,3417
49. 55 half duration of total darkness		0,0799

Subtract this from and add it to 16*h*. 26′. 57″, and it gives 15*h*. 37′. 2″ for the beginning of total darkness, and 17*h*. 16′. 52″ for the end.

From Cr=43′. 51″ subtract Cm=4′. 54″, and we get mr=38′. 57″, to which add tm=16′. 6″, and we get tr=55′. 3″ the parts deficient; hence, 16′. 6″ : 55′. 3″ :: 6[d], or 360′, : 20[d]. 31′ the digits eclipsed. The operation by logistic Logarithms is thus.

55′. 3″ log. + 1,	- - -	1,0374
16. 6	- - - -	0,5713
20[d]. 31′. 0	- - - -	0,4661

Hence, the times of this eclipse are December 3, 1797, the

Beginning at - - -	14[h]. 37′. 46″	apparent time at Greenwich.
Total darkness begins - -	15. 37 2	
Middle - - - -	16. 26. 57	
Total darkness ends - -	17. 16. 52	
End of the eclipse - -	18. 16. 8	
Duration of total darkness -	1. 39. 50	
Duration of the whole eclipse	3. 38. 22	
Digits eclipsed - - -	20[d]. 31. 0	

If the time corresponding to the difference between the meridian of Greenwich and that of any other place, be applied to the times here found, it will give the times at that place.

556. Instead of *computing* the first eclipse, it may be *constructed* thus. Having a scale of minutes and seconds, with the center C and radius CB=41′. 13″, the semidiameter of the shadow, describe a circle; draw Cn perpendicular to ABC and equal to 37′. 19″ the moon's latitude at the ecliptic conjunction; make the angle CnN=84°. 18′. 33″ the complement of the angle which the relative orbit makes with the ecliptic, and produce Nn to L; with a radius=56′. 37″, the sum of the semidiameters of the earth's shadow and moon, set off Cx, Cv; let fall the perpendicular Cm upon NL; and with the centers x, m, v, and radius

=15′ 21′, the semidiameter of the moon, describe the circles representing the moon. To find the beginning, middle and end, mark the point n, 12h 32′ the time of the ecliptic conjunction; and with a radius equal to the relative horary motion of the moon upon NL, set off that extent from n both ways, and divide each interval into as many equal parts as you conveniently can, and continue these divisions to z and x, and the times corresponding to the points z, m, x show the beginning, middle and end of the eclipse. And if tr be measured upon the scale, it will show the digits eclipsed. This method will give the time sufficiently near, when you only want to predict the eclipse, as you may depend upon the time to a minute, if the radius CB be six or seven inches. You may proceed in the same manner if the eclipse be total.

On an Eclipse of the Sun

557. An eclipse of the sun is caused by the interposition of the moon between the sun and spectator, or by the shadow of the moon falling on the earth at the place of the observer. The different kinds of eclipses will be best explained by a Figure. Let S be the sun, M the moon, AB or $A'B'$ the surface of the earth; draw tangents *pavs*, *qzvr* from the sun to the *same* side of the moon, and *vvz* will be the moon's *umbra*, in which no part of the sun can be seen; if tangents *ptbd*, *qwac* be drawn from the sun to the *opposite* sides of the moon, the space comprehended between the *umbra* and *wac*, *tbd*, is called the *penumbra*, in which part of the sun only is seen. Now it is manifest, that if AB be the surface of the earth, the space *mn* where the umbra falls will suffer a *total* eclipse; the part *am*, *bn* between the boundaries of the umbra and penumbra will suffer a *partial* eclipse; but to all the other parts of the earth there will be no eclipse. Now let $A'B'$ be the surface of the earth, the earth being, at different times, at different distances from the moon; then the space within *rs* will suffer an *annular* eclipse; for if tangents be drawn from any point *o* within *rs* to the moon, they must evidently fall within the sun, therefore the sun would appear all round about the moon in the form of a ring; the parts *cr*, *sd*, will suffer a *partial* eclipse; and the other parts of the earth will suffer no eclipse. In this case there can be no total eclipse any where, as the moon's umbra does not reach the earth. According to M. du SEJOUR, an eclipse can never be annular longer than 12′ 24″, nor total longer than 7′ 58″.

558. The umbra *avz* is a cone, and the penumbra *wcdt* the frustum of a cone whose vertex is V. Hence, if these be both cut through their common axis perpendicular to it, the section of each will be a circle having a common center in the line joining the centers of the sun and moon, and the penumbra includes the umbra.

559. The moon's mean motion about the center of the earth is at the rate of about 33' in an hour; but 33' of the moon's orbit is about 2280 miles, which therefore we may consider as the velocity with which the moon's shadow passes over the earth; but this is the velocity upon the surface of the earth where the shadow falls perpendicularly upon it, it being the velocity perpendicular to *Mv*; in every other place the velocity over the surface will be increased in the proportion of the sine of the angle which *Mv* makes with the surface in the direction of its motion, to radius. But the earth having a rotation about its axis, the relative velocity of the moon's shadow over any given point of the surface will be different from this; if the point be moving in the direction of the shadow, the velocity of the shadow in respect to that point will be diminished, and consequently the time the shadow is passing over it will be increased; but if the point be moving in a direction contrary to that of the shadow, as in the case when the shadow falls on the other side of the pole, the time will be diminished. The length of a solar eclipse is therefore affected by the earth's rotation about its axis.

560. Let *E* be the center of the earth *TV*, *S* the sun, *ES* the ecliptic, to which draw *ET* perpendicular; join *TS*, and draw *Tv* parallel to *ES*; let *Lm* be the distance of the center *m* of the moon *xy* in conjunction from the ecliptic at the time it touches the circle *rs* representing the apparent magnitude of the sun, whose center *S* is in *TS*; and let *Lm* intersect *Tv* in *t*. Then $Lm = Lt - ts + sm$; or if we take the angles under which these lines appear as seen from the earth, *Lt* is measured by the angle *LTt*, or *TLE* the hor. par. of the moon, *ts* is measured by *tTs*, or *TSE* the hor. par. of the sun, and *sm* is the sum of the apparent semidiameters of the sun and moon; hence, the angular distance *Lm* = hor. par. ☾ − hor. par. ☉ + semid. ☉ + semid. ☾. According to M. Cassini, if the latitude of the moon at the time of the ecliptic conjunction exceed this quantity by 28", there can be no eclipse; but if it do not exceed it by more than 20" there must be an eclipse. Or the ecliptic limits may be found thus. Find (548) the time of the mean conjunction, and at that time find the sun's mean longitude, and also the longitude of the moon's node; and if the difference of these be less than 21°, but greater than 15°, there *may* be an eclipse; but if the difference be less than 15°, there *must* be an eclipse, according to M. Cassini. The ecliptic limits may also be found in this manner. Let *NE* be the ecliptic, *NM* the moon's orbit, *Lr* the radius of the earth, and *mr* the radius of the moon's penumbra just passing by the earth and touching it, and draw *Lm* perpendicular to *LN*; then *Lm* when greatest is 1°. 34'. 27", and taking the angle *N* = 5°. 17', we have sin. 5°. 17' : rad. :: sin. 1°. 34'. 27" : sin. *NL* = 17°. 21'. 27", and as the value of *mr* is only 9', we may take this value of *LN* to be the earth's distance from the node at the time of the ecliptic conjunction; if therefore that distance be less than 17°. 21'. 27 at the time of the ecliptic conjunction, there may be an eclipse.

561. An eclipse of the sun, or rather of the earth, without respect to any particular place, may be calculated exactly in the same manner as an eclipse of the moon, that is, the times when the moon's umbra or penumbra first touches and leaves the earth, but to find the times of the beginning, middle and end at any particular place, the apparent place of the moon, as seen from thence, must be determined, and consequently its parallax in latitude and longitude must be computed, which renders the calculation of a solar eclipse extremely long and tedious We shall endeavour to render the whole operation as clear as possible, by precept and example

To calculate an Eclipse of the Sun *for any particular Place*

562 Having determined (560) that there will be an eclipse somewhere upon the earth, compute, by the Astronomical Tables, the true longitudes of the sun and moon, and the moon's true latitude, at the time of mean conjunction (544), find also the horary motions of the sun and moon in longitude and the moon's horary motion in latitude, and compute the time of the ecliptic conjunction of the sun and moon, in the same manner (516) as the time of the ecliptic opposition was computed At the time of the ecliptic conjunction, compute (547) the sun's and moon's longitude, and the moon's latitude, find also the equatorial horizontal parallax of the moon from the Tables of the moon's motion, and reduce it (173) to the horizontal parallax for the given latitude, from which subtract the sun's horizontal parallax, and you get the horizontal parallax of the moon from the sun, reduce also (173) the apparent latitude of the place on the spheroid to the latitude on a sphere

563 To this reduced latitude of the place, and the corresponding horizontal parallax of the moon from the sun, (which we here use instead of the horizontal parallax of the moon, as we want to find what effect the parallax has in altering their apparent relative situations,) at the time of the ecliptic conjunction, compute (164) the moon's parallax in latitude and longitude from the sun, the parallax in latitude applied to the true latitude gives the apparent latitude (L) of the moon from the sun, and the parallax in longitude shows the apparent difference (D) of the longitudes of the sun and moon

564 Let S be the sun, EC the ecliptic, take $SM=D$, draw MN perpendicular to MS, and take it $=L$, then N is the apparent place of the moon, and $SN=\sqrt{D^2+L^2}$ is the apparent distance of the moon from the sun

565 If the moon be to the *east* of the nonagesimal degree, the parallax *increases* the longitude, if to the *west*, it *diminishes* it, hence if the *true* longitudes of the sun and moon be equal, in the former case the apparent place will lie from S towards E, and in the latter towards C. To some time, as an hour,

after the true conjunction if the apparent place be towards C, or if the moon be to the *west* of the nonagesimal degree, or *before* the true conjunction if the apparent place be towards E, or if the moon be to the *east* of the nonagesimal degree, find the sun's and moon's true longitude, and the moon's true latitude, from their horary motions; and to the same time compute the moon's parallax in latitude and longitude from the sun; apply the parallax in latitude to the true latitude, and it gives the apparent latitude (l) of the moon from the sun; take the difference of the sun's and moon's true longitude, and apply the parallax in longitude, and it gives the apparent distance (d) of the moon from the sun in longitude. From S set off $SP=d$, and to EC erect the perpendicular PQ equal to l, and Q is the apparent place of the moon at one hour from the true conjunction; and $SQ=\sqrt{d^2+l^2}$ is the apparent distance of the moon from the sun; draw the straight line NQ, and it will nearly represent the relative apparent path of the moon, considered as a straight line, in general it being very nearly so; its value also represents the relative horary motion of the moon in the apparent orbit, the relative horary motion in longitude being MP.

566. The difference between the moon's apparent distance in longitude from the sun at the time of the true ecliptic conjunction, and at the interval of an hour, gives the apparent horary motion (r) in longitude of the moon from the sun; the difference (D) between the true longitude at the ecliptic conjunction, and the moon's apparent longitude is the apparent distance of the moon from the sun in longitude at the true time of the ecliptic conjunction; hence, $r : D :: 1$ hour : the time from the true to the apparent conjunction, consequently we know the time of the apparent conjunction. To find whether this time is accurate, we may compute (from the horary motions of the sun and moon) their true longitudes, and the moon's parallax in longitude from the sun, and apply it to the true longitude and it gives the apparent longitude, and if this be the same as the sun's longitude, the time of the apparent conjunction is truly found; if they be not the same, find from thence the true time, as before. To the true time of the apparent conjunction, find the moon's true latitude from its horary motion, and compute the parallax in latitude, and you get the apparent latitude at the time of the apparent conjunction. Draw SA perpendicular to CE and equal to this apparent latitude; then the point A will probably not fall in NQ, first let it fall in QN, to which draw SB perpendicular, and NR parallel to PM, meeting PQ in R. Then knowing NR $(=PM)$, and QR $(=QP\backsim MN)$, we have

$$NR : RQ :: \text{rad.} : \text{tan.}\ QNR, \text{ or } ASB$$
$$\text{Sin.}\ QNR : \text{rad.} :: QR : QN$$

The time of describing NQ in the apparent orbit being equal to the time from M to P in longitude, NQ is the horary motion in the apparent orbit,

Rad sin ASB AS AB
Rad cos ASB AS SB

567 At the apparent conjunction the moon appears at A, which time (566) is known, when the moon appears at B, it is at its nearest distance from the sun, and consequently the time is that of the greatest obscuration, (usually called the time of the middle,) provided there is an eclipse, which will always be the case when SB is less than the *sum* of the apparent semidiameters of the sun and moon If therefore it appears that there will be an eclipse, we proceed thus to find its quantity, and the beginning and end As we may consider the motion to be uniform, QN AB the time of describing NQ the time of describing AB, which added to or subtracted from the time at A, (according as the apparent latitude is decreasing or increasing,) gives the time of the greatest obscuration Or instead of taking QN, and the time of describing it, we may take An (569) and the corresponding time, which will be more accurate

568 From the sum of the apparent semidiameters of the sun and moon subtract BS, and the remainder shows how much of the sun is covered by the moon, or the parts deficient, hence, semid ☉ parts deficient 6 digits the digits eclipsed If SB be less than the *difference* of the semidiameters of the sun and moon, and the moon's semidiameter be the *greater*, the eclipse will be *total*, but if it be the *less*, the eclipse will be *annular*, the sun appearing all round the moon, if B and S coincide, the eclipse will be *central*

569 Let A fall out of QN, and to increase the accuracy, near to the apparent conjunction, that is within 10 or 15 minutes, calculate the apparent longitude mS of the moon from the sun, and the apparent latitude mn, draw n parallel to Sm, and in the triangle Am, find the angle Am which is equal to ASB, and compute SB, AB as before But except in cases where very great accuracy is required, this is unnecessary, If NQ were a perfect straight line, the first operation would give the correct values of AB, BS KEPLER, in an eclipse in 1598, found a curvature of more than 3′ in three hours, because the moon was very near the nonagesimal In the eclipse in 1764, M de la LANDE found a curvature of 26″, but he does not say in what time. It is owing to this circumstance, that is, the curvature of NQ, that it is necessary to find another point near to A, in order to determine accurately the values of AB, SB. Having determined the value of SB, and the time of the greatest obscuration, we thus find the beginning and end

570 Produce, if necessary, QN, and take SV, SW equal to the sum of the apparent semidiameters of the sun and moon, at the beginning and end, respectively, then $BV=\sqrt{SV^2-SB^2}$ (B being now supposed in QN), and $BW=\sqrt{SW^2-SB^2}$, then to find the times of describing these spaces, say, as the

hourly motion of the moon in the apparent orbit, or NQ, : BV :: 1 hour : the time of describing VB; and NQ : BW :: 1 hour : the time of describing BW, which times respectively subtracted from and added to the time of the greatest obscuration, give nearly the times of the beginning and end. But if accuracy be required, this method will not do; for it supposes VW to be a straight line, which supposition will cause errors, too considerable in general to be neglected, and will not do where great accuracy is required. It may, however, always serve as a rule to assume the time of the beginning and end. Hence it follows, that the time of the greatest obscuration at B, is not necessarily equidistant from the beginning and end.

571. If the eclipse be total, consider SV, SW equal to the difference of the semidiameters of the sun and moon, and then $BV = BW = \sqrt{SV^2 - SB^2}$, from whence we may find the times of describing BV, BW, as before, which we may consider as equal, and which applied to the time of the greatest obscuration at B, give the time of the beginning and end of the total darkness.

572. To find more accurately the time of the beginning and end of the eclipse, we must proceed thus. At the estimated time of the beginning, find, from the horary motions, and the computed parallaxes, the apparent latitude MN of the moon, and its apparent longitude MS from the sun, and we have $SN = \sqrt{SM^2 + MN^2}$, and if this be equal to the apparent semid. ☾ + semid. ☉ (which sum call S) the estimated time is the time of the beginning; but if SN be not equal to S, assume (as the error directs) another time at a small interval from it, *before*, if SN be less than S, but *after*, if it be *greater*; to that time compute again the moon's apparent latitude mn, and apparent longitude Sm from the sun, and find $Sn = \sqrt{Sm^2 + mn^2}$; and if this be not equal to S, proceed thus; as the difference of Sn and SN : the difference of Sn and SL ($=S$) :: the above assumed interval of time, or time of the motion through Nn, : the time through nL, which added to or subtracted from the time at n, according as Sn is greater or less than SL, gives the time of the beginning. The reason of this operation is, that as Nn, nL are very small, they will (266) be very nearly proportional to the differences of SN, Sn, and Sn, SL. But as the variation of the apparent distance of the sun from the moon is not exactly in proportion to the variation of the differences of the apparent longitudes and latitudes, in cases where the utmost accuracy is required, the time of the beginning thus found (if it appear to be not correct) may be corrected, by assuming it for a third time, and proceeding as before. This correction, however, will never be necessary, except where extreme accuracy is required in order to deduce some consequences from it. But the time thus found is to be considered as accurate, only so far as the Tables of the sun and moon can be depended upon for their accuracy; and the best lunar Tables are subject to an error of 10″ in longitude, which, in this eclipse, would make an error of about half a minute in the

time of the beginning and end Hence, accurate observations of an eclipse compared with the computed time, furnishes the means of correcting the lunar Tables, as will be afterwards explained In the same manner the end of the eclipse may be computed

EXAMPLE

To Compute the Times of the Solar *Eclipse on April* 3, 1791, *for the Royal Observatory at Greenwich*

The time of the mean conjunction (544) is April 3, $2h$ $58'$ $15''$ mean time, at which time we find

Mean long of the sun	-	0^s	$11°$	$51'$	$16'$
Long of the moon's desc node	-	0	22	11	44
Mean long of ☉ from ☾'s node	-	0	10	23	28

Hence (560) there must be an eclipse somewhere upon the earth

To the mean time of the new moon, compute the sun's and moon's true longitudes, and they will be found to be 0^s $13°$. $47'$ $43''$, and 0^s. $14°$ $49'$. $24''$, compute also the moon's true latitude, and it will be found to be $38'$ $49''$ N descending At the same time, the sun's horary motion is found to be $2'$ $28''$, the moon's horary motion in longitude is $30'$ $12''$, and in latitude $2'$ $46''$ decreasing, hence, the moon's horary motion in longitude from the sun is $27'$ $44''$

By proceeding as directed in Article 547, we find the mean time of the ecliptic conjunction of the sun and moon to be $3d$ $0h$ $44'$ $48''$, from which subtract $3'$ $18''$ the equation of time, and it gives the apparent time $3d$ $0h$ $41'$ $30''$, at which time, the sun's and moon's longitude in the ecliptic is 0^s. $13°$ $42'$ $14''$, and the moon's true latitude is $44'$ $59''$ N descending The horizontal parallax of the moon is $54'$ $46''$, and of the sun, $9''$; hence, the horizontal parallax of the moon from the sun is $54'$ $37''$; therefore (178, 164) the moon's parallax in longitude from the sun is $-20'$ $56''$, and its parallax in latitude from the sun is $-33'$ $44''$, hence, $-20'$ $56''$ is the apparent distance of the moon from the sun in longitude, also the apparent latitude from the sun is $11'$ $15''$ north

As the moon is to the west of the nonagesimal degree, assume 1 hour after

or $3d.\ 1h.\ 41'.\ 30''$, at which time (from the horary motions of the sun and moon) the sun's true longitude is found to be $0.\ 13°.\ 44'.\ 42''$, the moon's true longitude on the ecliptic $0^s.\ 14°.\ 12'.\ 26''$, and true latitude $42'.\ 13''$ north descending. The moon's parallax in latitude is $-30'.\ 41''$; hence, the moon's apparent latitude is $11'.\ 32''$; also, its parallax in longitude from the sun is $-28'.\ 50''$; but the moon's true longitude exceeds the sun's by $0°.\ 27'.\ 44''$, therefore the apparent distance of the moon from the sun in longitude is $-1'.\ 6''$. Hence,

Moon's apparent dist. in long. at $0^h.\ 41'.\ 30''$	$= -20'.\ 56''$
———————— $1.\ 41.\ 30$	$= -1.\ 6$
Apparent hor. mot. ☾ from ☉ in long. - -	$19.\ 50 = MP$.

Hence, $19'.\ 50'' : 20'.\ 56'' :: 1$ hour : $1h.\ 3'.\ 20''$, which added to the time of the true conjunction $0h.\ 41'.\ 30''$, gives $1h.\ 44'.\ 50''$, the time of the apparent conjunction. Also, the apparent horary motion in latitude is $17'' = RQ$; hence, QN is very nearly equal to MP.

At this time (from the horary motions) the sun's true longitude is found to be $0^s.\ 13°.\ 44'.\ 50''$, the moon's $0^s.\ 14°.\ 14'.\ 7''$, and the moon's true latitude $42'.\ 4''$; hence, the moon's true longitude is greater than the sun's by $29'.\ 17''$. The moon's parallax in latitude from the sun is $-30'.\ 32''$, and in longitude $-29'.\ 15''$; hence, the moon's apparent latitude is $11'.\ 32''$ north; also, the apparent longitude from the sun is $29'.\ 17'' - 29'.\ 15'' = 2''$, which is what the moon's apparent longitude exceeds the sun's true longitude.

This difference shows the apparent conjunction, found above, to be very nearly true; and to get it more accurately, say, $19'.\ 50'' : 2'' :: 1$ hour : $6''$, which (as the moon's apparent longitude is the greater) subtracted from $1h.\ 44'.\ 50''$ gives $1h.\ 44'.\ 44''$, the true time of the apparent conjunction, at which time the moon's apparent longitude is $0^s.\ 13°.\ 44'.\ 50''$, the same as the sun's true longitude, that not having sensibly varied in $6''$ of time. The apparent latitude is $11'.\ 32'',25$. Now at $1h.\ 41'.\ 30''$ the moon's apparent distance in longitude from the sun has been shown to be $1'.\ 6''$; and at $1h.\ 44'.\ 44''$ the longitude of the sun, and the moon's apparent longitude are equal; therefore in $3'.\ 14''$ the apparent motion of the moon from the sun was $1'.\ 6'' = 66''$; let this $= Sm$, or nr; also, at $1h.\ 41'.\ 30''$, the apparent latitude $mn = 11'.\ 32''$, and at $1h.\ 44'.\ 44''$ it was $11'.\ 32'',25 = SA$; therefore $Ar = 0'',25$. Hence,

$$66'' : 0'',25 :: \text{rad.} : \text{tan.}\ Anr, \text{ or } SAB = 13'.\ 1''.$$

As the angle Anr is so very small, we may take $An = rn = 66'$ without any sensible error, and for the same reason SB may be taken $= SA = 11'\ 32'$

$$\text{Rad} \quad \sin 13'\ 1'' \quad 11'\ 32' \quad AB = 2'',6$$

Hence, $An = 66''$ $AB = 2'',6$ $3'\ 14''$ $8''$ the time through BA, which taken from $1h\ 44'\ 44''$ gives $1h\ 44'\ 36''$ the time of the greatest obscuration at B

The moon's horizontal semidiameter is $14'\ 56'$, and its altitude at the time of the greatest obscuration (determined by a globe, which is sufficiently near for this purpose) is about $38°$, hence, the augmentation of the diameter is $9''$, consequently the apparent semidiameter of the moon is $15'\ 5''$, which added to $15'\ 59''$ the sun's semidiameter, gives $31'\ 4''$, from which subtract $SB = 11'\ 32''$, and the remainder is $19'\ 32''$ the parts deficient, hence, $15'\ 59''$ $19\ 32''$ 6 digits $7d\ 19'\ 57''$ the digits eclipsed at the time of the greatest obscuration

To find the time of the beginning, we must first get the time (570) nearly The value of $SB = 11'\ 32'' = 692''$, and as the apparent semidiameter of the moon is now $15'\ 6''$, we have $SV = 31'\ 5'' = 1865''$, hence, $BV = 1732''$ Now as MP is, in this case, nearly equal to QN, we may, for the purpose we here want it, assume the apparent horary motion of the moon from the sun in the apparent orbit equal to that in longitude, which is $19'\ 50'' = 1190'$, hence, $1190''$ $1732''$ 1 hour $1h.\ 27'\ 20''$, which subtracted from $1h\ 44'.\ 36''$ (the time at B) gives $0h\ 17'\ 16''$ the time of the beginning, nearly Let us therefore assume the beginning at $0h.\ 17'$, at which time we find (from the horary motions of the sun and moon) the sun's true longitude to be $0^s\ 13°\ 41'\ 15''$, and the moon's $0^s\ 13°\ 29'\ 55''$, whose difference is $11'\ 20''$ their true distance in longitude, but the moon's parallax in longitude is $-17'\ 45''$, hence, their apparent distance in longitude is $29'\ 5'' = 1745''$. At the same time the moon's true latitude is $46'\ 7''$, and its parallax in latitude $-35'.\ 10''$, hence, the apparent latitude of the moon from the sun is $10'\ 57''$, therefore $SN = \sqrt{1745^2 + 657^2} = 1864'' = 31'\ 4''$, which being less than $31'\ 5''$ shows that the eclipse is begun

Let us next assume $0h\ 16'$, and by proceeding in the same manner, we find $Sn = 1883'' = 31'\ 23''$, therefore the eclipse is not begun

Hence, $31'\ 23'' - 31'\ 4'' = 19''$ $31'\ 5'' - 31'\ 4'' = 1''$ 1 minute , $3''$, which subtracted from $0h\ 17'$, gives $0h\ 16'\ 57''$ for the beginning of the eclipse

If to $1h\ 44'\ 36''$ we add $1h\ 27'\ 20''$, we have $3h\ 11'.\ 56''$, we will therefore assume $3h\ 12'$ for the end, and by proceeding as before, we find the apparent distance of the moon from the sun in longitude to be $30'\ 37''$, and the moon's apparent latitude $10'\ 48''$, hence, the moon's apparent distance from the sun is $\sqrt{1837^2 + 648^2} = 1948'' = 32'\ 28''$, but the sum of the apparent semidiameters of the sun and moon is now $31'\ 2''$, consequently the eclipse is ended

Let us next assume the time $3h. 6'$, and the apparent distance of the moon from the sun in longitude is $28'. 28''$, and in latitude $10'. 55''$, hence, the moon's apparent distance from the sun is $\sqrt{1708^2+655^2}=1829''=30'. 29''$, therefore the eclipse is not ended.

Hence, $32'. 28''-30'. 29''=1'. 59'' : 31'. 2''-30'. 29''=33'' : 6' : 1'. 39''$, which added to $3h. 6'$, gives $3h. 7'. 39''$ for the end.

Hence, at the Royal Observatory at Greenwich, the Tables give the times of the eclipse on April 3, 1791.

Beginning - - - -	$0^h. 16'. 57''$	apparent time.
Greatest obscuration -	1. 44. 36	
End - - - - - - - -	3. 7. 39	
Digits eclipsed - - -	$7^d. 19. 57$	

If it be required to compute the eclipse for any other place, instead of the latitude of Greenwich use the latitude of the place; and reduce the apparent time at Greenwich to the apparent time at the place, according to the difference of the meridians.

573. To find what point of the sun's limb will first be touched by the moon, let P be the pole of the ecliptic ES, Z the zenith, S, M, the centers of the sun and moon when their limbs are in contact at a, and draw MD perpendicular to ES. By Art. 164, PZ is the altitude of the nonagesimal degree, and SPZ is the sun's distance from that point, both which are found in the computation of the parallax; also MD is the apparent latitude of the moon; hence,

$$\text{Rad.} : \tan. PZ :: \sin. SPZ : \tan. PSZ$$
$$\text{Tan.} SM : \text{rad.} :: \tan. DS : \cos. DSM$$

If L be the longitude of the nonagesimal degree, then $ZSD=90°-PSZ$, when the sun's longitude is between L and $L+180°$; otherwise $ZSD=90°+PSZ$; and $ZSD\pm MSD$ (according as the moon's visible latitude is south or north) gives ZSM the distance of the point of the limb of the sun first touched by the moon from the highest point of the sun's disc.

In this eclipse, $PZ=50°. 7'$, and $SPZ=25°. 16'$; hence, $PSZ=27°. 3'$, which (in this case) added to $90°$ gives $117°. 3'=ZSD$; also, $DSM=20°. 42'$, which (as the moon's apparent latitude is north) subtracted from $117°. 3'$ gives $ZSM=96°. 21'$, the moon's distance from the zenith of the sun at the beginning of the eclipse. In like manner, the distance at the middle and end of the eclipse may be found, and thence the apparent path of the moon over the sun's disc in respect to the horizon may be described (578).

574. In the computation of this eclipse, the moon's true latitude and longitude

were at first computed from the Tables, and afterwards determined from the horary motions; but as the horary motions may be subject to a small variation in the duration of an eclipse, in cases where the utmost accuracy is required, the true latitude and longitude should be computed every time from the Tables; in such cases, the decimals of the seconds should also be taken into consideration, which in this Example were omitted. When we want only to predict an eclipse, the method here practised will always be sufficiently accurate. We have followed the same method in computing the occultation of a fixed star by the moon; that computation therefore may, if necessary, be rendered more correct, in the same manner.

To construct a Solar Eclipse, by the Principles of Projection delivered in the last Chapter.

575. According to this projection (541), the apparent ellipse described by any point on the earth's surface, to any eye at the center of the sun, is projected upon a plane at the moon perpendicular to a line joining the earth and sun; and the point of the ellipse of projection, corresponding to any point of the other ellipse where the spectator is, is the point where the center of the sun appears to the spectator. The center of projection is in the ecliptic. If the lunar orbit be properly laid down and divided, showing where the center of the moon is at any time, we shall then have the relative situation of the centers of the sun and moon at any time seen from the given place of the spectator. From these principles of projection, we thus construct the solar eclipse which we have here calculated, assuming such elements as are necessary, from that calculation.

576. Take (541) a radius OE equal to $54' \; 37''$, the difference of the sun's and moon's horizontal parallaxes, and divide it into minutes, and describe the semicircle EGC representing half the circle of projection, EOC representing the ecliptic, to which draw OG perpendicular. Find (536) P the projected north pole; from the scale OE, take $Or = 54' \; 37'' \times \sin$ lat $\times \cos$ dec, and in a line perpendicular to Or set off both ways $r6 = h \times \cos$ lat, and $rm = rn = h \times \cos$ lat $\times \sin$ dec, and describe the ellipse $m6n6$, and divide it into hours, by Art. 536, and then subdivide those hours which you will want to make use of, as far as you conveniently can for the size of the figure. From the scale take Ov equal $44' \; 59''$, the moon's true latitude north descending at the time of the ecliptic conjunction, and draw LvM making an angle with Ov equal to $84° \; 18'$, the complement of the angle which the relative orbit makes with the ecliptic, on the *left* side, if the latitude be north or south *decreasing*, and on the *right*, if *increasing*; in this Example, it is on the left side; and LvM will represent the moon's relative orbit. Mark upon the moon's orbit at the point v,

$41'. 30''$, that being the time after 12 o'clock at which the true ecliptic conjunction happens; and with an extent $= 27'. 52''$, the horary motion of the moon from the sun in its relative orbit, set off the hours each way from *v*, and subdivide them into minutes, or as far as the size of the figure will permit. Now to find the time of the middle of the eclipse, take the compass, and find, by trial, what two corresponding times, as at *z* and *x*, upon the ellipse and moon's orbit are nearest together, which will give the time of the *greatest obscuration*, because the centers of the sun and moon are then at the least distance. To find the time of the beginning, take, with the compass, from the scale, an extent equal to $31'. 5''$, the sum of the semidiameters of the sun and moon, and, by trial, find two corresponding times, as at *s* and *t*, at that distance, and it gives the time of the *beginning*; and if you find two corresponding times, as at *y* and *w*, at the distance $31'. 2''$, the sum of the semidiameters at the end, it gives the time of the *end*; or you may omit the variation of the diameter of the moon in the interval. For the beginning must be when the centers of the sun and moon arrive at the distance of the sum of their semidiameters; and the end must be when they have receded till they have got to that distance. To find the digits eclipsed at the greatest obscuration, take *eu* from the scale, and say, *ze* : *eu* :: 6 digits : the digits eclipsed. To find the digits eclipsed at any other time, take, with the compass, the interval at that time on the ellipse and on the moon's orbit, and apply it to the scale, and then say, *ze* : that distance :: 6 digits : the digits eclipsed. If by taking the interval of two corresponding times, it appears that it is always greater than the sum of the semidiameters of the sun and moon, it shows that there will be no eclipse at that place.

577. From this construction, the position of the moon in respect to the zenith of the sun's disc may be found, and thence the apparent path of the moon over the sun in respect to the horizon. For (540) a line drawn from *O* to any point of the ellipse, where the spectator is, being vertical, from the principles of the projection, the angles *Ots*, *Ozx*, *Owy*, show the angular distance about the center of the sun from its vertex to the center of the moon at the beginning, middle and end of the eclipse. Hence, let *C* be the center of the sun *Zbda*, *Z* its zenith; and make the angles *ZCP*, *ZCQd*, *ZCR* $=$ *Ots*, *Ozx*, *Owy* in Fig. 134. respectively; take *CP*, *CR* equal to the sum of the semidiameters of the sun and moon, *dCs* $=$ the digits eclipsed, and *SQ* $=$ *aP*, and with the centers *P*, *Q*, *R* and radii *Pa* $=$ *Qs* $=$ *Rb* equal to the semidiameter of the moon describe three circles, and they will represent the situation of the moon at the beginning, middle and end of the eclipse in respect to the vertex *Z* of the sun, and consequently in respect to the horizon; hence, if we describe a circle through *P*, *Q*, *R*, and with the same radius describe *rst* parallel to it, it must very nearly represent the boundary of the eclipse, or of the extreme part of the moon's limb as it passes over the sun, in respect to the horizon.

578 The eclipse may also be thus calculated from the projection. Assume the time at t of beginning, as determined by the construction; draw tc perpendicular to OP, and join Ot, Os. The time from t to m being given, convert it into degrees a°; then (537) sin $a^\circ \times rb = tc$, and cos $a^\circ \times rm = rc$, but (535) Or is known, hence, Oc is known; therefore in the right angled triangle Oct, we know Oc, ct, to find cOt, and Ot; but (536) POv is given, therefore $cOt \underset{\sim}{+} POv = tOv$ is known; also, Ov and the angle Ovs are known by the construction; and the time from s to v being given, and also the moon's relative horary motion in LM, we know vs; hence, in the triangle Ovs, we know Ov, vs and the angle Ovs, to find Os, and the angle vOs; hence we find $tOs = tOv \underset{\sim}{+} vOs$; and lastly, in the triangle tOs, we know tO, Os and the angle tOs, to find ts, and if this be equal to the sum of the semidiameters of the sun and moon, the assumed time is true; if it be not equal to the sum, assume another time for the beginning, and find another value of ts, and proceed with these two as in Article 572. In like manner we may find the end. But this method is not (541) subject to the same accuracy as the method of calculation which we have already given.

579 Sir I. NEWTON supposes that the aberration of rays in the focus of a telescope makes the image appear greater than it ought, and hence different telescopes will give different measures of the sun's diameter, and consequently make the eclipse appear to begin at different times. That telescope which gives the diameter the least, is the most perfect instrument. The excellent transit telescope at Greenwich makes the diameter of the sun less by 6″ than that given by MAYER in his Tables, as Dr. MASKELYNE has found by his observations. The diameter of the sun assumed in these calculations has therefore been taken 6″ less than that which MAYER determined. M. du SEJOUR supposes that the rays of light coming from the sun are inflected as they pass by the moon, which he attributes to the refraction which they suffer in passing through the moon's atmosphere; on this account the apparent contact of the limbs will not take place so soon as it otherwise would; this would be the same as a diminution of the moon's diameter; which of these hypotheses ought to be admitted M. du SEJOUR endeavoured to determine from the observations of Mr. SHORT, on the solar eclipse, April 1, 1764, upon the distance of the horns of the moon, but he could deduce nothing satisfactory from thence. He supposed the inflection 3″,291, and the diameter of the moon to be diminished by the same quantity, and calculated upon each supposition a great many distances of the horns, and compared them with the observed distances; but he could not decide between the two hypotheses. An inflection of 1″,8, and a diminution of 1″,5 of the semidiameter, he found would satisfy some observations, and he seemed to think this conclusion most likely to be nearest the truth, but he came at last to

no determination upon the subject. All the requisite observations seem not to be capable of being made to that degree of accuracy which is necessary to settle so nice a matter. M. du SEJOUR therefore proposed the following method to determine whether the rays of light passing by the limb of the moon suffer any deviation. Take a telescope mounted upon a polar axis, with a wire micrometer annexed to it. When two stars come into the field of view together, and one of them is to be eclipsed by the moon, open the wires and bring one star upon one of the wires and the other star upon the other, and thus follow the stars until one of them be eclipsed, and at the instant before it disappears, observe whether its distance from the other star is changed, that is, whether it be off the wire, the other star remaining upon its wire; if this be found to be the case, the rays must have suffered a deviation *Traité* Analytique, pag. 420. I do not find that an observation of this kind has been ever made.

MAYER'S *Method of Computing* SOLAR ECLIPSES, *taken from his* OPERA INEDITA, *Vol. I. To which are added, such observations as were judged necessary for explaining the grounds of the Operations.*

580. It being determined that there will be an eclipse, assume three moments of time at equal intervals, as nearly as you can conjecture, for the beginning, middle and end of the eclipse; to which times compute the true longitude of the sun, the true longitude and latitude of the moon, her horizontal semidiameter, and her equatorial parallax.

At the assumed times for the beginning, middle and end of the eclipse, find the moon's altitude, which may be done with sufficient accuracy by a globe, and thence find her apparent semidiameter from her horizontal semidiameter.

Reduce the latitude of the place to that at the earth's center; also, reduce the equatorial parallax to that at the given place, by Tab. Art. 173.

To the latitude of the place so reduced, at the three assumed times, find the nonagesimal degree of the ecliptic and its altitude; hence, the distance of the moon from it is known. Then compute the moon's parallaxes in latitude and longitude at the said three times; thus you get the *apparent* latitudes and longitudes of the moon, and the differences of the apparent longitudes of the sun and moon, at these times; also, the moon's apparent latitude.

By interpolation find the differences of the apparent longitudes for every 5′ or 10′ (in our computation it is for 10′); corresponding to which, put the sum of the apparent semidiameters of the sun and moon.

By comparing the apparent latitude and differences of longitude with the sum of the semidiameters (c), estimate what apparent latitude (x) answers nearly to the beginning or end of the eclipse; and this may be easily done, though

the precise times are not yet known, since the moon's latitude does not often vary above a few minutes in an hour. Then compute $\sqrt{c^2-a^2}$, and see in the Table amongst the differences of longitude, whether it has a latitude answering to a, in which case, it will be the true difference of longitudes at the beginning or end of the eclipse. But if there be any difference between these latitudes, call it a', and say, $\sqrt{c^2-a^2}$: a :: a' : a fourth term, which added to $\sqrt{c^2-a^2}$ if the latitude a be too great, or subtracted if too little, gives the correct difference of longitudes corresponding to the beginning or end. Corresponding to this difference, find the time by the Table, and you have the times of the beginning or end.

To find the time of the greatest obscuration, and the digits eclipsed, assume the latitude at that time to be a, and let its increment or decrement in 5′ or 10′ about that time be a', and the increment or decrement of the difference of the longitudes be y', and let y' : a' :: a : a fourth term y, which will be the difference of longitudes at the time of the greatest obscuration, the time corresponding to which gives the time of the greatest obscuration; where it ought to be observed, that y is to be sought for amongst the differences of longitudes *before* the apparent conjunction if the latitude be *increasing*, and *after* if *decreasing*.

From the time of the greatest obscuration, find from the Table the latitude (l), and $\sqrt{l^2+y^2}$ is the nearest distance of the centers of the sun and moon, which taken from the sum of their semidiameters, the remainder (reckoning the sun's semidiameter 6 digits), gives the digits eclipsed.

EXAMPLE.

To calculate an Eclipse of the Sun which happened 26 October, 1753, for Gottingen, according to Mayer's Lunar Tables inserted in Tom. II. Comm.

Three assumed Times (true time)	9*h*. 20'. 0"	10*h*. 30'. 0"	11*h*. 40'. 0"
Sun's true longitude - - -	7ˢ. 3°. 6. 19	7ˢ. 3°. 9. 14	7ˢ. 3°. 12. 9
Moon's true longitude - -	7. 1. 58. 36	7. 2. 39. 43	7. 3. 20 40
——— latitude north - -	27. 49	31. 37	35 24
Sun's semidiameter - - -	16. 10	16. 10	16 10
Moon's equatorial parallax - -	59. 8	59. 7,5	59. 7
——— hor. semidiameter - -	16. 7,5	16. 7	16. 7
Altitude of the moon - -	— —	— —	— —
Augm. of moon's semid. - -	3,5	6	8
Moon's app. semid. in altitude -	16. 11	16. 13	16. 15
Sum of semids. of sun and moon	32. 21	32 23	32. 25
Latitude of Gottingen - -	51°. 32'	51°. 32'	51°. 32'
Reduction for ditto - -	16	16	16
Latitude reduced - - -	51. 16	51. 16	51. 16
Cor. of equatorial parallax - -	10	10	10
Moon's hor. parallax - - -	58'. 58"	58'. 57",5	58'. 57"
Moon's dist. from merid. eastward	40°. 0'	22°. 30'	5°. 0'
Sun's right ascension - - -	210. 53	210. 56	210. 58
Right ascen. of mid-heaven - -	170. 53	188. 26	205. 58
Obliquity of ecliptic 23°. 29' - -			
Nonagesimal degree - - -	4ˢ. 26. 57	5ˢ. 9. 52	5ˢ. 23. 56
Alt. of non. degree - - -	47. 29	41. 15	34. 27
Moon's true dist. from nonag. -	65. 2	52. 48	39. 25
Sun's hor. parallax - - -	0'. 11"	0'. 11"	0'. 11"
Dif. hor. par. ☉ and ☾ - - -	58. 47	58. 46,5	58. 46
Moon's par. in longitude - -	39. 29	31. 5	21. 16
——— latitude - -	39. 48	44 16	48. 32
∴ appar. *long*. moon - - -	7ˢ. 2°. 38'. 5"	7ˢ. 3°. 10'. 48"	7ˢ. 3°. 42'. 5"
Diff. of appar. *long*. of ☉ and ☾ -	West 28. 14	East 1. 34	East 29. 56
Appar. *lat*. of the moon south -	11. 59	12. 39	13. 8

Hence, by the Rules for interpolation, the following Table is computed, in which those times are passed over that are of no use.

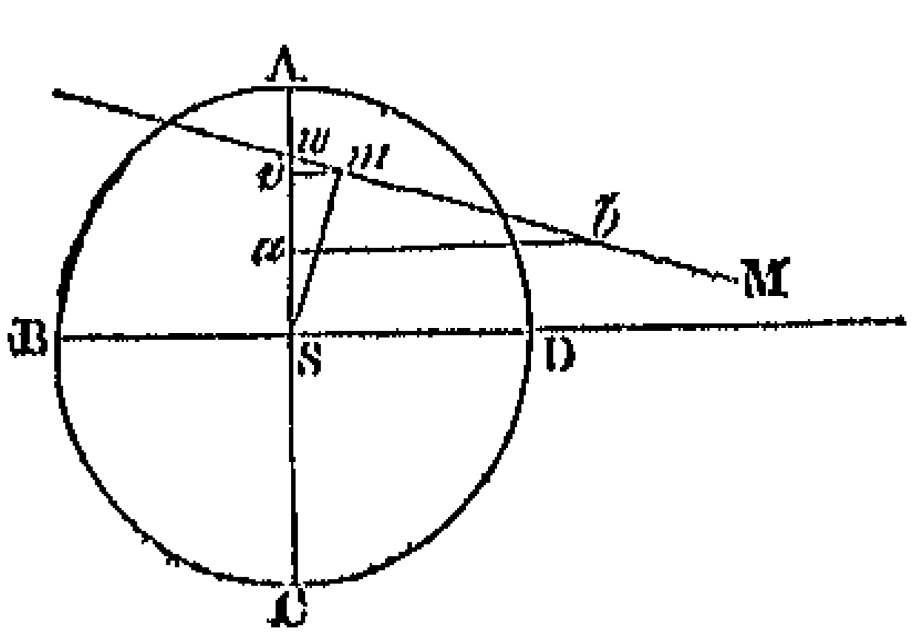
Time.	App. dif. long.	App. lat. S.	Sun's semid.
h ' "	"	' "	' "
9 10 0	− 32 36	11 52	32 21
9 20 0	− 28 14	11 59	32 21
9 30 0	− 23 51	12 5,5	32 21
10 20 0	− 2 36	12 31	32 23
10 30 0	+ 1 34	12 39	32 23
10 40 0	+ 5 42	12 44	32 23
11 30 0	+ 25 58	13 4,5	32 25
11 40 0	+ 29 56	13 8	32 25
11 50 0	+ 33 52	13 11	32 25

Suppose the latitude (x)* at the *beginning* to be 12', $c = 32'\ 21''$, then $\sqrt{c^2 - x^2} = 30'\ 2'',5$ the difference of longitudes, which difference of longitudes does not answer to the assumed latitude 12'. Now the decrease of the apparent difference of longitudes in 10' is 32' 36" − 28' 14" = 4' 22", and the difference between the apparent longitude 32' 36" and 30' 2",5 is 2' 34" (omitting the 0",5 as of no consequence); also, the increase of the apparent latitude in 10' is 7"; hence, 4' 22" : 2' 34" :: 7" : 4", which added to 11' 52" gives 11' 56", we may therefore consider this as the apparent latitude at the beginning of the eclipse; and to find the corresponding apparent difference of longitudes, let AC (fig. 31) represent the ecliptic, A the sun, with the center A and radius $AC = 32'\ 21''$ describe the circle Cn, let nw (perpendicular to AC) = 12', take nr = 4", draw rm perpendicular to nw, and mv to AC, then mv = 11' 56", wv is the corresponding decrease of the difference of the apparent longitudes; to find which, as the triangles Anw, nmr are similar (mn being very small), Aw (30' 2",5) : nw (12') :: nr (4") : rm = 1",5, which added to 30' 2",5 gives 30' 4" the difference Av of the apparent longitudes at the beginning of the eclipse. And to find the corresponding time, as the variation of the difference of the longitudes vary as the time, 4' 22" : 2' 32" (32' 36" − 30' 4") :: 10' : 5' 48", which added to 9h. 10' gives 9h 15' 48" the time of the *beginning* of the eclipse.

* The ecliptic conjunction happens between the two apparent differences of longitudes when the sign changes from − to +, the sign − showing the apparent difference of longitudes of the moon to be *west* of the sun, and + to the *east*.

The *end* of the eclipse being found on the same principle, it will be necessary to put down only the operation. Assume the latitude (x) at the end to be 13, $c=32'.\ 25''$, then $\sqrt{c^2-x^2}=29'.\ 42''$, which difference of longitudes does not answer to the assumed latitude. Now $29'.\ 56'-25'.\ 58''=3'.\ 58''$, and $29'.\ 56''-29'.\ 42'=14''$, and the increase of apparent latitude in 10′ is $3'',5$; hence, $3'.\ 58''$. $14''$ :: $3'',5$: $0''$, which shews that there is nothing to be subtracted from $13'.\ 8''$ which may therefore be considered as the apparent latitude at the end of the eclipse, and this differs $8''$ from the assumed apparent latitude; hence, $29'.\ 42''$: $13'$ :: $8''$: $3'',5$, which taken from $25'.\ 42''$, leaves $29'.\ 38'',5$ for the difference of longitudes at the end; and to find the corresponding time, say, $3'.\ 58''$: $17'',5$ $(29'.\ 56''-29'.\ 38'',5)$ :: $10'$ $47''$, which subtracted from $11h.\ 40'$ leaves $11h.\ 39'.\ 13''$ the end of the eclipse.

The time of the *greatest obscuration* happens not far from the time when the difference of the apparent longitudes is equal to nothing, and consequently between $10h.\ 20'$ and $10h.\ 30'$; and to find the apparent latitude nearly when this happens, we have $4'.\ 10''$ $(2'.\ 36''+1'.\ 34''$ the whole change of ap. dif. long.) : $2'.\ 36''$ (the change of ap. dif. long. from $9h.\ 10'$ till the dif. $=0$) .: $5''$ (the change of ap. lat. in $10'$) : $3''$ the change of ap. lat. from $9h.\ 10'$ till the ap. dif. long. $=0$; hence, at that time the lat. $=12'.\ 37''$.



Let $ABCD$ represent the sun, S its center, BSD the ecliptic $AwSC$, perpendicular to BSD, wM a portion of the moon's apparent path, take $Sv=12'.\ 37''$, draw Sm perpendicular to wM, and vm to AS. Now in $10'$ the change of latitude is $5''$; let $wa=5''$, and draw ab parallel to SD, then ab the corresponding variation of the dif. of ap. longitudes $=4'.\ 10''$, and the triangles baw, Svm being similar, (ab) $4'.\ 10''$: aw $(5'')$ :: Sv $(12'.\ 37'')$: $vm=15'$ the apparent difference of longitudes at the time of the greatest obscuration, that is, it is $2'.\ 21''$ less than at $10h.\ 20'$; hence, $4'.\ 10''$. $2'.\ 21''$.: $10'$: $5'.\ 38''$, which therefore added to $10h.\ 20'$ gives $10h.\ 25'.\ 38''$ the time of the greatest obscuration.

Lastly, as $Sv=12'.\ 37''$ and $vm=15''$, $Sm=\sqrt{\overline{12'.\ 37''}^2+\overline{15''}^2}=12'.\ 37'',5$, which subtracted from $32'.\ 23''$ there remains $19'.\ 45'',5$ the greatest quantity of

the eclipse; hence, 16′ 10″ (the sun's semid.) 19′ 45″,5 6 digits 7 dig. 20 the digits eclipsed.

The calculations may be made by proportional logarithms and logarithmic sines thus:

At the beginning $\sqrt{c^2 - i^2} = \sqrt{32'\ 21''^2 - 12'^2} = \sqrt{32'\ 21'' + 12' \times 32'\ 21'' - 12'}$ $= \sqrt{44'\ 21' \times 20'\ 21'}$,

By prop. log. -	44′ 21′	- p. l.	,6084
	20 21	- p. l.	,9167
			2)1,5551
	32′ 2″$\frac{1}{2}$	p. l.	,77755

At the time of the greatest obscuration, we have $\sqrt{12'\ 37'^2 + 15'^2}$, which is thus computed:

12 37′	- - - - -	p. l.	1,1543
15	- - -	p. l.	2,8573
Dif. log. + 10 = log. tan. 88° 52′	- -		11,7030
Log. cos. 88° 52′ 0″	- - -		8,2962
15	- - - -	p. l.	2,8573
12 37,5	- - -	p. l.	1,1535

Thus all other like computations are made.

Where there are proportions, the computations are also made by prop. logs. by the known rules. See my Treatise on *Plane and Spherical Trigonometry*.

In making the above calculations, you have in the *Nautical Almanac*, the sun's and moon's longitudes and the moon's latitude for every noon and midnight at Greenwich, and hence from four of them nearest the times of the phases of the eclipse, you may find the longitudes and latitudes for the assumed times. Then compute the parallaxes in longitude and latitude, and you get the *apparent* longitudes and latitude. With the moon's altitude (which being very nearly the same with the sun's, you may, if you do not use a globe, find by

computing the altitude of the latter) take out the augmentation of the moon's semidiameter from the XLIV of the Lunar Tables, Vol. III which added to the sum of the horizontal semidiameters of the sun and moon, will give the sum of the apparent semidiameters. In your first researches it will be best to compute for the noon of the day when the eclipse is expected to happen, as you will thence be enabled to guess more nearly the times of the phases, remarking that parallax always carries the moon further from the nonagesimal degree, that is, west, if the moon be to the west of that point, and east, if to the east. For Greenwich, the latitude reduced to the earth's center, is 51°. 11'. 7". The nearest approach of the centers *precedes* the ecliptic conjunction, if the latitude be *increasing*, but *follows*, if *decreasing*. When regard is had to the sun's parallax in computing the parallax in longitude and latitude, the moon's horizontal parallax must be diminished by 9".

To trace out the Path on the Surface of the Earth, where the Eclipse will be central, or for any number of digits.

581. Let $EACD$ be the enlightened hemisphere of the earth, O the center of the disc, or that point to which the sun is vertical; EOC the plane of the ecliptic, OG perpendicular to it, P the north pole; join PO, and let LM represent the path of the center of the penumbra, conceived to be upon the plane into which the disc would be orthographically projected; then from the nature of that projection, the angles at O upon the surface will be equal to the angles into which they are projected. Now the place upon which the center of the moon is projected is manifestly that point on the earth where the eclipse is central, because the projection is made by lines drawn to the center of the sun. Let Z be the projected center of the moon at any time, or the real center of the penumbra, and PB any given meridian. Now we know Ov the moon's latitude at the time of conjunction, and (576) the angle OvZ, and as the time, when the center of the penumbra is at Z, is given, the time through Zv is known, and the relative horary motion of the moon being known, vZ will be known; hence, we can find ZO, and the angle ZOv; find (536) the angle POv, and we shall have the angle POZ. Now consider PO and ZO as two circles upon the earth's surface, then the angle POZ between them is equal to the angle POZ of projection, and therefore known; also, the arc PO is the complement of the sun's declination; and to find the arc ZO, we must consider ZO in the projection to be the sine of the arc projected; hence, the arc ZO is that whose sine is to radius as OZ to OA, therefore we know the sine of the arc ZO, and consequently we get ZO itself, hence, in the spherical triangle OPZ,

we know PO, OZ and the angle POZ, to find PZ the complement of the latitude of the place where the eclipse is central. Find also the angle, OPZ, then the time at the meridian PB being known, the angle OPB (the sun's distance from the meridian) is known; hence we know the angle BPZ the longitude of the point Z from the meridian PB, therefore the latitude and longitude of Z being known, the point Z is determined where the eclipse was central at the given time. Make this calculation for every quarter or half hour, for all the time the penumbra is describing dc, and you will trace out upon the surface of the earth the path of the center of the penumbra, or that tract where the eclipse is central. If we bring Z to d, we get the place where the sun rises centrally eclipsed, and if Z be brought to c, we shall find where the sun sets centrally eclipsed. If Z coincide with r, we get the place where the sun is centrally eclipsed upon the meridian. Let y be the center of the penumbra when it first touches the earth, and z the center when it leaves the earth, and draw Ow perpendicular to LM. Then knowing Ov and the angle Ovw, we can find Ow and vOw; also, Oy = semid ⊖ + semid penumb is known; hence, in the right angled triangle yOw, we get the angle yOw, and therefore we know yOv, and POv being already found, we know bOP; hence, in the triangle bOP, we know bO (= 90°), PO, and the angle POb, hence we find Pb the complement of the *latitude* of b, find also OPb, and we get bPB the longitude of b, from the given meridian PB; thus we get the place b where the eclipse first begins at the sun rising. In like manner we get the place a where the eclipse last ends at sun setting.

Ex. In the first solar eclipse which we have here computed, let it be required to find that place upon the earth's surface where the sun is centrally eclipsed at one o'clock, apparent time at Greenwich. In this case, $Ov = 14'\ 59''$, and the angle $OvZ = 5°\ 42'$, and as the time at v is $41'\ 30''$, and the center of the penumbra is at Z at one o'clock, the time through $vZ = 19'\ 30''$ which gives $vZ = 9'\ 3''$; hence, $ZO = 35'\ 59''$, the angle $OZv = 81°\ 22'$, and $ZOv = 13°\ 56'$. Now the radius $OG = 54'\ 56''$, hence, the arc OZ upon the surface (corresponding to its projection $= 35'\ 59''$) $= 41°\ 4'$; also, $PO = 51°\ 30'$, and (536) $POv = 23°\ 22'$; hence, $POZ = 9°\ 26'$, consequently $PZ = 45°\ 43'$, the complement of which is 44° 17' the latitude of the place; also $ZPO = 8°\ 51'$; but at one o'clock apparent time at Greenwich, its meridian BP makes an angle of 15° with PO, Greenwich being upon that meridian at 12 o'clock; hence, $BPZ = 23°\ 51'$ the longitude of the place, west from Greenwich. In like manner may any of the other phænomena be calculated.

582. Draw OW perpendicular to LM, and take $ve = ve$ equal to the sum of the semidiameters of the sun and moon, and draw def, xey parallel to LM, then

df and xy will mark out the boundaries of the eclipse, or the places where the limbs of the sun and moon just appear in contact. So that if we take the moon at any place Z, and draw Zr perpendicular to df, and compute the latitude and longitude of the point r in the same manner as we did that of Z in the last Article, it will give the place where the limbs of the sun and moon appear in contact outwardly. If we take Zr on the other side of LM, we shall, on that side, get the place where they appear in contact. If we do this for every quarter or half hour, we shall trace the path over the surface of the earth where the limbs of the sun and moon appear in contact, or the boundaries of the eclipse; thus we can lay down upon the earth's surface that tract over which the penumbra passes. If ev be divided into twelve equal parts, and ek be taken equal to three of them, for instance, and tks be drawn parallel to LM, and the place of r' be computed, it gives the place where the sun will be three digits eclipsed; and in like manner as before may the tract on the earth's surface be marked out where the sun will appear three digits eclipsed; in the same manner, we may trace out the path for any number of digits. If $va = vb$, and ab be the difference between the apparent diameters of the sun and moon, and wax, gbh be drawn parallel to LM; then if the diameter of the moon be greater than that of the sun, the space between wx and gh is the limit for the total eclipse; but if the diameter of the sun be the greater, it will be the limit of the annular eclipse. This method of delineating the lines of the phases upon the earth's surface supposes that the apparent nearest distance of the centers of the sun and moon, or of the corresponding hours upon the relative orbit of the moon and upon the ellipse, is in a line perpendicular to the moon's relative orbit, but this is not accurately true, and therefore the delineation cannot be accurate. This line will have different positions at different places for the same phase. M. du Séjour proposes therefore to take the mean angle. He found an error of 3°. 33',5 in longitude, and of 35'. 49" in time of the contact, for the latitude of 16°. 57', by supposing it to happen upon a perpendicular to the relative orbit.

583. M. de la Lande has given the following graphical method. Draw LM on a separate piece of paper, and divide it, so that by moving it you may bring any hour to v. Then if, for instance, the orbit be moved to the right till the time of falling on v be one hour later than that for which the construction was first made, it is in a proper position for a place 15° to the east of that place. Let pq be any parallel of latitude, and divide it into hours, &c. then move the orbit LM until, with an extent of compass equal to the sum of the semidiameters of the sun and moon, you can make the points fall on the same hour both on pq and LM, and at the same time that it shall be the shortest distance between any two corresponding hours on pq and LM; then the difference of the hours shown at v in consequence of the removing of the orbit, shows the longitude of the place from that for which the projection was made, and the pa-

rallel pq shows the latitude; therefore the place on the earth's surface is determined where the limbs just come into contact, and also the time. This we may repeat for as many parallels of latitude as we please, and thus trace out the curve on the surface of the earth where the limbs just come into contact. In like manner, if we take an extent of compass $\frac{1}{12}$ of the sun's diameter less, we may trace out the path where the sun is one digit eclipsed; and so for any other phase.

581. But the general phænomena may be much more easily, and with sufficient accuracy, determined by a common globe thus. Bring the sun's place S to the brazen meridian PV, set the hour index to 12, and bring the sun to the zenith; and from that point on the globe, extend a thread SW perpendicular to the ecliptic LC. Upon a small straight rod LM let there be a moveable circle $adbm$, whose center r is in the middle of the rod, and let its radius be to the radius of the globe as the semidiameter of the penumbra is to the horizontal parallax of the moon. Let there be two moveable upright pieces attached to the horizon of the globe, and capable of being fixed at any two points, at a distance Sv equal to the moon's latitude at the time of the ecliptic conjunction; let the rod LM cut the thread in a plane touching the globe at S, and making an angle MvS equal to the angle which the relative orbit makes with a perpendicular to the ecliptic, as before directed; and in this situation let the rod be fixed to the above mentioned pieces attached to the horizon; let also LM be divided (as in Art. 576.) into time, the time at the point v being that of the ecliptic conjunction; and let the horizon of the globe be fixed exactly parallel to the horizon; and upon a fine thread ab suspend a small conical weight, with an acute vertex; every thing being thus prepared, we proceed thus; first premising, that the circle $TEVC$ (coinciding with the horizon of the globe) contains all the places where the sun is in the horizon, the western hemisphere containing those where the sun is rising, and the eastern where the sun is setting.

Carry the center of the penumbra to r where you find, by suspending the thread from its periphery, that its projection would just touch the earth at t; then turn the globe till the index shows the same hour as the point r, and the point t at that time is the place first touched by the penumbra, and the eclipse there begins at sun rise. Carry the center of the penumbra to s, a quarter of an hour forwards for instance, and set the globe again to that time, and with the thread find the points z, w, where the projection of the circumference of the penumbra would intersect the circle $TCVE$, and z, w, are two other points where the eclipse begins at sun rise. By proceeding thus through the western part, we may find as many points as we please where the eclipse begins at sun rise, and by drawing a curve through them, we get the tract on the earth's surface where the eclipse begins at the rising of the sun. But when z comes into

the eastern part, we get the places where the eclipse begins at sun set; consequently the curve passing through them is a continuation of the curve which passes through all the places where the eclipse begins at sun rising, provided the whole of the penumbra does not fall upon the earth at the middle of the eclipse, that is, if O be the middle of the general eclipse, and OT be perpendicular to LM in the projection, when OT is less than the radius (R) of the penumbra. If $OT=R$, the curves touch, and if OT be greater than R, they are separated; because the penumbra then passes over a part of the earth after it leaves the western side before it comes to the eastern, and therefore in that interval the sun is not eclipsed to any part of the circumference $TEVC$.

Draw bra perpendicular to LM, and ac parallel to it; and when w comes to c, the intersection manifestly begins to go back, and the part adb will then cut TCV, and therefore the intersections will show the points in the horizon where the eclipse ends at sun set; for the velocity of the penumbra being greater than that of the earth about its axis, the penumbra must leave the point which it then touches. And for this reason the eclipse begins where bta cuts TCV. Hence, as c is a point where the eclipse ceases to begin at sun rise and begins to end at sun set, the curve showing the places when the eclipse begins at sun rising is continued into the curve which shows the places where the eclipse ends at sun rising; and as a and w meet again on the eastern side of the earth, the curves there meet again. When therefore OT is less than R, we have a curve in the form of Fig. 139. The velocity of the parts of the earth, from its rotation, being greatest towards the equator, the ovals will be more open towards A and B than towards C.

If $OT=R$, the two ovals touch as in Fig. 140; but that nearest the pole is double.

If OT be greater than R, the two ovals will be detached as in Fig. 141; but if the perpendicular Dz on LM (in the projection,) be less than R, the oval nearest the pole will be double. Two curves AB, CD touching the ovals, and formed by the projection of a and b, will show where the limbs of the sun and moon were in contact.

If LM fall beyond the earth, or if O lie on the other side of T, the curve will be like Fig. 139. until Dz be greater than R, in which case, the curve becomes a simple oval; and when $OT=R$, the oval vanishes. These circumstances, which were given by M. de LAMBRE, appear from the above method of delineation.

The projection of ba will give the middle of the eclipse. The western part of TCV which is cut by ba in the projection, is the point where the eclipse is in the middle at sun rise; and where the eastern part of the horizon is cut by it, it shows the point where the middle of the eclipse is at sun set. When a comes to c, the globe being adjusted to that time, d is a point of the former kind, but

here the eclipse is only for a moment, or rather there is no eclipse, the moon's limb only touching that of the sun. As the umbra advances, *ab* will cut *TCV* at some other point, as *w*, to the time denoted by the center *r*, adjust the globe, and *w* is the place upon the earth where the middle of the eclipse is at sun rising. And thus we may find any number of such places, and draw a curve, as *BDA*, passing through all the places where the middle of the eclipse is at the rising of the sun. As the line, in this case, must be suspended from *ba*, it will be more convenient to have the penumbra divided into two parts through *ab*, so that they may be separated.

To trace out the center of the penumbra over the earth, bring, by the line, the center to coincide in the projection with *TCV* at *e*, and adjust the globe to that time, and *e* will be the place where the center of the umbra first touches the earth. Carry on the penumbra, a quarter of an hour for instance, and adjust the globe to the time, and project the center upon the earth, and it gives the point where the eclipse is central at that time. Find thus as many points as you please, and draw a curve through them all, and you get the path of the center of the penumbra over the earth, showing all those places where the eclipse was central.

If the penumbra be formed by 12 equidistant concentric wires, the phænomena of any one of the digits may be traced out in the same manner; that is, we can find, for instance, all the places where the sun is three digits eclipsed at its rising and setting, and the tract where the sun is three digits for the time of the eclipse. The globe here used should be one which has the hours marked on the equator.

The method of tracing out the different curves was, I believe, first given by M. de la CAILLE in his Astronomy. M. du SEJOUR has given an analytical method of laying down the curves in his *Traité Analytique*. But these are matters rather of curiosity, than of any real use in Astronomy. If we place the circle *ETCV* perpendicular to the horizon, and vertical to *S* a strong lamp be fixed in the principal focus of a double convex lens, so that the rays may be thrown parallel upon the globe and perpendicular to *ETCV*, the shadow of the penumbra will give the points of projection required, instead of the plumb line. Thus we make a common globe answer the purpose of an Eclipsareon, invented by Mr. FERGUSON, and described in his Astronomy.

585. As there are not many persons who have an opportunity of seeing a total eclipse of the sun, we shall here give the phænomena which attended that on April 22, 1715. Captain STANNYAN, at Bern in Switzerland, says, "the sun was totally dark for four minutes and an half; that a fixed star and planet appeared very bright; and that its getting out of the eclipse was preceded by a blood-red streak of light, from its left limb, which continued not longer than six or seven seconds of time; then part of the sun's disc appeared, all on a sud

den, as bright as *Venus* was ever seen in the night; nay brighter, and in that very instant gave a light and shadow to things, as strong as moon light used to do." The inference drawn from these phænomena, is, that the moon has an atmosphere.

J. C. Facis, at Geneva, says, "there was seen, during the whole time of the total immersion, a whiteness, which seemed to break out from behind the moon, and to encompass it on all sides equally; its breadth was not the twelfth part of the moon's diameter. *Venus*, *Saturn*, and *Mercury* were seen by many; and if the sky had been clear, many more stars might have been seen, and with them *Jupiter* and *Mars*. Some gentlewomen in the country saw more than 16 stars; and many people on the mountains saw the sky starry, in some places where it was not overcast, as during the night at the time of the full moon. The duration of the total darkness was three minutes."

Dr. J. J. Scheuchzer, at Zurich, says, that "both planets and fixed stars were seen; the birds went to roost; the bats came out of their holes; and the fishes swam about; we experienced a manifest sense of cold; and the dew fell upon the grass. The total darkness lasted four minutes."

Dr. Halley*, who observed this eclipse at London, has thus given the phænomena attending it. "It was *universally* observed, that when the last part of the sun remained on its east side, it grew very faint, and was easily supportable to the naked eye, even through the telescope, for above a minute of time before the total darkness; whereas on the contrary, my eye could not endure the splendor of the emerging beams in the telescope from the first moment. To this perhaps two causes concurred; the one, that the pupil of the eye did necessarily dilate itself during the darkness, which before had been much contracted by looking on the sun. The other, that the eastern parts of the moon, having been heated with a day near as long as thirty of ours, must of necessity have that part of its atmosphere replete with vapours, raised by the long continued action of the sun; and by consequence, it was more dense near the moon's surface, and more capable of obstructing the lustre of the sun's beams. Whereas at the same time the western edge of the moon had suffered as long a night, during which time there might fall in dews, all the vapours that were raised in the preceding long day; and for that reason, that part of its atmosphere might be seen much more pure and transparent.

* The Dr. begins his account thus: Though it be certain from the principles of Astronomy, that there happens necessarily a central eclipse of the sun, in some part or other of the terraqueous globe, about twenty-eight times in each period of eighteen years, and that of these, no less than eight do pass over the parallel of London, three of which eight are total with continuance; yet from the great variety of the elements, whereof the *calculus* of eclipses consists, it has so happened, that since March 20, 1140, I cannot find that there has been a total eclipse of the sun seen at London, though in the mean time the shade of the moon has often passed over other parts of Great Britain.

"About two minutes before the total immersion, the remaining part of the sun was reduced to a very fine horn, whose extremities seemed to lose their acuteness, and to become round like stars. And for the space of about a quarter of a minute, a small piece of the southern horn of the eclipse seemed to be cut off from the rest by a good interval, and appeared like an oblong star rounded at both ends; which appearance could proceed from no other cause, but the inequalities of the moon's surface, there being some elevated parts thereof near the moon's southern pole, by which interposition, part of that exceedingly fine filament of light was intercepted.

"A few seconds before the sun was totally hid, there discovered itself round the moon a luminous ring, about a digit or perhaps a tenth part of the moon's diameter in breadth. It was of a pale whiteness, or rather pearl colour, seeming to me a little tinged with the colours of the *iris*, and to be concentric with the moon, whence I concluded it was the moon's atmosphere. But the great height of it, far exceeding that of our earth's atmosphere, and the observations of some who found the breadth of the ring to increase on the west side of the moon, as the emersion approached; together with the contrary sentiments of those, whose judgment I shall always revere, make me less confident, especially in a matter whereto I gave not all the attention requisite.

"Whatever it was, this ring appeared much brighter and whiter near the body of the moon, than at a distance from it, and its outward circumference, which was ill defined, seemed terminated only by the extreme rarity of the matter it was composed of, and in all respects resembled the appearance of an enlightened atmosphere, viewed from far; but whether it belonged to the sun or the moon, I shall not at present undertake to decide.

"During the whole time of the total eclipse, I kept my telescope constantly fixt on the moon, in order to observe, what might occur in this uncommon appearance, and I saw perpetual flashes or coruscation of light, which seemed for a moment to dart out from behind the moon, now here, now there, on all sides, but more especially on the western side, a little before the emersion; and about two or three seconds before it, on the same western side, where the sun was just coming out, a long and very narrow streak of a dusky, but strong red light, seemed to colour the dark edge of the moon, though nothing like it had been seen immediately after the emersion. But this instantly vanished upon the first appearance of the sun, as did also the aforesaid luminous ring.

"As to the degree of darkness, it was such, that one might have expected to have seen many more stars than were seen in London; the planets, *Jupiter*, *Mercury* and *Venus* were all that were seen by the gentlemen of the Society from the top of their house, where they had a free horizon; and I do not hear that any one in town saw more than *Capella* and *Aldebaran* of the fixed stars. Nor was the light of the ring round the moon capable of effacing the lustre of

the stars, for it was vastly inferior to that of the full moon, and so weak that I did not observe it cast a shade. But the under parts of the hemisphere, particularly in the south-east under the sun, had a crepuscular brightness; and all round us so much of the segment of our atmosphere as was above the horizon, and was without the cone of the moon's shadow, was more or less enlightened by the sun's beams; and its reflection gave a diffused light, which made the air seem hazy, and hindered the appearance of the stars. And that this was the real cause thereof, is manifest by the darkness being more perfect in those places near which the center of the shade past, where many more stars were seen, and in some, not less than twenty, though the light of the ring was to all alike.

"I forbear to mention the chill and damp, with which the darkness of this eclipse was attended, of which most spectators were sensible, and equally judges; or the concern that appeared in all sorts of animals, birds, beasts and fishes upon the extinction of the sun, since ourselves could not behold it without some sense of horror."

586. At an eclipse of the sun, the distance between the centers of the sun and moon may be found at any time with a micrometer thus. Let ACB be the sun, S its center, ABD the moon, M its center; take the distance AB of the horns with the micrometer; then we know Ae half that distance, and knowing SA from the Tables, we have $Se=\sqrt{SA^2-Ae^2}$; for the same reason knowing AM from the Tables, we have $Me=\sqrt{AM^2-Ae^2}$; hence, $SM=Se+eM$ is known. If we thus take SN, SQ the distances of the sun and moon at any times, and calculate the apparent motion NQ of the moon in the interval, we may find the apparent time of conjunction. M. du Sejour found it necessary to subtract $3'',5$ from the semidiameter of the sun as given in the Tables he used, in order to make his calculations agree with observations, independently of the diminution of the moon's semidiameter by inflection (580, 587). In our calculations, we have taken the semidiameter of the sun $3''$ less than that given in Mayer's Tables, according to Dr. Maskelyne's determination. The distance of the centers may also be found by measuring the breadth Cx, which taken from Cz leaves xz; hence, $MS=Mx+Sz-2xz$ is known.

587. Admitting the inflection of the rays of the sun at the moon, an eclipse will begin later and end sooner, and therefore the duration will be diminished. For if S be the sun, M the moon, T the spectator; draw Tma a tangent to the sun, and let a ray abT be inflected at b; then the eclipse does not begin till the moon's limb gets to b; whereas, without inflection, it would have begun at the line Sma; for the same reason it ends sooner. The duration however of an annular eclipse, and the breadth of the annulus is increased by this cause. This M. du Sejour has found to agree with observation.

588. Let $Nanb$ represent the orbit of the earth, $Ncnd$ the plane of the moon's orbit inclined to it; take $vN=Nw=xn=nz=17°.\ 21'$, $pN=Nr=sn=nt=11°$.

31′, then (552, 560) all the time the earth is moving from *v* to *w*, and from *x* to *z*, it is within the solar ecliptic limits, and whilst it moves from *p* to *r*, and from *s* to *t*, it is within the lunar ecliptic limits Now if a conjunction happen at or very near the node, there will be a great solar eclipse, but at the preceding opposition, the earth may not be got into the lunar ecliptic limits, and at the next opposition it may be got beyond it, hence at each node there may happen only one solar eclipse, and therefore in a year there *may* happen only two solar eclipses

There must be one conjunction happen in the time in which the earth is passing through the solar ecliptic limits, and consequently there *must* be one solar eclipse happen at each node, hence there *must* be two solar eclipses at least in a year

If an opposition happen just before the earth gets into the lunar ecliptic limit, the next opposition may not happen till the earth is got beyond the limit on the other side of the node, consequently there may not be a lunar eclipse at the node, hence, there *may not* be an eclipse of the moon in the course of a year When therefore there are only two eclipses in a year, they must be both of the sun

If there be an eclipse of the moon as soon as the earth gets within the lunar ecliptic limit, it will be got out of the limit before the next opposition, consequently there can be only one lunar eclipse at the same node But as the lunar nodes move backwards about 19° in a year, the earth may come within the lunar ecliptic limits, at the same node, a second time in the course of a year, and therefore there *may* be three lunar eclipses in a year, and there can be no more

If an eclipse of the moon happen at or very near to the node, a conjunction may happen before and after, whilst the earth is within the solar ecliptic limits, hence there may, at each node, happen two eclipses of the sun and one of the moon, and in this case, the eclipses of the sun will be small, and that of the moon large Thus when the eclipses happen at each node only once, there may be six eclipses in a year, four of which must be of the sun and two of the moon But if, as in the last case, an eclipse should happen at the return of the earth within the lunar ecliptic limits at the same node a second time within the year, there may be six eclipses, three of the sun and three of the moon

There may be seven eclipses in a year For twelve lunations are performed in 354 days, or in eleven days less time than a common year If therefore an eclipse of the sun should happen before January 11, and there be at that, and at the next node, two solar and one lunar eclipse at each node, then the twelfth lunation from the first eclipse will give a new moon within the year, and (on account of the retrograde motion of the moon's nodes) the earth may be got within the solar ecliptic limits, and there may be another solar eclipse.

Hence, when there are seven eclipses in a year, five will be of the sun and two of the moon. This is upon supposition that the first eclipse is a solar one; but if the first eclipse should be that of the moon, and very near the beginning of the year, there may be three eclipses of the moon and four of the sun.

As there are but seldom seven eclipses in a year, the mean number will be about four.

The nodes of the moon move backwards about 19° in a year, which are the earth describes in about 19 days, consequently the middle of the seasons of the eclipses happens every year about 19 days sooner than in the preceding year.

589. The ecliptic limits of the sun (560) are greater than those of the moon (552), and hence there will be more solar than lunar eclipses, in about the same proportion as the limit is greater, that is as 3 : 2 nearly. But more lunar than solar eclipses are seen at any given place, because a lunar eclipse is visible to a whole hemisphere at once; whereas a solar eclipse is visible only to a part, and therefore there is a greater probability of seeing a lunar than a solar eclipse. Since the moon is as long above the horizon as below, every spectator may expect to see half the number of lunar eclipses which happen.

To compute the Time of an Occultation of a Fixed Star *by the* Moon.

590. Find, from the precepts and Tables (given in the course of this work), the apparent longitude and latitude of the star at the time when an occultation is expected *.

591. Compute (as will be explained in the Introduction to the Tables in the third Volume) the time of the *mean* conjunction, and at that time the moon's true latitude; then (according to M. Cassini) if the difference of the latitudes of the moon and star exceed 1°. 37′, there can be no occultation, but if the difference be less than 51′, there must be one, somewhere on the earth; hence between 1°. 37′ and 51′ it is doubtful.

592. Compute the time of the true conjunction from that of the mean, from the moon's true longitude at the mean conjunction, and its horary motion. But if, from the equation of the moon's orbit, it appears that there is a considerable interval between the mean and true conjunction, it will be better to assume

* Here, as in solar eclipses, no method, previous to the calculation of the apparent place of the moon at the ecliptic conjunction, can be given, to determine whether there will be an occultation at any particular place, on account of the moon's parallax, from the course of the moon, however, we may conjecture when it is probable such a phænomenon may happen According to M. Cassini, those stars whose north and south latitudes do not exceed 6°. 36′, may suffer an occultation *somewhere* upon the earth, and if the latitude do not exceed 4°. 32′, they may be eclipsed on *any* part of the earth.

some time as near as you can conjecture to the true conjunction, at which time compute the moon's true longitude and its horary motion, and then by applying the horary motion you will get the time of the true conjunction to a great degree of accuracy; whereas from the considerable variation of the horary motion in the course of a few hours, the time of the true conjunction found in this manner from the mean, when the interval is considerable, will be subject to a proportional degree of error. If the true longitude be computed from the Tables every time it is used, there will be no occasion for this assumption of a nearer time; but by this assumption, we may either avoid a certain degree of error, or the trouble of computing the longitude again from the Tables. If at the time of the true conjunction, the moon's true latitude be computed, and the difference between its latitude and that of the star exceed 1° 19′, there can (according to M. Cassini) be no occultation; but if it be less than 1° 7′ there must be one; hence, between 1° 19′ and 1° 7′ it is doubtful. It being determined that there may be an occultation, we proceed thus to compute it.

593. Having found, at the time of the ecliptic conjunction, the moon's true longitude, its horary motion in longitude, and its true latitude, compute its horary motion in latitude; also its parallax in latitude and longitude, (as in solar eclipses,) and its semidiameter.

594. The parallax in longitude at the true time of the ecliptic conjunction shows the apparent distance of the moon from the star in longitude; and the parallax in latitude applied to the true latitude gives the apparent latitude, the difference between which and the star's latitude, gives their apparent difference of latitudes; and if this be less than the moon's semidiameter, there will probably be an occultation, in which case, we proceed thus to find the time of immersion and emersion.

595. To find nearly the time of the *apparent* conjunction, say, as the moon's horary motion : the parallax in longitude :: 1 hour : the time from the true to the apparent conjunction nearly*, which added to or subtracted from the time of the true conjunction, according as the moon is to the west or east of the nonagesimal degree, gives the time nearly of the apparent conjunction. If at the time of the true conjunction, the difference of the apparent latitudes of the moon and star should be very nearly equal to the semidiameter of the moon, it will be necessary to compute the difference at the apparent conjunction, in order to be sure whether, or not, there will be an occultation.

* This is not accurately the time, because the parallax in longitude has varied a little, and therefore it must have made a small variation of the horary motion in longitude; but great accuracy here is not necessary, as we only want the apparent time of conjunction in order to assume nearly the time of immersion and emersion.

596. It being found that there will be an occultation, we must ascertain, as nearly as possible, the beginning and end; for this purpose, the Table at the end of this subject is very useful, it was constructed and computed by the Rev. MALACHY HITCHINS, a gentleman well conversant in the theory and practice of Astronomy, who had the goodness to communicate it to me, with permission to publish it; its construction and use we shall here explain.

597. Let C be the center of the moon, LM that diameter which is parallel to the ecliptic EP, to which draw dCF perpendicular; let vw represent the path of the star behind the moon*, z the place at the apparent ecliptic conjunction, i at the immersion, and e at the emersion, assume a point r a little before the immersion, and draw rx parallel to LM, and ram, xtn, zsp perpendicular to EP, or secondaries to the ecliptic, and join Ct, Cr, and Cx; draw also zo parallel to LM, and join Co. Now the construction of the Table is to represent the value of ct corresponding to any semidiameter Cx of the moon, and to any difference xt of the apparent latitudes of the moon and star, by entering, with the former, the head of the Table, and with the latter, its side.

598. To find nearly the time of immersion and emersion, with the moon's semidiameter, and the difference Cz of the apparent latitudes at the apparent conjunction, enter the Table, and it gives zo; and as vz makes but a small angle with zo, zo is generally nearly equal to zi, and also to ze; take therefore the horary motion of the moon, and find the time of describing zo, and subtract it from and add it to the time at z, and it generally gives the time of the beginning and end sufficiently near for the purpose here wanted†.

599. By applying this Rule, let us suppose that it gives the beginning at r instead of i. The true longitude and latitude of the moon at the time of the true conjunction being known, and their horary motions, find its true latitude and longitude at the assumed time of beginning, and to that time compute the parallaxes ‡ (173) in latitude and longitude, and apply them to the true latitude

* We here represent the moon as being fixed, and the star in motion, making the same angle with the ecliptic as the apparent path of the moon does, which makes no alteration in the relative situation of the two bodies, and the representation is more simple.

† More accurately, zo should be reduced to the ecliptic, by (108) multiplying it by the secant of the apparent latitude dz, before the time of describing it is found, but this is unnecessary. Also zi and ze are not accurately equal, and the nearer z is to F the greater will be the error. But accuracy here is not necessary, and, in general, the rule will be sufficiently accurate, and always be a very useful guide for estimating the time of beginning and end.

‡ The horizontal parallax of the moon is not here diminished (as it was in solar eclipses by the sun's parallax), the star not having any parallax.

and longitude and we get the apparent latitude and longitude, the differences between which and the apparent latitude and longitude of the star, give the apparent distance of the moon from the star in latitude and longitude, or they give ar and dm, also, cos Cd app lat ☾ $\times dm = Ca$, hence, $Cr = \sqrt{Ca^2 + ar^2}$ is known, and if this be *equal* to the moon's semidiameter, the assumed is the true time of beginning, if *greater*, the occultation has not taken place, if *less*, it has

600 With the difference ra $(= tr)$ of apparent latitudes, and the moon's semidiameter Cr, take out Ct from the Tables, and we get $ta = Ca \sim Ct$, hence, sec app lat d☾ $\times ta = mn$, and from the horary motion find the time of describing mn, which added to or subtracted from the above assumed time of beginning, will give very nearly the true time of beginning

601 To this time find, as before, the apparent difference (d) of latitudes, and apparent difference (D) of longitudes at the moon, whence we get $\sqrt{D^2 + d^2} = m$, the apparent distance of the star from the moon's center, if this distance be *equal* to the moon's semidiameter, this second assumed time is the time of the immersion, if *greater*, the occultation has not taken place, if *less*, the immersion is past If therefore this second assumed time be not true, we proceed as in Art 572, and say, $Cr \sim m$ $Cr \sim m$ the interval of the assumed times the interval between the second assumed time and the time of the immersion, this interval therefore applied to the second assumed time, gives the time of the *Immersion*

The time of the *Emersion* is found exactly in the same manner

EXAMPLE

To find the Time of the Occultation of Aldebaran *by the Moon, on January 2, 1795, at Greenwich*

The apparent longitude of *Aldebaran* on January 2, 1795, is found to be 2s. 6° 55′ 35″, and its latitude 5° 28′ 50″ south

The time of the *mean* conjunction is at 9*h* 9′ 53″, at which time the difference of the moon's and star's latitudes is 1°. 1′, consequently (591) there may be an occultation But from the equation of the moon's orbit, the difference of the times of the mean and true conjunction will probably happen five hours sooner; let us therefore assume 4*h*. at which time the moon's true longitude is found to be 2s 6° 47′ 45″, and its horary motion 35′ 39″, hence, the time of the true ecliptic conjunction of the moon and Aldebaran is found to be 2*d*, 4*h* 13′ 21″ mean time, from which subtract 4′ 41″ the equation of time, and

we get the true ecliptic conjunction at $4h.\ 8'.\ 40''$ apparent time, at which time the moon's true longitude is $2^s.\ 6^\circ.\ 55'.\ 35''$.

At the time of the true conjunction, the moon's true latitude is found by calculation to be $4^\circ.\ 36'$ south, and its horary motion in latitude is $1'.\ 35''$ decreasing; also, the moon's horizontal parallax at Greenwich is found to be $59'.\ 16''$, and hence (173) the parallax in longitude is $29'.\ 17''$ *additive* to the true to get the apparent longitude, the star being to the east of the nonagesimal degree, also the parallax in latitude is $48'.\ 4''$, increasing the true latitude. The horizontal semidiameter of the moon is $16'.\ 11''$, which increased by $5''$, the augmentation of the semidiameter on account of the moon's altitude, gives $16'.\ 16''$ for the apparent semidiameter.

The parallax $29'.\ 17''$ in longitude (at the time of the true conjunction) shows (594) the moon's apparent distance from the star in longitude, and the parallax $48'.\ 4''$ in latitude applied to the true latitude $4^\circ.\ 36'$. gives $5^\circ.\ 24'.\ 4''$ for the moon's apparent latitude, which differs from $5^\circ.\ 28'.\ 50''$, the star's latitude, by $4'.\ 46''$, which being less than $16'.\ 16''$ the moon's apparent semidiameter, there will be an occultation.

To find (595) nearly the time of the apparent conjunction, say, $35'.\ 19'' : 29'.\ 17'' :: 1$ hour . $49'.\ 18''$ the time nearly between the true and the apparent conjunction; and as the moon is to the *east* of the nonagesimal degree, this subtracted from the time $4h.\ 8'.\ 40'$ of the true conjunction leaves $3h.\ 19'.\ 22''$ for the time of the apparent conjunction nearly.

With the moon's semidiameter $16'.\ 16''$, and the difference $4'.\ 46''$ of the star's and moon's apparent latitudes, enter the Table, and it gives $zo = 15'.\ 32''$; hence $35'.\ 39'' : 15'.\ 32'' :: 1$ hour : $26'.\ 9''$, which subtracted from and added to $3h.\ 19'.\ 22''$, give $2h.\ 53'.\ 13''$ for the beginning, and $3h.\ 45'.\ 31''$ for the end, nearly.

The true longitude of the moon at $4h.\ 8'.\ 40''$ being $2^s.\ 6^\circ.\ 55'.\ 35''$, and the horary motion $35'.\ 39''$, the true longitude at $2h.\ 53'.\ 13''$ is $2^s.\ 6^\circ.\ 10'.\ 44''$, and the parallax in longitude is $28'.\ 43''$; hence, the apparent longitude at that time is $2^s.\ 6^\circ.\ 39'.\ 27''$, which subtracted from $2^s.\ 6^\circ.\ 55'.\ 35''$ the star's longitude, gives $16'.\ 8'' = dm$ for the apparent difference of longitudes of the moon and star, which multiplied by 0,9955 (the cosine of the moon's apparent latitude) gives $16'.\ 4'' = 964'' = Ca$. Also, the moon's true latitude at $4h.\ 8'.\ 40''$ is $4^\circ.\ 36'$, and the horary motion in latitude being $1'.\ 35''$ decreasing, the true latitude at $2h.\ 53'.\ 13''$ is $4^\circ.\ 37'.\ 55''$, and the parallax in latitude being $51'.\ 27''$, the apparent latitude is $5^\circ.\ 29'.\ 22''$ which differs from $5^\circ.\ 28'.\ 50''$, the star's latitude, by $32'' = ra$; hence, $\sqrt{964^2 + 32^2} = 965'' = 16'.\ 5'' = cr$, which is less than $16'.\ 16''$ the moon's apparent semidiameter by $11''$; therefore the occultation at this time must have taken place.

With 32″ the difference of the apparent latitudes and 16′ 16′ the moon's semidiameter, enter the Table, and we get 16′ 15′, the difference between which and Ca (=16′ 4″) is la=11″*, the time of describing which is 22″, which subtracted from $2h$ 53′ 13″ gives $2h$ 52′ 51″ for the next assumed time of immersion

At this time we find (exactly as we found the same for the first assumed time) the difference Ca of the moon's apparent longitude and that of the star, at the moon, to be 16′ 18″=978′, and the difference ra of the apparent latitudes to be 33′, hence, $\sqrt{978^2+33^2}=979$ =16 19′=Cr, which is greater than 16 16′ by 3′, consequently the occultation has not taken place

Hence (601), 11′+3′=11′ 3′ 22″ 5, which added to $2h$ 52 51′ gives $2h$ 52′ 56′ for the time of *Immersion*

At $3h$ 45′ 31″, the assumed time of the emersion, compute as before the apparent longitude and latitude of the moon, and we find Ca=15′ 44″=944″ (a now lying on the other side of C), and ar=2′ 59 =179′, hence, $\sqrt{944^2+179^2}$ =961″=16′ 1″, which is less than 16′ 16′ by 15′, consequently the emersion is not yet arrived

With 179″ the apparent difference of latitudes, and 16′ 16″ the moon's semidiameter, enter the Table, and we get 16, the difference between which and Ca=15′ 44″ is 16″, the time of describing which is 32′, which added to $3h$ 45′ 31″ gives $3h$ 46 3′ for the next assumed time of emersion

At this time, compute, as before, the apparent longitude and latitude of the moon, and we find Ca=16′ 3″=963″, and ar=2′ 59″=179″, hence, $\sqrt{963^2+179^2}$ =977″=16′ 17″, which is greater than 16′ 16″ by 1″, consequently the emersion has taken place

Hence (601), 15″+1″=16″ 1″ 32′ 2′, which subtracted from $3h$ 46′ 3″ leaves $3h$ 46′ 1″ for the time of *Emersion*

Hence, the apparent times at Greenwich, are,

Immersion	2^h	52′	56′
Emersion	3	46	1
Duration	0	53	5

The times thus calculated must be subject to the error of the Tables, as in solar eclipses, but in somewhat a less degree, as the horary motion of the moon

* This quantity is so small, that if we were to reduce it to the ecliptic before we found the time of describing it, it would make no sensible difference, the reduction is therefore here unnecessary, as it is also in finding nearly the time of the emersion

in respect to the star is greater than that in respect to the sun. Hence, the computed times compared with the times by observation, afford the means of correcting the Tables.

The immersion at *i* is about 30″ north of the moon's center, and the emersion at *e*, is about 3′ north.

To determine by Construction, *the Time of an Occultation of a Fixed Star by the Moon.*

602. The moon's latitude and longitude being computed for the true time of conjunction, and the horary motion of the moon in latitude and longitude, find the latitude and longitude for one hour before or after, according as the occultation happens before or after. Take also the star's latitude and declination, and find (105) the time of passing the meridian; find also the moon's semidiameter and horizontal parallax. Now it is manifest, that the star may be used as the sun, only instead of 12 upon the ellipse we must put the hour of the star's passage over the meridian; and as the star has no parallax, the radius of projection will be equal to the moon's horizontal parallax. Hence, with that radius describe the semicircle *EGC*, erect *GO* perpendicular to *EC*, find (536) the position of the pole *P*, and describe the ellipse for the latitude of the place, and declination of the star; and to find the moon's orbit, take *Ov* equal to the difference of the moon and star's latitude at the time of the true conjunction, and take also *Ox* equal to the moon's horary motion in longitude, and draw *xy* perpendicular to *EC* and equal to the difference of the moon's and star's latitude at one hour from conjunction, and the straight line *MyvL* will represent the moon's orbit. Then with an extent of compass equal to the moon's semidiameter, find two points *c* and *d* marked the same instant, and it gives the time of immersion; find also two other points *s* and *t* denoting the same point of time, and you have the time of emersion; also if the nearest distance *wc* of the corresponding points of time be taken and measured upon the scale, it will give the nearest distance of the star to the moon's center in the time of occultation.

Ex. To construct the occultation which we before computed. The time of conjunction was at 4*h*. 8′. 40″ in the morning, in longitude 2ˢ. 6°. 55′. 35″; the moon's latitude was 4°. 36′ S. its horary motion in longitude 35′. 39″, and in latitude 1′. 35″ decreasing; its semidiameter was 16′. 16″, and horizontal parallax 59′. 16″; also, *Aldebàran's* latitude was 5°. 28′. 50″ south, its declination 16°. 5′. 6″, and it passed the meridian at 9*h*. 29′. 24″ in the afternoon. Hence, take *OC* = 59°. 16′, find (536) the pole *P* at the given time, and describe the ellipse

for the latitude of Greenwich and the star's declination, and at the point n mark $9h$ 29 (neglecting the 24″), being the time when the star comes to the meridian, and from it divide the ellipse into hours, and subdivide them as far as you can conveniently. Take $Ov = 52'$ $50''$ the difference of the star's and moon's latitude at the time of conjunction, and $Oi = 35'$ $39''$ the moon's horary motion in longitude, and erect the perpendicular $ay = 51'$ $15'$ (the moon's latitude decreasing 1′ 35′ in an hour), draw the right line $LvyM$, and it will represent the moon's orbit, and vy its horary motion in it; at the point v mark $4h$ 9′, and vy representing one hour, divide LM into hours from it, and subdivide it as far as the scale will permit; and taking an extent of compass equal to 16 16′ the moon's semidiameter, it will give the points c, d corresponding to the time of *Immersion*, and the points s, t corresponding to the time of *Emersion*; and the corresponding nearest distance is found to be nearly 2′ north of the moon's center.

603. When an occultation of a star by the moon takes place, for three or four seconds of time before the star disappears, it sometimes appears to be projected upon the disc of the moon; M. du SEJOUR explains the phænomenon thus. Let S be the star, bmn the moon, abc the passage of a ray of light through the moon's atmosphere, and just passing by the limb of the moon at b; let cE be the direction of the ray after it emerges from the atmosphere, and produce Ec to s. Then to an eye at E the star would appear at s; but at the same time a ray of light from the moon's limb at b would be refracted through bc and then move to E and appear also at s; thus when the ray of light which comes from the star becomes a tangent to the moon, the star at that time *appears* also to be in contact with the moon. The refraction of the atmosphere alone therefore is not sufficient to account for this phænomenon, as some Astronomers have supposed. But if the light from the star suffer a different degree of refraction from the solar light refracted from d, for instance, if the star be higher than the center of the moon, and the refraction of the light from the moon be greater than the refraction of the light from the star, the point b being elevated by refraction more than the star, the star will appear upon the moon's disc before the occultation takes place. Or the same would happen if the star were lower than the moon's center, and the refraction of the light from the star the greater. From the different colours of the light from different stars, he thinks we may admit different degrees of refraction of their light. The irradiation of the light of the stars, by which some have conjectured they might appear to encroach a little upon the moon's limb before they disappear, would, he observes, affect all stars and at all altitudes, whereas this circumstance does not always take place. He states however this objection to his hypothesis, that the aberration of light from all the stars appears, from observation, to be the same, and

FIG. 147

therefore the velocities of their light must be all equal; consequently the light from all the stars suffers the same refraction, admitting that the refraction depends altogether on the velocity of the light. But there have been only a few stars whose aberrations have been determined by observation, and we are not assured but, in some stars, a difference in their aberrations might be found; future Astronomers may settle this.

604. Lunar eclipses are useful for finding the longitudes of places; solar eclipses, and occultations are useful for the same purpose, and also for correcting the errors on the lunar Tables. All those things will be explained when we treat on the methods of finding the longitude.

A TABLE

Showing the visible Difference of Latitudes between the Moon and Star, at the Star, at the instant of the Star's Immersion or Emersion in Occultations

Vis. diff. of lat. of ☾ and ✱		THE SEMIDIAMETERS OF THE MOON																									
M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S
0	0	14	40	14	50	15	0	15	10	15	20	15	30	15	40	15	50	16	0	16	10	16	20	16	30	16	40
1	0	14	38	14	48	14	58	15	8	15	18	15	28	15	38	15	48	15	58	16	8	16	18	16	28	16	38
2	0	14	32	14	42	14	52	15	2	15	12	15	22	15	32	15	42	15	52	16	3	16	13	16	23	16	33
3	0	14	22	14	32	14	42	14	52	15	2	15	12	15	22	15	32	15	43	15	54	16	4	16	14	16	24
4	0	14	7	14	17	14	27	14	38	14	48	14	58	15	9	15	19	15	29	15	40	15	50	16	1	16	11
4	30	13	58	14	8	14	18	14	29	14	39	14	50	15	0	15	10	15	21	15	31	15	42	15	52	16	3
5	0	13	47	13	58	14	9	14	19	14	30	14	40	14	51	15	1	15	12	15	22	15	33	15	43	15	54
5	20	13	40	13	51	14	2	14	12	14	23	14	34	14	44	14	55	15	5	15	16	15	26	15	37	15	47
5	40	13	32	13	43	13	54	14	4	14	15	14	26	14	36	14	47	14	58	15	9	15	19	15	30	15	40
6	0	13	23	13	34	13	45	13	56	14	7	14	18	14	29	14	39	14	50	15	1	15	12	15	23	15	33
6	20	13	14	13	25	13	36	13	47	13	58	14	9	14	20	14	31	14	42	14	52	15	3	15	14	15	25
6	40	13	4	13	15	13	26	13	37	13	48	14	0	14	11	14	22	14	33	14	44	14	55	15	6	15	17
7	0	12	54	13	5	13	16	13	27	13	35	13	50	14	1	14	12	14	23	14	34	14	45	14	56	15	7
7	20	12	42	12	54	13	5	13	16	13	27	13	39	13	50	14	2	14	13	14	24	14	35	14	46	14	57
7	40	12	30	12	42	12	54	13	5	13	16	13	28	13	39	13	51	14	2	14	14	14	25	14	36	14	47
8	0	12	18	12	30	12	42	12	54	13	5	13	17	13	28	13	40	13	51	14	3	14	14	14	26	14	37
8	15	12	8	12	20	12	32	12	44	12	55	13	7	13	19	13	31	13	42	13	54	14	5	14	17	14	29
8	30	11	57	12	9	12	21	12	34	12	45	12	57	13	9	13	21	13	33	13	45	13	56	14	8	14	20
8	45	11	46	11	58	12	11	12	23	12	35	12	47	13	0	13	12	13	24	13	36	13	47	13	59	14	11
9	0	11	35	11	47	12	0	12	12	12	25	12	37	12	49	13	2	13	14	13	26	13	38	13	50	14	2
9	15	11	23	11	35	11	48	12	1	12	14	12	26	12	39	12	51	13	3	13	15	13	27	13	40	13	52
9	30	11	10	11	23	11	36	11	49	12	2	12	15	12	28	12	40	12	52	13	4	13	16	13	29	13	41
9	45	10	57	11	11	11	24	11	37	11	50	12	3	12	16	12	29	12	41	12	53	13	5	13	18	13	31
10	0	10	44	10	57	11	11	11	24	11	38	11	51	12	4	12	17	12	29	12	42	12	54	13	7	13	20
10	15	10	29	10	43	10	57	11	11	11	25	11	38	11	51	12	4	12	17	12	30	12	43	12	56	13	9
10	30	10	14	10	29	10	43	10	57	11	11	11	24	11	38	11	51	12	4	12	18	12	31	12	44	12	57
10	45	9	59	10	13	10	28	10	42	10	56	11	10	11	24	11	37	11	51	12	5	12	18	12	31	12	44
11	0	9	42	9	57	10	12	10	27	10	41	10	55	11	0	11	23	11	37	11	51	12	4	12	18	12	31

THE TABLE CONTINUED.

i diff of lat of ☾ and ✱	*THE SEMIDIAMETERS OF THE MOON.*												
M S.	M S	M S	M S	M S	M S	M S	M S	M S.	M S	M S	M S	M. S	M S
0. 0	14. 40	14. 50	15 0	15 10	15 20	15 30	15. 40	15 50	16 0	16 10	16 20	16 30	16 40
11 15	9 25	9 40	9 55	10 11	10 25	10 40	10 54	11 8	11 22	11 37	11 50	12. 4	12 18
11 30	9 6	9 22	9 38	9 54	10 9	10 24	10 39	10 53	11 7	11 22	11. 36	11 50	12. 4
11. 45	8 46	9 3	9 19	9 36	9 51	10 7	10. 22	10 37	10 51	11 6	11 21	11. 35	11. 49
12 0	8 26	8. 43	9 0	9 17	9 33	9 49	10 5	10 20	10 35	10. 50	11 5	11. 19	11 33
12. 15	8 4	8 22	8 39	8 57	9 13	9 30	9 46	10 2	10 17	10 33	10. 48	11. 3	11. 17
12 30	7 40	7 59	8. 17	8 35	8 53	9 10	9 27	9. 43	9 59	10 15	10 31	10 46	11 1
12 45	7. 14	7 35	7. 54	8 13	8 31	8 49	9. 7	9. 23	9 40	9 57	10. 13	10 29	10 44
13 0	6. 47	7 9	7. 29	7 49	8 8	8 27	8 45	9 2	9 20	9. 37	9 54	10 10	10 26
13. 15	6 17	6 40	7. 2	7. 23	7 43	8 3	8 22	8. 40	8. 58	9 16	9. 33	9. 50	10. 6
13 30	5 44	6 9	6 32	6 55	7. 16	7 37	7. 57	8. 16	8 35	8 53	9. 11	9 29	9 46
13. 45	5 6	5 34	6. 0	6 24	6 47	7 9	7 30	7 51	8 11	8. 30	8 49	9. 7	9 25
14 0	4 22	4. 54	5 23	5 49	6 15	6 39	7 3	7 25	7. 45	8 5	8 25	8 44	9 3
14 10	3 48	4 24	4 56	5 25	5 52	6 17	6 41	7 4	7 26	7 47	8. 8	8 28	8. 47
14. 20	3 7	3 49	4 25	4 57	5 27	5 51	6 19	6 44	7 7	7. 29	7 50	8. 10	8. 30
14 30	2. 12	3. 8	3. 50	4 27	4 59	5 29	5 56	6 22	6 46	7 9	7 31	7 52	8 13
14 40	0. 0	2 13	3 9	3 52	4 28	5 1	5. 30	5. 58	6 24	6 48	7 11	7. 34	7 55
14. 50		0. 0	2 14	3. 10	3 53	4 30	5 2	5 32	6 0	6. 26	6 50	7 14	7 36
15 0			0 0	2 15	3 11	3 54	4 31	5. 4	5 34	6 2	6 28	6 52	7 16
15. 10				0 0	2. 15	3 12	3 56	4 33	5 6	5 36	6 4	6. 30	6. 55
15 20					0 0	2 16	3. 13	3 57	4. 34	5 7	5 38	6 6	6. 32
15. 30						0. 0	2. 17	3 14	3. 58	4 36	5 9	5 39	6 8
15 40							0. 0	2 17	3. 15	3 59	4 37	5 11	5. 41
15. 50								0 0	2. 18	3 16	4 1	4 39	5. 12
16 0									0. 0	2 19	3. 17	4 2	4. 40
16 10										0 0	2. 20	3 18	4. 3
16 20											0 0	2 20	3. 19
16 30												0. 0	2. 21
16. 40													0. 0

605 Sometimes the planets are eclipsed by the moon, the calculations of which are made in the same manner as for the sun, or a fixed star, considering the relative horary motion of the moon in respect to the planet, in latitude and longitude, in order to get the relative orbit

606 The planets sometimes eclipse the planets *Mars* eclipsed *Jupiter*, January 9, 1501, *Venus* eclipsed *Mars*, October 3, 1590, *Mercury* was eclipsed by *Venus*, May 17, 1737

607 The fixed stars are sometimes eclipsed by the planets GASSENDUS observed *Jupiter* eclipse a fixed star in the foot of *Gemini*, December 19, 1633 Mr POUND observed *Jupiter* eclipse α of *Gemini*, November 21, 1716, the middle of the eclipse was at 19*h* 55′ *Phil Trans* No 350 In 1672, *Mars* eclipsed one of the stars in *Aquarius* *Venus* eclipsed the *Lion's Heart*, in 1574, and 1598 The fixed stars are also observed to be sometimes eclipsed by *Comets*, which are very useful observations, as they serve to ascertain very accurately the place of the comet

CHAP. XXV.

ON THE TRANSITS OF MERCURY AND VENUS OVER THE SUN'S DISC.

Art. 608. WHEN Dr. Halley was at St. Helena, whither he went for the purpose of making a catalogue of the stars in the southern hemisphere, he observed a transit of *Mercury* over the sun's disc; and, by means of a good telescope, it appeared to him that he could determine the time of the ingress and egress, without its being subject to an error of 1″; * upon which he immediately concluded, that the sun's parallax might be determined by such observations, from the difference of the times of the transit over the sun, at different places upon the earth's surface. But this difference is so small in Mercury, that it would render the conclusion subject to a great degree of inaccuracy; in Venus however, whose parallax is nearly four times as great as that of the sun, there will be a very considerable difference between the times of the transits seen from different parts of the earth, by which the accuracy of the conclusion will be proportionably increased. The Dr. therefore proposed to determine the sun's parallax from the transit of Venus over the sun's disc, observed at different places on the earth; and as it was not probable that he himself should live to observe the next transits, which happened in 1761 and 1769, he very earnestly recommended the attention of them to the Astronomers who should be alive at that time. Astronomers were therefore sent from England and France to the most proper parts of the earth to observe both those transits, from the result of which, the parallax has been determined to a very great degree of accuracy.

609. Kepler was the first person who predicted the transits of Venus and Mercury over the sun's disc, he foretold the transit of Mercury in 1631, and the transits of Venus in 1631 and 1761. The first time *Venus* was ever seen upon the sun was in the year 1639, on November 24, at Hoole near Liverpool, by our countryman Mr. Horrox, who was educated at Emanuel College in this University. He was employed in calculating an Ephemeris from the *Lansberge* Tables, which gave, at the conjunction of Venus with the sun on

* Hence Dr Halley concluded, that by a transit of *Venus*, the sun's distance might be determined with certainty to the 500th part of the whole, but the observations upon the transits which happened in 1761 and 1769, showed that the time of contact of the limbs of the Sun and Venus could not be determined to that degree of certainty

that day, its apparent latitude less than the semidiameter of the sun But as these Tables had so often deceived him, he consulted the Tables constructed by KEPLER, according to which, the conjunction would be at $8h$ 1′ A M at Manchester, and the planet's latitude 14′ 10′ south, but, from his own corrections, he expected it to happen at $3h$ 57′ P M with 10′ south latitude He accordingly gave this information to his friend Mr CRABTREE at Manchester, desiring him to observe it, and he himself also prepared to make observations upon it, by transmitting the sun's image through a telescope into a dark chamber He described a circle of about six inches diameter, and divided the circumference into 360°, and the diameter into 120 equal parts, and caused the sun's image to fill up the circle He began to observe on the 23d, and repeated his observations on the 24th till one o'clock, when he was unfortunately called away by business, but returning at 15′ after three o'clock, he had the satisfaction of seeing Venus upon the sun's disc, just wholly entered on the left side, so that the limbs perfectly coincided At 35′ after three, he found the distance of Venus from the sun's center to be 13′ 30″, and at 45 after three, he found it to be 13′, and the sun setting at 50′ after 3 o'clock, put an end to his observations From these observations, Mr HORROX endeavoured to correct some of the elements of the orbit of Venus He found Venus had entered upon the disc at about 62° 30′ from the vertex towards the right on the image, which, by the telescope, was inverted He measured the diameter of Venus, and found it to be to that of the sun, as 1,12 30, as near as he could measure Mr CRABTREE, on account of the clouds, got only one sight of Venus, which was at $3h$ 45′ Mr HORROX* wrote a Treatise, entitled *Venus in Sole visa*, but did not live to publish it, it was however afterwards published by HEVELIUS. GASSENDUS observed the transit of *Mercury* which happened on November 7, 1631, and this was the first which had ever been observed, he made his observations in the same manner that HORROX did after him Since his time, several transits of Mercury have been observed, as they frequently happen, whereas only two transits of Venus have happened since the time of HORROX. If we know the time of the transit at one node, we can determine, in the following manner, when they will probably happen again at the same node

610 The mean time from conjunction to conjunction of Venus or Mercury being known (324), and the time of one mean conjunction, we shall know the time of all the future mean conjunctions, observe therefore those which happen near to the node, and compute the geocentric latitude of the planet at the

* The difficulties which this very extraordinary person had to encounter with in his astronomical pursuits, he himself has described, in a *Prolegomena* prefixed to his *Opera Posthuma*, published by Dr WALLIS

time of conjunction, in which case, if it be less than the apparent semidiameter of the sun, there will be a transit of the planet over the sun's disc; and we may determine the periods when such conjunctions happen, in the following manner. Let P = the periodic time of the earth, p that of Venus or Mercury. Now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position in respect to the node Let the earth perform x revolutions whilst the planet performs y revolutions; then will $Px=py$, therefore $\frac{x}{y}=\frac{p}{P}$. Now $P=365{,}256$, and for *Mercury*, $p=87{,}968$; therefore $\frac{x}{y}=\frac{p}{P}=\frac{87{,}968}{365{,}256}$ = (by resolving it into its continual fractions) $\frac{1}{4}$, $\frac{6}{25}$, $\frac{7}{29}$, $\frac{13}{54}$, $\frac{33}{137}$, $\frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a state of equality, the further you go. The first period, or that of one year, is not sufficiently exact; the period of six years will sometimes bring on a return of the transit at the *same* node; that of seven years more frequently; that of 13 years still more frequently, and so on. Now there was a transit of Mercury at its descending node, in May 1786; hence, by continually adding 6, 7, 13, 33, 46, &c. to it, you get all the years when the transit may be expected to happen at that node. In 1789 there was a transit at the ascending node, and therefore by adding the same numbers to that year you will get the years in which the transits may be expected to happen at that node. The next transits at the descending node will happen in 1799, 1832, 1845, 1878, 1891; and at the ascending node, in 1802, 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894. For *Venus*, $p=224{,}7$; hence $\frac{x}{y}=\frac{p}{P}=\frac{224{,}7}{365{,}256}=\frac{8}{13}$, $\frac{235}{382}$, $\frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The transits at the same node will therefore sometimes return at 8 years, but oftener in 235, and still oftener in 713, &c. Now in 1769 a transit happened at the descending node in June, and the next transits at the same node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741 and 2984. In 1639 a transit happened at the ascending node in November, and the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846 and 2854. These transits are found to happen, by continually adding the periods, and finding the years when they may be expected, and then computing, for each time, the shortest geocentric distance of Venus from the sun's center at the time of conjunction, and if it be less than the semidiameter of the sun, there will be a transit.

To compute the Time of the Transit of Venus *or* Mercury *over the Sun's disc, and the Duration thereof, to a Spectator at the center of the Earth*

611 Let A and P be two planets in conjunction, Pa, AQ, their cotemporary motions parallel to the ecliptic, and ab, QR perpendicular to it, then Pb, AR will be their real motions, take $Ar = Pa$, and $rs = ab$ and perpendicular to AQ, draw st parallel to AQ, and join sR, then sR will be the relative motion of A seen from P For their relative motions in any directions must always be the sum or difference of their real motions in those directions, according as they move in different or in the same directions In this figure, we have supposed them to move in the same direction, and Rt is the difference of their real motions in latitude, and ts in longitude, and by Trigonometry, ts Rt rad tan Rst, the inclination of the relative orbit described by A, also, cos Rst rad st sR the cotemporary motion in the relative orbit If we apply this to the sun and Venus, and AQ represent the horary motion of Venus in longitude, Ar that of the sun, and QR the horary motion of Venus in latitude, then the sun having no motion in latitude, rR is the relative motion of Venus in respect to the sun, hence, Qr QR rad tan RrQ, and cos RrQ rad rQ Rr FIG 148

Ex On July 3, 1769, at mean noon at Greenwich, the longitude of the earth was 8^s 13° 3′ 14″,8, and that of Venus 8^s 12° 47′ 53″,7, the difference of which is 15′ 21″, the horary motion of the sun (by the Tables) was 143″,46, the distance of Venus from the sun was 0,72626, hence (234), the horary motion of Venus in longitude was 237″,96, therefore the difference 94″,5 is the relative horary motion of Venus in respect to the earth in longitude, hence, 94″,5 15′ 21″ 1 hour 9h 44′ 45″, therefore the conjunction was on June 3, at 9h 44′ 45″ mean time, at which time the longitude of the earth was 8^s 13° 27′ 16″ The heliocentric latitude of Venus was also found (by the Tables) to be 6′ 27″ north decreasing, and its horary motion in latitude 14″,06, hence, 1 hour 9h 44′ 45″ 14″,06 2 17″, which subtracted from 6′ 27″ leaves 4′ 10″ the heliocentric latitude of Venus at the time of the ecliptic conjunction The distance of the earth from the sun was 1,01521, therefore the distance of Venus from the earth was 0,28895, the mean distance of the earth from the sun being unity, hence, 0,28895 0,72626 4′ 10″ 10′ 8″ the geocentric latitude of Venus at the ecliptic conjunction, which being less than the semidiameter of the sun, there must be a transit of Venus over the sun at the center of the earth, and consequently somewhere upon the surface, we have therefore, in this case, no occasion to compute the shortest distance of Venus from the center of the sun, in order to determine whether there will be a transit Also,

0,28895 : 0,72626 :: 94″,5 : 3′. 57″,52 the geocentric horary motion of Venus from the sun in longitude; and 0,28895 : 0,72626 :: 14″,06 : 35″,42 the geocentric horary motion of Venus in latitude.

Now let the circle *DOK* represent the sun, *C* its center, *DCN* the ecliptic; and on *CO* (perpendicular to *DCN*) take *CV*= 10′. 8″, and *V* is the apparent place of Venus in conjunction; and let *SVN* represent the orbit of Venus as seen from the earth. Now the geocentric horary motion of Venus from the sun in longitude is 3′. 57″,52, and the geocentric horary motion of Venus in latitude is 35″,42; hence (611), 3′. 57″,52 : 35″,42 : rad. : tan. of 8°. 28′. 54″ the inclination of the relative orbit to the ecliptic; draw therefore *SVN* making the angle *CVN* = 81°. 31′. 6″, and *SVN* will be the apparent path of Venus seen from the earth. Draw *CM* perpendicular to *SE* and it will bisect it, therefore *M* is the middle of the transit. Now as *CV* = 10′. 28″, and the angle *VCM* = 8°. 28′. 54″, we have, rad. : sin. 8°. 28′. 54″ :: 10′. 28″ : *VM* = 1′. 33″.

The horary motion of Venus in its relative orbit is found (611) by saying, cosine of inclination 8°. 28′. 54″ : rad. :: 3′. 57″,52 (the difference of the horary motions of Venus and the sun seen from the earth in longitude) : 4′. 0″,15 the horary motion in its relative orbit *SN*. Hence, 4′. 0″,15 : 1′. 33″ :: 1 hour : 23′. 14″ the time of describing *VN*, which added to 9*h*. 44′. 45″, gives 10*h*. 7′. 59″ for the middle of the transit.

In the triangle *VCM*, rad. : cos. *VCM* = 8°. 28′. 54″ :: *CV* = 10′. 28″ : *CM* = 10′. 21″ the nearest distance of Venus from the sun's center; hence, in the triangle *SCM*, *CM* = 10′. 21″, and *SC* = 15′. 46″, therefore *SM* = 11′. 53″,6 to find the time of describing which, say, 4′. 0″,15 : 1 hour :: *SM* = 11′. 53″,6 : 2*h*. 58′. 24″ the time of describing *SM*, which subtracted from 10*h*. 7′ 59″, the time when Venus was at *M*, gives 7*h*. 9′. 35″ for the *Beginning*, and added, gives 13*h*. 6′. 23″ for the *End*, mean time, according to the Tables. The effect of the menstrual parallax has not been here considered, as it is in the following calculations, it being so extremely small, compared with the errors to which the Tables are subject.

In the transit of *Mercury*, the variation of its distance may be so great between the times of the ingress and egress, as sensibly to affect its geocentric motion, and thereby render it necessary to be taken into computation. M. de l'Isle, in calculating the transit of Mercury, on November 7, 1756, found that the true middle of the passage was altered 11″,5 by this circumstance, so that *DB* was performed in 23″ less time than *AD*.

A New Method of computing the Effect of Parallax, in accelerating or retarding the Time of the Beginning or End of a Transit of Venus, *or* Mercury *over the Sun's disc. By Nevil Maskelyne, D. D. F. R. S. and* Astronomer Royal.

612. The scheme here given relates particularly to the transit of *Venus* over the sun which happened in 1769. Let C represent the center of the sun LQ, FIG. 155
P the celestial north pole of the equator, S the south pole, PCS a meridian passing through the sun, Z the zenith of the place, ADB☋ the relative path of Venus, ☋ being the relative place of the descending node, A the geocentric place of Venus at the ingress, B at the egress, and D at the nearest approach to the sun's center, as seen from the earth's center, and o the apparent place of Venus at the egress to an observer whose zenith is Z; draw ouZ, and u is the *true* place of Venus when the *apparent* place is at o, and uo is the parallax in altitude of Venus from the sun, and the time of contact will be diminished by the time which Venus takes to describe uB; draw $n'o'honE$ parallel to AB, meeting ZB produced in E, and Bn, An' tangents to the circle, and let ChD be perpendicular to AB. Now the trapezium $uoEB$, on account of the smallness of its sides, may be considered as rectilinear, and from the magnitude of ZB compared with Bu, BE may be considered as parallel to uo, and consequently $uoEB$ may be considered as a parallelogram, and therefore Eo may be taken equal to Bu. Now $Eo = En \pm no$, according as E falls without or within the circle LQ of the sun's disc; and by Trigonometry, $En : EB :: \sin EBn = \cos CBZ : \sin BnE = \sin BCD = \cos CBD$; hence, $En = \frac{EB \times \cos CBZ}{\cos CBD}$, and (by EUCLID) $no = \frac{Bn^2}{no} = \frac{Bn^2}{AB}$ very nearly; but $Bn : BE :: \sin BEn = \sin ZBD : \sin BnE = \cos CBD$, therefore $Bn^2 = \frac{BE^2 \times \sin ZBD^2}{\cos CBD^2}$; hence, $no = \frac{BE^2 \times \sin ZBD^2}{AB \times \cos CBD^2}$. Put h = horizontal parallax of Venus from the sun, and (154) $BE = h \times \sin Zo = h \times \sin ZB$; hence, $uB = oE = En \pm no = \frac{h \times \sin ZB \times \cos CBZ}{\cos CBD} \pm h^2 \times \sin ZB^2 \times \frac{\sin ZBD^2}{BA \times \cos CBD^2} = h \times \sin ZB \times \cos. CBZ \times \sec. CBD \pm \frac{h^2 \times \sin ZB^2 \times \sin ZBD^2 \times \sec CBD^2}{AB}$. Put t' = the time which Venus takes, by its geocentric relative motion, to describe the space h; to find which, let m be the relative horary motion of Venus; then, $m : h :: 1$ hour $= 3600'' : t' = \frac{h \times 3600''}{m}$. Hence, to find the time of describing uB, we have, $h : h \times \sin.$

$$ZB \times \cos.\ CBZ \times \sec.\ CBD \underset{\sim}{+} \frac{h^2 \times \sin.\ ZB^2 \times \sin.\ ZBD^2 \times \sec.\ CBD^2}{AB} :: t : t \times$$

$$\sin.\ ZB \times \cos.\ CBZ \times \sec.\ CBD \underset{\sim}{+} \frac{t \times h \times \sin.\ ZB^2 \times \sin.\ ZBD^2 \times \sec.\ CBD^2}{AB}$$ the

time of describing uB, or the effect of parallax in accelerating or retarding the time of contact; the upper sign is to be used when CBZ is acute, and the lower sign when it is obtuse. If CBZ be very nearly a right angle, but obtuse, it may happen that nE may be less than no, in which case, nE is to be taken from no, according to the rule. The principal part nE of the effect of parallax will increase or diminish the planet's distance from the sun's center, according as the angle ZBC is acute or obtuse; but the small part no of parallax will always increase the planet's distance from the center; take therefore the sum or difference of the effects, with the sign of the greater, as to increasing or decreasing the planet's distance from the center of the sun. Otherwise state the rule thus: Take the sum or difference of nE and no, according as ZBC is acute or obtuse, and the distance of the planet from the sun's center will always be increased in the first case and diminished in the second, except ZBC being obtuse, and near 90°, nE shall be less than no, and then the distance from the sun's center will be increased by the difference. If ZBC be acute, the part nE will retard the ingress and accelerate the egress, but if ZBC be obtuse, the part nE will accelerate the ingress and retard the egress. In like manner, the parallax affects the time of the planet's coming to any given distance from the sun's center before or after the middle of the transit The second part of the correction will not exceed 9″ or 10″ of time in the transits of Venus in 1761 and 1769, where the nearest approach of Venus to the sun's center was about 10′. In the transits of *Mercury*, the first part alone will be sufficient, except the nearest distance be much greater.

CALCULATION. As $t \times \sec.\ CBD$ is a constant quantity for the same transit, find its logarithm, and it will be constant; and as $\frac{t \times h \times \sec.\ CBD^2}{AB}$ is also constant, find its logarithm, and you get a second constant logarithm. Then to find the first, or principal effect of parallax in time, to the *first* constant logarithm, add the log. sine of the zenith distance ZB, and the log. cosine of CBZ, and the sum is the logarithm of the first part of the effect of parallax; and to the *second* constant logarithm, add twice the log. sine of the zenith distance ZB, and twice the log. sine of ZBD, and the sum is the logarithm of the second part of the effect of parallax.

613. From the Tables of the sun's motion, the distance of the sun from the earth at the time of the transit was 1,015214, the mean distance being unity;

and the sun's horary motion was 143″,457, uncorrected by the effect of the menstrual parallax. From the Tables of the motion of Venus, the distance of Venus from the sun was 0,7262648, its mean distance being 0,72333, and its mean horary motion was 240″,325, hence (233), its true heliocentric horary motion was 238″,381.

614. To explain the effect of the menstrual parallax, let S be the sun, vw the earth's orbit, C the center of gravity of the earth E and moon M, MHV the orbit of the moon, and Eu the orbit of the earth, which each describes about their center of gravity C, whilst that center describes the orbit vw; join SC, SE, and on SE let fall the perpendicular Ca; then the angle ESC is the menstrual parallax, which at its maximum, or when CE is perpendicular to CS, is 7″,1, the sun and moon being at their mean distances; let the former be represented by unity, and let $m=$ the mean distance of the moon. Then $CE : Ca :: 7''{,}1 : CSE = 7''{,}1 \times \frac{Ca}{CE} = 7''{,}1 \times$ sin. elong. ☾ from ☉ when the sun and moon are at their mean distances; but CSE must vary inversely as the distance of the sun, hence, $CS : 1 :: 7''{,}1 \times$ sin. elong. : $CSE = 7''{,}1 \times \frac{\text{sin. elong.}}{CS}$, at any distance CS; also, by varying CE, the angle CSE must vary in proportion; but CE varies as CM; hence, $m : CM :: 7''{,}1 \times \frac{\text{sin. elong.}}{CS} : CSE = 7''{,}1 \times$ sin. elong. $\times \frac{CM}{CS \times m} =$ (because the hor. parallax of ☾ varies inversely as its distance) $7''{,}1 \times$ sin. elong. $\times \frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$, hence, the increment of this angle (the angle itself only being supposed variable) $= 7''{,}1 \times \overline{\text{elong.}} \times$ cos. elong.* $\times \frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$, but in an hour, $\overline{\text{elong.}} =$ hor. mot. ☾ − hor. mot. ☉ $= H$; hence, the horary motion of the menstrual par. of ☉ in *Longitude* $= 7''{,}1 \times H \times$ cos. elong. $\times \frac{\text{mean hor. par. ☾}}{CS \times \text{hor. par. ☾}}$.

615. To find the menstrual horary motion in latitude, let xCv represent the ecliptic, E the earth, M the moon, C their center of gravity, and Ev perpendicular to Cv; then $CE : Ev :: 7''{,}1 : 7''{,}1 \times \frac{Ev}{CE} = 7''{,}1 \times$ sin. ☾'s lat. which is the sun's latitude at the mean distances of the sun and moon; and if $h =$ the moon's horary motion in latitude, by proceeding as before we get the horary motion of the menstrual parallax in *latitude* $= 7''{,}1 \times$ cos. lat. ☾ $\times h \times$

Fig. 150

Fig. 151

* Because $\overline{\text{sin.}}$: $\overline{\text{arc}}$ :: cos. : rad. and therefore $\overline{\text{sin.}} = \overline{\text{arc}} \times$ cos. the radius being unity.

$\frac{\text{mean hor. par. ☾}}{\text{☉'s dist.} \times \text{hor. par. ☾}}$, and will be the same way as the ☾ moves in latitude.

Hence, the computation of these quantities, at the time of the transit in 1769, when the hor. par. ☾ was 61′. 28″, ☾'s hor. mot. 37′. 57″,8; ☉'s hor. mot. 2′. 23″,5, gives $H = 35'.\ 34'',3$; hor. mot. ☾ lat. = 3′. 22″,5; ☉'s dist. = 1,015214; and the mean hor. par. ☾ = 56′. 59″; also at the time of this transit, cos. elong. = 1.

7″,1	log.	0,8513
56′. 59″	log. sin.	8,2194
61. 28	co-ar. log. sin.	1,7476
35. 34,3	log. sin.	8,0148
1,015214	co-ar. log.	9,9935
0″,0671 hor. mem. par. ☉ in *Long.*		8,8266

7″,1	log.	0,8513
56′. 59″	log. sin.	8,2194
61. 28	co-ar. log. sin.	1,7476
3. 22,5	log. sin.	6,9920
1,015214	co-ar. log.	9,9935
0,0064 hor. men. par. ☉ in *Lat.*		8,8038

Hence, the sun's true horary motion in longitude is 143″,524.

616. Let C be the sun, C ☊ P ☋ a semicircle in the plane of the orbit of *Venus*, ☊ Q ☋ a semicircle in the plane of the ecliptic; P the place of Venus, and draw Pd perpendicular to ☊ C ☋, and Pe perpendicular to the plane ☊ Q ☋, and join ed; now

Rad : cos. incl. ♀'s orb. :: Pd : de :: tan. PC☋ . tan. eC☋ = tan. PC☋ × cos. incl.

Take the increments, and we get $\dot{\overline{eC☋}} = \dot{\overline{PC☋}} \times$ cos. incl. $\times \overline{\text{sec.}\ PC☋}^2 \times \overline{\text{cos.}\ eC☋}^2$, because the increment of the tangent of an angle = increm. arc × $\overline{\text{sec.}}^2$, radius being unity. Also, PC : Pd :: sin. Pde = inclin. : sin. PCe =

helioc lat of ♀, but $PC : Pd ::$ rad : sin $P☊$ or $PC☊$, therefore rad : sin. $PC☊$ :: sin. inclin : sin lat PCe = sin incl × sin $PC☊$. Take the increments, and we get $\dot{\overline{PCe}} = \dot{\overline{PC☊}} \times \cos PC☊ \times \sec PCe \times \sin 3° \; 23' \; 20''$

617 Let S be the sun, T, V, two cotemporary places of the earth and Venus, and after a small space of time let u and X be their cotemporary positions, the place u being affected by the motion of the earth about the center of gravity of the earth and moon during the intermediate time, then $uSX = TSX - TSu$ = hel mot ♀ in long − hel mot ⊖ in long including the change of the menstrual parallax, hence, uSX (the app hel mot ♀ from ⊖ in long) : SuX (the app retrog geoc mot of ♀ from ☉ in long) :: $Xu = TV$: $SX = SV$, because the angles being small, they may be taken as their sines FIG. 15[illegible]

618 Let also Te be the motion of the earth in a small space of time from T to e perpendicular to the plane of the ecliptic, and Vu the corresponding motion of Venus, where the motion Te is that about the center of gravity of the earth and moon in latitude Then the heliocentric motion of Venus from the earth in latitude = $VSu + TSe$ (the figure being adapted to the circumstances of the transit in 1769) FIG. 154

619 Let VSd be the heliocentric motion of Venus from the earth in longitude (= uSX in Art 617), Vd being perpendicular to the SV, and let dSo be the heliocentric motion of Venus from the earth in latitude (= uSe in Art 618), od being perpendicular to the ecliptic, then the hypothenuse Vo will be the apparent heliocentric path of Venus relative to the earth, supposed to be at rest, and oVd will be the angle which Venus's apparent heliocentric motion from the earth makes with the ecliptic, or which is the same, the angle which its apparent geocentric motion from the sun makes with the ecliptic Now VSd (the hel mot of ♀ from ⊖ in long) : dSo (its hel mot from ⊖ in lat) :: Vd : do :: rad : tan oVd, and Vd : Vo :: rad : sec oVd FIG. 156

620 Lastly, let S, T, V be the centers of the sun, earth, and Venus, and TA = the semidiameter of the earth, then $VA = VT$: SV :: AST (= ☉'s hor par) : $SAV = AVT - AST$ = the diff of the hor par of ☉ and ♀ FIG. 157

Calculation from Article 616

Hel long ☊ by obs at this transit	.	8ˢ	4°	35′	35″
Venus's hel long at mid of transit	.	8	13	28	13
Arg of S lat of ♀, or dist of ♀ from ☊, it being so much short of it as seen from ☉		0	1	7	22

PC☊	$= 1°. \ 7'. \ 22''$	log. tan.	8,2922268
$Pde\angle$ ☊	$= 3. \ 23. \ 20$	log. cos.	9,9992399
eC☊	$= 1. \ 7. \ 15$	log. tan.	8,2914667

Log. ratio cos. eC☊ to cos. PC☊	0,0000003
Log. ratio of the square	0,0006006
Log. cos. 3°. 23'. 20"	9,9992399
Hel. hor. mot. ♀ in long. 238",381 - log.	2,377272
Hel. hor. mot. ♀ on ecl. 237",9644 - log.	2,376512

P☊	$= 1°. \ 7'. \ 22''$	log. sin.	8,2921434
$\angle$ ☊	$= 3. \ 23. \ 20$	log. sin.	8,7716814
Pe	$= 0. \ 3. \ 59$	log. sin.	7,0638248

By this formula, $\overline{P\dot{C}e} = \overline{P\dot{C}☊} \times \cos.\ PC☊ \times \sec.\ PCe \times \sin.\ 3°.\ 23'.\ 20''.$

238",381	log.	2,377272
3°. 23'. 20"	log. sin.	8,7716814
1. 7. 22	log. cos.	9,9999166
		11,1488700
0. 3. 59	log. cos.	9,9999997
14",0887 hel. mot. of ♀ in lat.	log.	1,1488703

Hel hor mot ♀ red to ecliptic	- -	237″,9644
Hor mot ⊙, including effect menst par	-	143, 524
Hel hor mot ♀ from ⊙ in long	-	94, 4404

Calculation from Article 618, *and* 619

94″,4404	- - - -	log.	1,9751578
Hel hor mot ♀ in lat 14″,0887			
——————— ⊖ —— 0,0064			
—— —— ♀ { ⊖ —— 14,0951	-	log	1,1490682
8° 29′ 19″,17 ∠ rel orb makes with ecl	-	tan	9,1739104

8° 29′ 19″,17	- - -	co ar log cos	0,0047838
94″,4404	- - - -	log	1,9751578
95″,4864 app. hel. hor mot ♀ á ⊖	-	co ar log cos	1,9799416

Calculation from Article 617

95″,4864	- - -	log	1,9799416
TV = 0,288949	-	co-ar log	0,5391787
SV = 0,726265	- - - -	log	9,861095
240″,0023 app geoc. mot ♀ á ⊙	-	log.	2,3802153

Calculation from Article 620.

Assume the sun's mean horizontal parallax 8″,83, agreeable to what was determined from the observations of the transit in 1761, see the Precepts to MAYER's Tables, page 61 and 114.

8″,83 - - - - - -	log.	0,945961
1,015214 ⊙'s dist. from ⊖ - - -	log.	0,006515
8″,6985 hor. par. ⊙ on day of transit -	log.	0,939446
TV=0,288949 - - -	co-ar. log.	0,539179
SV=0,726265 - - - -	log.	9,861095
21″,864 ♀'s hor. par. á ⊙ during transit -	log.	1,339720

To find the apparent time taken by Venus to move over its horizontal parallax from the sun.

As 240″,0023 ♀'s hor. mot. á ⊙ mean time in rel. orb. -	co-ar. log.	7,6197848
is to 3600″ - - - - - -	log.	3,5563025
so is 1″ - - - - - -	log.	0,0000000
to 14″,99986 time ♀ takes to move 1″ from ⊙ in rel. orb. - -	log.	1,1760873
21″,864 hor. par. ♀ á ⊙ - - -	log.	1,339720
327″,95 mean time ♀ moves over its hor. par. from ⊙ - -	log.	2,515807

But 24 hours of apparent time=24*h*. 0′. 10″ of mean time; hence, to reduce the mean to apparent time,

$\frac{\text{App. time}}{\text{Mean time}}=\frac{24h.\ 0'.\ 0''}{24h.\ 0'.\ 10''}=\frac{8640}{8641}$ - -	log.	0,000050
327″,95 - - - - - -	log.	2,515807
327″,912 app. time ♀ moves over its hor. par. á ⊙, which put $=t$. -	log.	2,515757

Time of obs. of *first* int. cont. at *Wardhus*, red. to ☉'s center, by some former calculations - -	9h 40' 40" - D° *second* int. cont.	15h 23' 4"
Diff. mer. E. of Greenwich	2 4 17	2 4 17
App. times at Greenwich	7 36 23	13 18 47
☉'s declinations -	22° 25 50 N	22° 27 35 N
☉'s dist. *PC* from N. pole	67 34 10	67 32 25
☉'s dist. *SC* from S. pole	112 25 50	112 27 35
Angle betw. ecliptic and parallel to equator	7 5 20	6 59 27
App. incl. ♀'s rel. orb. on ☉ to ecliptic *C*☊*B*	8 29 19	8 29 19
Sum = ∠ bet. par. to equ. and ♀'s rel. orb. *PCD*	15 34 39	15 28 46

FIG. 158

Otherwise,

LCP ∠ of ecl. and mer. -	82° 54' 40"	83° 0' 33"
Its supp. ☊*CP* - -	97 5 20	96 59 27
☊*CD* = comp. *C*☊*D* -	81 30 41	81 30 41
Diff. = *PCD* - -	15 34 39	15 28 46

621 We shall take the difference of semidiameters of the sun and Venus with M. de la LANDE = 15' 15",1, which is what he found necessary to reconcile the total durations of the transits in 1761 and 1769 with the motion of the node of Venus's orbit in the interval, known nearly. By some calculations of this transit, we had found the chord described by Venus over the sun between the two internal contacts, reduced to the center of the earth, to be = 1368',57. Hence, the semi chord is 684',285, with which, and the difference of semidiameters of the sun and Venus 15' 15",1 above mentioned, we find the nearest approach of Venus to the sun's center, and the angle which Venus's path over the sun seen from the center of the earth makes with the radius of the sun's disc at the two internal contacts, as follows,

Calculation from Article 620.

Assume the sun's mean horizontal parallax 8",83, agreeable to what was determined from the observations of the transit in 1769, see the Precepts to MAYER's Tables, page 61 and 114.

8",83 - - - - - -		log. 0,945961
1,015214 ☉'s dist. from ♀ - - -		log. 0,006545
8",6985 hor. par. ☉ on day of transit -		log. 0,939416
TT' 0,288919 - - -	co.ar.	log. 0,539179
SI . 0,719267 - - - -		log. 9,861035
21",864 ♀'s hor. par. ☉ during transit -		log. 1,339730

To find the apparent time taken by Venus to move over its horizontal parallax from the sun.

As 240',0023 ♀'s hor. mot. ☉ mean time in rel. orb. } -	co-ar.	log. 7,6197844
is to 3600" - - - - -		log. 3,5563025
so is 1" - - - - - -		log. 0,0000000
to 14",99986 time ♀ takes to move 1" from ☉ in rel. orb. } - -		log. 1,1760873
21",864 hor. par. ♀ ☉ - - -		log. 1,339730
327",95 mean time ♀ moves over its hor. par. from ☉ } - -		log. 2,515807

But 24 hours of apparent time = 24*h.* 0'. 10" of mean time; hence, to reduce the mean to apparent time,

App. time 24*h.* 0. 0" = 86400 - -		log. 0,000050
Mean time 24*h.* 0. 10" = 86410		
327",95 - - - - - -		log. 2,515807
327",912 app. time ♀ moves over its hor. par. ☉, which put = *t*. } -		log. 2,515757

Time of obs. of *first* int. cont. at *Wardhus*, red. to ☉'s center, by some former calculations	9h 40′ 40″	D° *second* int. cont.	15h 23′ 4″
Diff. mer. E. of Greenwich	2 4 17		2 4 17
App. times at Greenwich	7 36 23		13 18 47
☉'s declinations	22° 25 50 N		22° 27 35 N
☉'s dist. *PC* from N. pole	67 34 10		67 32 25
☉'s dist. *SC* from S. pole	112 25 50		112 27 35
Angle betw. ecliptic and parallel to equator	7 5 20		6 59 27
App. incl. ♀'s rel. orb. on ☉ to ecliptic *C*☊*B*	8 29 19		8 29 19
Sum = ∠ bet. par. to equ. and ♀'s rel. orb. *PCD*	15 34 39		15 28 46

FIG. 158

Otherwise,

LCP ∠ of ecl. and mer.	82° 54′ 40″	83° 0′ 33″
Its supp. ☊*CP*	97 5 20	96 59 27
☊*CD* = comp. *C*☊*D*	81 30 41	81 30 41
Diff. = *PCD*	15 34 39	15 28 46

621 We shall take the difference of semidiameters of the sun and Venus with M. de la LANDE = 15′ 15″,1, which is what he found necessary to reconcile the total durations of the transits in 1761 and 1769 with the motion of the node of Venus's orbit in the interval, known nearly. By some calculations of this transit, we had found the chord described by Venus over the sun between the two internal contacts, reduced to the center of the earth, to be = 1368″,57. Hence, the semichord is 684′,285, with which, and the difference of semidiameters of the sun and Venus 15′ 15″,1 above mentioned, we find the nearest approach of Venus to the sun's center, and the angle which Venus's path over the sun seen from the center of the earth makes with the radius of the sun's disc at the two internal contacts, as follows,

Calculation from Article 620.

Assume the sun's mean horizontal parallax 8″,83, agreeable to what was determined from the observations of the transit in 1761; see the Precepts to MAYER's Tables, page 61 and 114.

8″,83 - - - - - -	log.	0,945961
1,015214 ⊙'s dist. from ⊖ - - -	log.	0,006515
8″,6985 hor. par. ⊙ on day of transit -	log.	0,939446
TV=0,288949 - - -	co-ar. log.	0,539179
SV =0,726265 - - - -	log.	9,861095
21″,864 ♀'s hor. par. á ⊙ during transit -	log.	1,339720

To find the apparent time taken by Venus to move over its horizontal parallax from the sun.

As 240″,0023 ♀'s hor. mot. á ⊙ mean time in rel. orb. -	co-ar. log.	7,6197848
is to 3600″ - - - - -	log.	3,5563025
so is 1″ - - - - - -	log.	0,0000000
to 14″,99986 time ♀ takes to move 1″ from ⊙ in rel. orb. - -	log.	1,1760873
21″,864 hor. par. ♀ á ⊙ - - -	log.	1,339720
327″,95 mean time ♀ moves over its hor. par. from ⊙ - -	log.	2,515807

But 24 hours of apparent time=24*h*. 0′. 10″ of mean time; hence, to reduce the mean to apparent time,

$\frac{\text{App. time}}{\text{Mean time}}=\frac{24h.\,0'.\ 0''}{24h.\,0'.\,10''}=\frac{8640}{8641}$ - -	log.	0,000050
327″,95 - - - - - -	log.	2,515807
327″,912 app. time ♀ moves over its hor. par. á ⊙, which put=t. -	log.	2,515757

Time of obs. of *first* int. cont. at *Wardhus*, red. to ☉'s center, by some former calculations	9^h	40'	40"	D° *second* int. cont.	15^h	23'	4"
Diff. mer. E. of Greenwich	2	4	17		2	4	17
App. times at Greenwich	7	36	23		13	18	47
☉'s declinations	22°	25	50 N		22°	27	35 N
☉'s dist. *PC* from N. pole	67	34	10		67	32	25
☉'s dist. *SC* from S. pole	112	25	50		112	27	35
Angle betw. ecliptic and parallel to equator	7	5	20		6	59	27
App. incl. ♀'s rel. orb. on ☉ to ecliptic *C☊B*	8	29	19		8	29	19
Sum = ∠ bet. par. to equ. and ♀'s rel. orb. *PCD*	15	34	39		15	28	46

FIG. 158.

Otherwise,

LCP ∠ of ecl. and mer.	82°	54'	40"	83°	0'	33"
Its supp. ☊*CP*	97	5	20	96	59	27
☊*CD* = comp. *C*☊*D*	81	30	41	81	30	41
Diff. = *PCD*	15	34	39	15	28	46

621. We shall take the difference of semidiameters of the sun and Venus with M. de la LANDE = 15' 15",1, which is what he found necessary to reconcile the total durations of the transits in 1761 and 1769 with the motion of the node of Venus's orbit in the interval, known nearly. By some calculations of this transit, we had found the chord described by Venus over the sun between the two internal contacts, reduced to the center of the earth, to be = 1368",57. Hence, the semi chord is 684',285, with which, and the difference of semidiameters of the sun and Venus 15' 15",1 above mentioned, we find the nearest approach of Venus to the sun's center, and the angle which Venus's path over the sun seen from the center of the earth makes with the radius of the sun's disc at the two internal contacts, as follows,

915",1 + 684",287 = 1599",387	- -	log. 3,2039541
915",1 − 684",287 = 230",813	- -	log. 2,3632630
		½ s. 5,5672171
607",587 = 10'. 7",587 = CD	- - -	2,7836085
915",1	- - - - - - -	2,9614856
41°. 36'. 8",3 = CAD = CBD	- -	log. sin. 9,8221229
l = 327",912	- - - - - -	log. 2,51576
CAD = 41°. 36'. 8"	- - -	co-ar. cos. 0,12621
First constant logarithm	- - - -	2,64199
l = 327",912	- - - - - -	log. 2,51576
2 × co-ar. cos. CAD	- - - - -	0,25246
h = 21",864	- - - - - -	log. 1,33972
		4,10794
AB = 1368",77	- - - - - -	3,13637
Second constant logarithm	- - - -	0,97157

	At 1st int. cont.		At 2d int. cont.
LCP = angle of ecl. and merid.	82°. 54'. 40"		83°. 0'. 33"
πCP its supplement -	97. 5. 20		96. 59. 27
πCD = 90°. − 8°. 29'. 19" -	− 81. 30. 41		81. 30. 41
PCD = - - -	15. 34. 39		15. 28. 46
ACD = 90° − 41°. 36'. 8" -	48. 23. 52	BCD =	48. 23. 52
Diff. = ACP = - -	32. 49. 13	- sum BCP	63. 52. 38

Now in the spherical triangle ACP, there is given AC = 14°. 11', PC = 67°. 34'. 10", and ACP = 32°. 49'. 13"; and in BCP, there is given BC = 15'. 15", PC = 67°. 34'. 10", and BCP = 63°. 52'. 38". Let Q be the point where a perpendicular from A cuts CP; and R the point where a perpendicular from B cuts CP.

$ACP = 32°. 49. 13"$ - cos 9,92417 - tan 9,80953
$AC = 0. 15 13$ - tan. 7,64700

$CQ = 0. 12. 49$ tan 7,57147 - - sin. 7,57150
$PC = 67. 34. 10$

$PQ = 67. 21 21$ - - - - co ar sin 0,03484

$CPA = 0. 8 57,1$ - - - tan 7,41587

$BCP = 64° 59' 38$ - cos. 9,64374 - - tan. 10,30946
$BC = 0. 13 15$ - tan 7,64700

$CR = 0 6 43$ - tan. 7,29074 - - sin. 7,29074
$CP = 67. 34 10$

$PR = 67 27 27$ - - - - co ar sin. 0,03452

$BPC = 0 14. 49,7$ - - - - - tan. 7,63172

Let fall the perpendicular PX from P, upon CA produced.

$ACP = 32°. 49 13"$ - cos. 9,92447 - tan. 9,80953
$CP = 67 34 10$ - tan 10,38427 - - - - cos 9,581567

$CX = 64 30 21$ - tan 10,30874 - sin 9,95307 - co-ar. cos. 0,355676

$CA = 0. 13 13$

$AX = 64 17. 8$ - - co ar. sin. 0,04788 - cos. 9,648224

$PAC = 147. 7 21,6$ - tan. 9,81048 $PA = 67° 21' 21"$ cos 9,585467

Let fall the perpendicular PY from P, upon CB produced

BCP = 63°. 52'. 38"	cos. 9,644743	tan. 10,309160		
CP = 67. 34. 10	tan. 10,384266	. . . .		cos. 9,581467
CY = 46. 50. 47	tan. 10,028011	sin. 9,863049	.	co-ar. cos. 0,164971
CB = 0. 13. 15				
BY = 46. 37. 32	. .	co-ar. sin 0,138777	.	cos. 9,837074
PBC = 116. 1. 42	. . .	tan. 10,311274	PB 66°. 37'. 28"	cos. 9,598612

To compute the effect of parallax on the first internal contact at *Wardhus*.

ZPC = 113°. 32'. 39"			
APC = 0. 8. 57			
ZPA = 113. 23. 42	.	cos. 9,601528	
Co-lat. ZP = 19. 37. 27	.	tan. 9,552118	. . cos. 9,974014
PN = 15. 38. 21	.	tan. 9,446706	. co-ar. cos. 0,017009
PA = 67. 41. 21			
AN = 83. 19. 42	. . . .		cos. 9,065130
ZA = 83. 27. 51	. . . .		cos. 9,056341
ZA			sin. 9,99717
First constant logarithm			2,64199
From the next operation, CAZ = 158°. 44'. 6" .			cos. 9,96942
Acceleration of contact, first part, = 406",07 .			2,60858

ZPA = 114° 21′ 42″	-	-	tan.	9,870872
PN = 15 58 21	-	-	sin.	9,439611
AN = 83 19 42	-		co ar. sin.	0,002951
ZAP = 11 37 11	-	-	tan	9,313434
PAC = 147 7 22				

ZAC = 158 44 33, hence $ZAC - CAD$ = 117° 8′ 58″ = ZAD

Second constant logarithm - -	0,972
log sin ZA - - - -	9,994
2 log sin ZAD - - - -	9,898
Retardation of contact, second part, = 7″,31	0,864

Hence, 806″,05 − 7″,31 = 798″,74 = 13′ 18″,74 the whole effect of parallax in accelerating the ingress.

To compute the effect on the second internal contact.

ZPC = 128° 8′ 41″				
BPC = 0 14 40				
ZPB = 127 54 1	cos 9,788979			
Co lat. PZ = 19 37 23	tan. 9,552118	-	cos.	9,974014
PX = 12 21 15	tan. 9,340191		co ar. cos	0,010174
PB = 67 27 28				
BX = 79 48 43	- - - -		cos.	9,247678
ZB = 80 10 47	- - - -		cos.	9,231866
ZB	- - - - - -		sin	9,99359
First constant logarithm	- - - -			2,64199
From the next operation, ZBC = 131° 37′ 48″	-		cos.	9,82238
Retardation of contact, first part, = 287″,05	- -			2,45796

$ZPB = 127°.54'.1"$	. .	tan.	10,108749
$PN = 12.21.15$	. . .	sin.	9,330120
$BN = 79.48.43$	. .	co-ar. sin.	0,006902
$PBZ = 13.36.6$	. . .	tan.	9,44597
$PBC = 116.1.42$			

$ZBC = 131.37.48$; hence, $ZBC - DBC = 90°.1'.40 = ZBD$.

Second constant logarithm	0,972
$2 \times$ log. sin. ZB	9,978
$2 \times$ log. sin. ZBD	0,000
Acceleration of contact, second part, $= 8",91$.	0,950

Hence, $287",05 - 8",91 = 278",14 = 4'.38",14$ the whole effect of parallax, in retarding the egress. Hence, the whole duration was lengthened 11'.16",88 by parallax.

To compute the effect of parallax on the first internal contact at *Otaheite*.

$ZSC = 33°.57'.0"$		
$CSA = 0.8.57$		
$ZSA = 34.7.57$	cos. 9,917895	
Co-lat. $ZS = 72.30.43$	tan. 10,501594	. cos. 9,477853
$SN = 69.9.41$	tan. 10,419489	. co-ar. cos. 0,448871
$SA = 112.38.39$		
$NA = 43.28.58$		cos. 9,860686
$ZA = 52.11.54$	. . .	cos. 9,787419

ZA	sin. 9,89770
First constant logarithm	9,64159
From the next operation, $ZAC = 9°.45'.37"$.	cos. 9,99367
Retardation of contact, first part, $= 341",48$ . .	2,53336

$ZSA = 34°\ 7'\ 37''$	. . . . .	tan	9,831152
$SN = 69\ 9\ 41$	. . . . .	sin	9,970619
$NA = 43\ 28\ 58$	. . . .	co ar. sin.	0,162325
$SAZ = 40\ 18\ 4$	. . . . .	tan.	9,964096
$SAC = 30\ 32\ 37$			

$ZAC = 9\ 45.\ 27$, hence $CAD - CAZ = 31°\ 50'.\ 41'' = ZAD$.

Second constant logarithm	0,9717
2 × log sin ∠ *A*	9,7954
2 × log sin ∠ *AD*	9,4446
Retardation of contact, second part, = 1",63 . .	0,2117

Hence, the ingress is retarded 341",48 + 1",63 = 343",11 = 5'. 43",11.

To compute the effect of parallax on the second internal contact.

$ZSC = 48°.\ 32'.\ 0''$			
$CSB.\ \ 0\ 14.\ 50$			
$ZSB = 48.\ 46.\ 50$	. cos. 9,818849		
Co-lat. $ZS = 72.\ 30.\ 44$	. tan 10,501594	. .	cos. 9,477855
$SN = 64\ 26.\ 44$	. tan. 10,320443	.	co ar. cos. 0,365151
$SB = 112.\ 32\ 32$			
$BN = 48.\ 5\ 48$	. . . . . .		cos. 9,824696
$ZB = 62.\ 16.\ 21$	. . . . . .		cos. 9,667702

ZB	sin. 9,94708
First constant logarithm	2,64199
From the next operation, $CBZ = 9°.\ 49'.\ 39''$ . .	cos. 9,99358
Acceleration of contact, first part, = 382",47 . .	2,58260

$ZSB = 48°. 10. 30$	. . . . .	tan.	10,057479
$SN = 61. 26. 11$	. . . . .	sin.	9,943791
$BN = 18. \ . 48$	. . . .	co-ar. sin	0,129864
$SBZ = 54. 8. 29$	. . . . .	tan.	10,141018
$SBC = 64. 58. 18$			

$CBZ = 9. 49. 39$, hence $CBA + CBZ = 31°. 25. 47 = ZBD$.

Second constant logarithm	. . . . .	0,9717
2 × log. sin. ZB	. . . .	9,8941
2 × log. sin. ZBD	. . . . . .	9,7862
Acceleration of contact, second part, $= 4'',49$	.	0,6520

Hence, the egress is accelerated $382'',47 + 4'',49 = 386'',96 = 6'. 26'',96$. Therefore the whole duration was diminished $12'. 10'',07$.

The total duration at Wardhus was lengthened by parallax $11'. 16'',88$, and diminished at Otaheite by $12'. 10'',07$; hence, the computed difference of the times is $23'. 26'',95$; but the observed difference was $23'. 10''$.

622. Hence, the correct parallax may be accurately found as follows. Because the observed difference of the total durations at Wardhus and Otaheite is $23'. 10''$, and the computed difference, from the assumed mean horizontal parallax of the sun $8'',83$, is $23'. 26'',95$, the true parallax of the sun is less than that assumed. Let the true parallax be to that assumed as $1 - e$ to 1, and (612) the first parts of the computed parallax will be lessened in the ratio of $1 - e : 1$; and the second parts, in the ratio of $\overline{1 - e}^2$ to 1, or of $1 - 2e$ to 1 nearly. All the first parts, viz. $406'',05$; $287'',05$; $341'',48$; $382'',47$, in all $1417'',05$, combine the same way to make the total duration longer at Wardhus than at Otaheite. As to the second parts, the effects at Wardhus are $-7'',31$ and $-8'',91$, and at Otaheite are $+1'',63$ and $+4'',49$, in all $= -10'',10$. Therefore $1417'',05 \times \overline{1 - e} - 10'',10 \times \overline{1 - 2e} = 1390''$ the excess of the observed total duration at Wardhus above that at Otaheite; or $1417'',05 - 10'',10 - 1390'' = 1417'',05 - 20'',20 \times e$; and $e = \frac{16'',95}{1396'',85} = 0,0121$. Hence, the mean horizontal parallax of the sun $= 8'',83 \times \overline{1 - 0,0121} = 8'',72316$; and t corrected $= 327'',919 \times \overline{1 - 0,0121} = 323'',944$, whose logarithm is 2,510049; and the log.

of 127,912—log of 123,944=2,515757—2,510169. The log of the ratio of 1—e or 0,98,9 to 1, or 9[illegible],[illegible]41 to 127,912, is 1,99171, and the log of the ratio of the squares of these quantities is 1,98941, or the correction of the *first* constant logarithm before used is —0,00529, and that of the *second* constant logarithm before used is —0,0106.

We shall now calculate the effects of parallax over again at Wardhus and Otaheite, by means of these correcting logarithms.

First internal contact at Wardhus.

2,60898   0,864
—0,00529   —0,011

401",13=6'. 41",13   2,60329   7",13   0,853
— 7, 13

6'. 34, 00

Second internal contact at Wardhus.

2,45796   0,950
—0,00529   —0,011

253",58=4'. 13, 58   2,45267   [illegible],939
— [illegible], 69

4'. [illegible]4, 89

6'. 34",00 first internal contact accelerated.
4. 34, 89 second internal contact retarded.

11'. 8, 89 total duration lengthened by parallax.
5h.53'. 14 ———— observed.

5h. 42'. 5, 11 total duration reduced to the earth's center.

First internal contact at Otaheite.				*Second* internal contact at Otaheite.			
	2,55336		0,2117		2,58260		0,6520
	−0,00529		−0,0106		−0,00529		−0,0106
337″,34 = 5′. 37″,34	2,52807	1″,59	0,2011	377″,84 = 6′. 17″,84	2,57731	4″,33	0,6414
+ 1, 59				− 4, 33			
5. 38, 93				6. 22, 22			

5′. 38″,93	first internal contact retarded.
6. 22, 22	second internal contact accelerated.
12. 1, 15	total duration shortened by parallax.
5ʰ. 30. 4	———— observed.
5. 42. 5, 15	total duration reduced to the earth's center.
5. 42. 5, 11	———— from obs. at Wardhus.

5. 42. 5, 13 mean = 20525″,15 log.	4,3122[illegible]
Venus's motion from the sun in 1″ of apparent time	2,4[illegible]
1368″,51 length of the path described by Venus on the Sun	[illegible]
684″,26 = the semichord of Venus on the Sun.	
915″,1 + 684″,26 = 1599″,36	log. [illegible]
915, 1 − 684, 26 = 230, 84	log. [illegible]
	[illegible]
607″,61 = 10′. 7″,61 nearest approach of ♀ to ☉'s center	[illegible]

First internal contact at Wardhus	- -	$9^h, 34' 10'',6$
Effect of parallax	- - -	$+6\ 34,0$
Reduced to the earth's center	- - -	$9\ 40\ 44,6$

Second internal contact	- - - -	$15^h\ 27'\ 24'',6$
Effect of parallax	- - -	$-4\ 34,89$
Reduced to the earth's center	- - -	$15\ 22\ 49,71$

First internal contact at Otaheite	- -	$21^h, 44', 4''$
Effect of parallax	- - - - -	$-5, 38, 93$
Reduced to the earth's center	- -	$21\ 38\ 25,07$

Second internal contact	- -	$3^h, 14'\ 8''$
Effect of parallax	-	$+6\ 22,22$
Reduced to the earth's center	- -	$3, 20, 30,22$

Describe a small circle upon paper, representing the sun's disc, and draw a line upon it, representing the path of *Venus*, and let the center of the circle be laid upon a globe on the sun's place in the ecliptic, with the path of Venus pointing to her descending node, then from the horary angle, and the latitude of the place, the situation of the place upon the globe in respect to the sun and Venus, will immediately appear

623. If we would calculate the parallaxes for any other places, the constant logarithms to be used will be found, by subtracting 0,00529 from 264199 the first constant logarithm before used, and 0,1006 from 0,9717 the second constant logarithm before used, which gives the first constant logarithm corrected 2,63670, and the second constant logarithm corrected 0,9611.

624. The further the planet passes from the center of the sun, the greater will be the angle *CBD*, and therefore (612) the greater will be the parallax, cæteris paribus. Hence, the transit in 1769 is better to deduce the parallax from, than that in 1761; for by gaining a greater difference of times of the transit seen from different parts of the earth, any given error therein must less affect the conclusion.

625. Having explained the method of determining the sun's parallax from the transit of Venus, we shall proceed to give the results deduced from the transits in 1761 and 1769. And first we shall give those of Mr. J. Short, A. M. F. R. S. from a comparison of the times of the same contact observed at the Cape of Good Hope, with those in different parts of Europe.

626. The time of the internal contact at the Cape, in the transit of Venus in 1761, was 9*h*. 39'. 50", and the difference of the longitudes of the Cape and Greenwich is 1*h*. 13'. 35; hence, the time at Greenwich of the contact at the Cape was 8*h*. 26'. 15"; but the observed time of the same contact at Greenwich was 8*h*. 19', the difference of which, 7'. 15", is the effect of parallax between the two places. Now if we suppose the sun's horizontal parallax to be 8",5, it appears, by computation, that the effect of the parallax at the Cape is 6'. 8", by which time an observer at the Cape would see the contact *later* than at the center of the earth; and the effect of parallax at Greenwich is 1. 12", by which time an observer would see the contact *sooner* than at the center of the earth; therefore the sum, 7'. 20", is the whole effect of parallax between the two places. Hence, 7'. 20" : 7'. 15" :: 8",5 : 8",4 the sun's horizontal parallax from these observations. Thus, knowing the difference of longitudes, by comparing the times at the Cape with the following places, Mr. Short deduced these horizontal parallaxes of the sun. *Phil. Trans.* 1763.

Greenwich	8 ,52
Leskeard	8, 69
Rome	8, 61
Stockholm	8, 48
Abo	8, 74
Shirbourn	8, 19
Paris	8, 42
Dronthein	8, 23
Hernosand	8, 68
Torneo	8, 07
Savile House	8, 77
Bologna	8, 41
Upsal	8, 50
Calmar	8, 86
Cajaneburg	8, 33

The mean of these results is 8",47.

627. Mr. Short also determined the parallax from the whole time of the duration, in the following manner. He found the least apparent distance of

the center of Venus from the center of the sun, to be 9′ 32″, from which, and the horary motion of Venus, he found the total time of duration at the center of the earth to be 5*h* 58′ 1″. And from an assumed parallax of 8″,5, he computed the effect upon the observed time of the transit, and thence found the total duration at the center of the earth, which he compared with 5*h* 58′ 1″, and thence deduced the parallax. *Phil. Trans.* 1762.

628. At Calcutta, the duration observed was 5*h* 50′ 36″. Now upon supposition that the horizontal parallax of the sun was 8″,5, the effect of the parallax was 7′ 30″ to *shorten* the duration, hence the duration at the center, from this assumed parallax, was 5*h* 58′ 6″. But the true time was found to be 5*h.* 58′ 1″, this assumed parallax therefore gave the time too great by 5″. Now if we alter the parallax 1″, the time of duration will be altered 53″; hence, 53″ : 5″ :: 1″ : 0″,094 the change of parallax corresponding to the difference 5″ of duration, this subtracted from 8″,5 gives 8″,4 for the parallax from this observation; and from a mean of sixteen observations of this kind, Mr. SHORT determined the parallax to be 8″,48. From the mean of all the observations computed by Mr. SHORT, he determined the parallax to be 8″,557.

629. Dr. HORNSBY, Savilian Professor of Astronomy in the University of Oxford, from the mean of a great number of computations of the same transit, found the parallax to be 9″,73. But from a mean of nine observations of the transit in 1769, he found the parallax to be 8″,65. Hence, the mean of Mr. SHORT's and Dr. HORNSBY's conclusions give 8″,92 for the parallax. But if we take only those observations the most to be depended upon, from which Dr. HORNSBY computed in the first transit, the parallax at that time will be found to be only 8″,69, hence, the mean result from the DOCTOR's two conclusions 8″,69, 8″,92, and of Mr. SHORT's 8″,557, is 8″,72 for the parallax at the times of the transits, and assuming 1,0152 for the distance of the sun from the earth at the time of the transit, we have 1 : 1,0152 :: 8″,72 : 8″,85 for the parallax at the mean distance. Euler made the mean parallax, 8″,68; M. PINGRE, 8″,8; M. LEXELL, 8″,63; M. du SEJOUR, 8″,81; M. de la LANDE, 8″,6. The mean of all these determinations is 8″,74; which agrees (page 418) very nearly with Dr. MASKELYNE's calculation from the observations at Wardhus and Otaheite. We may therefore suppose the mean horizontal parallax of the sun to be 8″¾, with a great probability of its being extremely near to the truth. Hence, the radius of the earth : the distance of the sun :: sin. 8″¾ : rad. :: 1 : 23575.

630. The elements made use of by Mr. SHORT in his calculations were, the diameter of the sun = 31′ 31″, the diameter of Venus = 59″, the horary motion of Venus in its path = 3′. 59″,8, the angle of the apparent orbit of Venus with the ecliptic = 8°. 30′. 10″, the nearest distance of the centers of Venus and the sun seen from the earth = 9′. 32″, and the difference of the horizontal parallaxes of Venus and the sun = 21″,35.

631. The effect of the parallax being determined, the transit affords a very ready method of finding the difference of the longitudes of two places where the same observations were made. For compute the effect of parallax in time, and reduce the observations at each place, to the time it seen from the center of the earth, and the difference of the times is the difference of the longitudes. For example, we have shown (page 416) that the times at Wardhus and Otaheite, at which the first internal contact would take place at the earth's center, are 9*h*. 40′. 44″,6 and 21*h*. 38′. 25″,07, the difference of which is 12*h*. 2′. 19″,33 = 180°. 34′. 53″ the difference of the meridians. From the mean of 64 results from the transits of Mercury, Mr. SHORT found the difference of the meridians of Greenwich and Paris to be 9′. 17″; and from the transit of Venus in 1761, to be 9′. 10″, in time.

632. The transit of Venus affords a very accurate method of finding the place of the node. For by the observations made by Mr. RITTENHOUSE at Norriton in the United States of America, the least distance CM was observed to be 10′. 10″; hence, cos. MCP = 8°. 28′. 31″ : rad. :: CM = 10′. 10″ : CP = 10′. 17″ the geocentric latitude of Venus at the time of conjunction; and 0,72626 : 0,28895 :: 10′. 17″ : 4′. 5″ the heliocentric latitude CP of Venus; hence, tan. PNC = 3°. 23′. 33″ : rad. :: the heliocentric latitude CP = 4′. 5″ : CN = 1°. 8′. 52″, which added to 2s. 13°. 26′. 34″ gives 2s. 14°. 35′. 26″ for the place of the ascending node of the orbit of Venus.

633. The time of the ecliptic conjunction may be thus found. Find at any time (t) the difference (d) of longitudes of Venus and the sun's center (Art. 630); find also the apparent geocentric horary motion (m) of Venus *from* the sun in longitude, and then say, m : 1 hour :: d : the interval between the time t and the conjunction, which interval is to be added to or subtracted from t, according as the observation was made before or after the conjunction. In the transit in 1761, at 6*h*. 31′. 46″ apparent time at Paris, M. de la LANDE found d = 2′. 34″,4, and m = 3′. 57″,4; hence, 3′. 57″,4 : 2′. 34″,4 :: 1 hour : 39′. 1″, which subtracted from 6*h*. 31′. 46″, because at that time the conjunction was past, gives 5*h*. 52′. 45″ for the time of conjunction from this observation. We may also thus find the latitude at conjunction. The horary motion of Venus in latitude was 35″,4; hence, 60 : 39,1 :: 35″,4 : 23″ the motion in latitude in 39′,1, which subtracted from 10′. 1″,2 the latitude observed at 6*h*. 31′. 46″, gives 9′. 38″,2 for the latitude at the time of conjunction.

On the Necessary Observations to be made in the Transit of Venus over the Sun's Disc.

634. Previous to the time of the beginning of the transit, the observer should have his telescope properly fixed, and prepared with black glasses to defend the

eye, and should know, from his computations, the point of the sun's limb where Venus is expected to enter. Upon that part of the limb he should keep his eye steadily fixed, and at the instant he suspects the contact to take place, he must note the time, and proceed to observe, in order to be certain that he was not mistaken. If he find that he was mistaken, he must continue to wait for it, always noting the time when he suspects it, in order that he may not miss it when it really does happen. Venus having entered the sun's disc, wait for the internal contact, and note its time. Do the same for the internal and external contact, when Venus passes off the disc. In the transit in 1761, the Rev. Mr Hirst, F. R. S. at Madras, observed a kind of penumbra, or dusky shade, which preceded the first external contact two or three seconds of time, and was so remarkable, that he was thereby assured that the contact was near, which happened accordingly. In the transit in 1769, Dr Maskelyne was very attentive to observe if this circumstance took place, but he could perceive no such effect. When Venus was a little more than half immerged into the sun's disc, he saw its whole circumference completed, by means of a vivid, but narrow ill defined border of light, which illuminated that part of its circumference which was off the sun, but this disappeared about 2″ or 3″ before the internal contact. In the transit in 1769, Mr Hirst had warning of the approach of Venus to the external contact, by the sudden appearance of a violent coruscation, ebullition, or agitation of the upper edge of the sun, five or six minutes before the limb of Venus broke in upon the sun. This he thinks might be owing to the atmosphere of Venus. He did not, however, observe any kind of penumbra, as in the other transit. Some observers perceived, at the first external contact, a kind of watery pointed shadow, appearing to give a tremulous motion to that part of the sun's limb. Most of the observers took notice of a tremulous motion of the sun's limb, which rendered the true time of the contact uncertain to several seconds. Some Astronomers, at the last transit, observed a luminous crescent at the times of the ingress and egress, which enlightened that part of Venus's circumference which was off the sun, so that the whole circumference was visible. At the internal contact, the limb of Venus seemed, to most of the observers, to be united to the sun's limb by a black protuberance or ligament, which was not broken by the thread of light, till some seconds after the regular circumference of Venus seemed to have coincided with the sun's. Others observed that the thread of light between the limbs did not break instantaneously, the points of the threads darting into each other, and parting again, in a quivering manner, several times before they finally adhered. Perhaps the best way to get the time of the internal contact, is to judge by the eye, from that part of the circumference of Venus which is not disturbed, when the regular circumference of Venus would just touch the sun's limb. Hence it appears, that Dr Halley was mistaken, in supposing

that the contacts could be observed to a second of time; and accordingly the observations made by different observers, and reduced to the same meridian, differed more than was expected. The mean of all the observations however have counterbalanced this, and rendered the determination of the parallax of the sun to be depended upon to very great accuracy.

685. After the first internal contact, the next observations are to determine the nearest approach of their centers. This is best done with a micrometer fixed to the telescope, by measuring the horizontal diameter of the sun, the diameter of Venus, and the nearest distance of the exterior limb of Venus from the nearest point of the sun's limb; and this is done by bringing the limb of Venus up to the sun's limb in different parts till you find that you have got the nearest distance; or by Mr. DOLLOND's divided object glass micrometer, it is done by turning about the micrometer in its own plane, and when, during this motion, Venus is carried parallel to a tangent to the nearest point of the limb, or so as to continue to form a perfect internal contact, this is the position to measure the least distance of their limbs; then subtract the semidiameter of Venus from the radius of the sun, and you have the distance of their centers at that time. If the sun be so near to the horizon, that its vertical diameter is shortened by refraction, then, from the position of Venus, compute (207) how much that radius of the sun, in which she is, is shortened, and subtract the semidiameter of Venus from it. In those countries where the middle can be observed, continue to observe the distance of Venus from the nearest point of the sun's limb till that distance increases no longer, and you then get the nearest approach of their centers. If you cannot observe the middle, the least distance may be thus found. Let vC, wC be two observed distances of the centers of Venus and the sun; note the time when each observation was made, and you have the time through vw, and knowing the horary motion of Venus in its apparent orbit, you will know vw. Hence you know Cv, Cw, vw, from which, compute the angle Cwv; therefore in the right angled triangle Chw, you know Cw, and the angle Cwh, to find Ch the least distance required. If several observations of this kind be made, the mean of the results will give the least distance more accurately. If the telescope be mounted on a polar axis, it will be more convenient.

686. Secondly, the distance of Venus from the sun's center may be found by a wire micrometer adapted to a telescope to measure the difference of right ascensions and declinations, or by Mr. DOLLOND's divided object glass micrometer; see my *Treatise on Practical Astronomy*, Chap. vi. Let P be the pole of the equator EQ, v the place of Venus; draw the great circles, PCF, PvD, and Cx parallel to DF. Then having determined the difference vx of declinations of Venus and the sun's center, and the difference DF of their right ascensions, multiply DF by the cosine of PC the sun's declination, and (13)

you get Cr. knowing therefore wr, rC, in the right angled triangle wrC, you know wC. The distance of Venus from the sun's center being twice taken, and the time between, you get the least distance Ch as before. Having determined the difference of the right ascensions and declinations of the sun and Venus at any time, you may find the difference of their longitudes by Art. 384.

637. The parallax of the sun, from the transit of Venus, being determined from the difference of the times of the transits at two places, the conclusion will be most accurate when that difference is the greatest possible. The places therefore to be chosen for the two observations should be upon opposite meridians, and such, that the middle of the transit may be when the sun is upon the meridian; for under these circumstances, the ingress at one place will be accelerated and the egress retarded, increasing thereby the time of the transit, and the ingress at the other place will be retarded and the egress accelerated, by which the time of the transit will be diminished; the difference therefore of the times of the transits at the two places will thus become the greatest. As the transit must be observed under opposite meridians, it must happen in the day at one of the places, and at night at the other; the place therefore where it happens in the night must be so near to the north or south pole, according as the declination of the sun is north or south, that the ingress may be observed before the sun sets, and the egress the next morning after it rises. Hence, the transits of Venus which happen in June are more convenient than those which happen in December, because there is a great choice of situations towards the north pole, which is not the case towards the south. Dr. HALLEY made a mistake, by setting off the axis of the planet's orbit on the *same* side of the ecliptic that the axis of the equator was situated, instead of the *contrary* side. By using therefore the difference of these two angles instead of their sum, he made the difference of the times of the transit in 1761 seen at the Ganges and Port Nelson (two places recommended by him for observing this transit) longer by 20 than it ought, as computed by Dr. HORNSBY; see the *Phil. Trans.* 1763.

To determine at what Countries the Ingress and Egress are visible.

638. Elevate the north or south pole of the terrestrial globe above the horizon equal to the sun's declination at the time of the transit, according as the declination is north or south. Bring Greenwich, for instance, to the meridian, and set the index to twelve. Now for the ingress, turn the globe and set it to the hour the ingress happens, and the globe will be in a proper position for that time, the sun being vertical to that hemisphere of the earth above the horizon of the globe. The beginning of the transit is therefore visible to that hemi-

sphere. To those places under the *western* semicircle of the horizon, the sun is then rising, and therefore to such places the transit begins at sun rise. To those places under the *eastern* semicircle of the horizon, the sun is then setting, and therefore the transit there begins at sun set. To those places lying under the meridian, the ingress begins at twelve o'clock.

Set the index, by turning the globe, to the time of the middle of the transit, and proceed as before. Then in the *western* semicircle of the horizon you will see all those places where the middle is at sun rise; and under the *eastern* semicircle of the horizon, those where the middle is at sun set. And the places under the meridian are those where the transit is at the middle at twelve o'clock.

Turn the globe, and set the index to the time of the egress, and the egress will be visible to all the countries above the horizon. The places lying under the meridian are those where the transit ends at twelve o'clock. Under the *western* semicircle of the horizon, lie the places where the end is at sun rise. And under the *eastern* semicircle of the horizon, lie the places where the end is at sun set.

The times when any of these appearances happen at any other place, may be found, by taking the difference between the meridians, and converting it into time, and applying that difference to the time at Greenwich.

CHAP. XXVI.

ON THE NATURE AND MOTION OF COMETS.

Art. 639. COMETS are solid bodies, revolving in very excentric ellipses about the sun in one of the foci, and are therefore subject to the same laws as the planets, but differ in appearance from them; for as they approach the sun, a tail of light, in some of them, begins to appear, which increases till the comet comes to its perihelion, and then it decreases again, and vanishes; others have a light encompassing the nucleus, or body of the comet, without any tail. The most ancient philosophers supposed comets to be like planets, performing their revolution in stated times. ARISTOTLE, in his first book of *Meteors*, speaking of comets, says, "But some of the Italians, called Pythagoreans, say, that a *Comet* is one of the *Planets*, but that they do not appear unless after a long time, and are seen but a small time, which happens also to *Mercury*." SENECA also in *Nat. Quest.* Lib. VII. says, "APOLLONIUS affirmed, that the *Comets* were, by the Chaldeans, reckoned among the *Planets*, and had their periods like them." SENECA himself also, having considered the phænomena of two remarkable comets, believed them to be stars of equal duration with the world, though he was ignorant of the laws that governed them; and foretold, that after ages would unfold all these mysteries. He recommended it to Astronomers to keep a catalogue of the comets, in order to be able to determine whether they returned at certain periods. Notwithstanding this, most Astronomers from his time till TYCHO BRAHE, considered them only as meteors, existing in our atmosphere. But that Astronomer, finding from his own observations on a comet, that it had no diurnal parallax, placed them above the moon. Afterwards KEPLER had an opportunity of observing two comets, one of which was very remarkable; and from his observations, which afforded sufficient indications of an annual parallax, he concluded, "that comets moved freely through the planetary orbs, with a motion not much different from a rectilinear one; but of what kind he could not precisely determine." HEVELIUS embraced the same hypothesis of a rectilinear motion; but finding his calculations did not perfectly agree with his observations, he concluded, "that the path of a comet was bent in a curve line, concave towards the sun." He supposed a comet to be generated in the atmosphere of a planet, and to be discharged from it, partly by the rotation of the planet, and then to revolve about the sun in a parabola by the force of projection and its tendency to the sun, in the same manner as a projectile upon the earth's surface describes a parabola. At length came the famous comet in 1680, which descending nearly in a right line towards the sun,

arose again from it in like manner, which proved its motion in a curve about the sun. G. S. Dörfell, Minister at Plaven in Upper Saxony, made observations upon this comet, and found that its motion might be very well represented by a parabola, having the sun in its focus. He was ignorant however of all the laws by which the motion of a body in a parabola is regulated, and erred considerably in his parabola, making the perihelion distance about twelve times greater than it was. This was published five years before the *Principia*, in which work Sir I. Newton having proved that Kepler's law, by which the motions of the planets are regulated, was a necessary consequence of his theory of gravity, it immediately followed that comets were governed by the same law; and the observations upon them agreed so accurately with his theory, as to leave no doubt of its truth. That the comets describe ellipses, and not parabolas or hyperbolas, Dr. Halley, (see his *Synopsis of the Astronomy of Comets*) advances the following reasons.

"Hitherto I have considered the orbits of comets as exactly parabolic; upon which supposition it would follow, that comets, being impelled towards the sun by a centripetal force, would descend as from spaces infinitely distant; and, by their so falling, acquire such a velocity, as that they may again fly off into the remotest parts of the universe, moving upwards with a perpetual tendency, so as never to return again to the sun. But since they appear frequently enough, and since none of them can be found to move with an hyperbolic motion, or a motion swifter than what a comet might acquire by its gravity to the sun, it is highly probable they rather move in very excentric elliptic orbits, and make their returns after long periods of time: for so their number will be determinate, and, perhaps, not so very great. Besides, the space between the sun and the fixed stars is so immense, that there is room enough for a comet to revolve, though the period of its revolution be vastly long. Now, the *latus rectum* of an ellipsis is to the *latus rectum* of a parabola, which has the same distance in its perihelion, as the distance in the aphelion, in the ellipsis, is to the whole axis of the ellipsis. And the velocities are in a subduplicate ratio of the same; wherefore, in very excentric orbits, the ratio comes very near to a ratio of equality; and the very small difference which happens, on account of the greater velocity in the parabola, is easily compensated in determining the situation of the orbit. The principal use therefore of the Table of the elements of their motions, and that which indeed induced me to construct it, is, that whenever a new comet shall appear, we may be able to know, by comparing together the elements, whether it be any of those which has appeared before, and consequently to determine its period, and the axis of its orbit, and to foretel its return. And, indeed, there are many things which make me believe, that the comet which Apian observed in the year 1531, was the same with that which Kepler and Longomontanus more accurately described in the year 1607; and which I my-

self have seen return, and observed in the year 1682. All the elements agree, and nothing seems to contradict this my opinion, besides the inequality of the periodic revolutions; which inequality is not so great neither, as that it may not be owing to physical causes. For the motion of Saturn is so disturbed by the rest of the planets, especially Jupiter, that the periodic time of that planet is uncertain for some whole days together. How much more therefore will a comet be subject to such like errors, which rises almost four times higher than Saturn, and whose velocity, though increased but a very little, would be sufficient to change its orbit, from an elliptical to a parabolical one. And I am the more confirmed in my opinion of its being the same; for, in the year 1456, in the summer time, a comet was seen passing retrograde between the earth and the sun, much after the same manner; which, though nobody made observations upon it, yet, from its period, and the manner of its transit, I cannot think different from those I have just now mentioned. And since looking over the histories of comets, I find, at an equal interval of time, a comet to have been seen about Easter in the year 1305, which is another double period of 151 years before the former. Hence, I think, I may venture to foretell that it will return again in the year 1758."

640. Dr. HALLEY computed the effect of *Jupiter* upon this comet in 1682, and found that it would increase its periodic time above a year, in consequence of which he predicted its return at the end of the year 1758 or the beginning of 1759. He did not make his computations with the utmost accuracy, but, as he himself informs, *levi calamo*. M. CLAIRAUT computed the effects both of Saturn and Jupiter, and found that the former would retard its return in the last period 100 days, and the latter 511 days; and he determined the time when the comet would come to its perihelion to be on April 15, 1759, observing that he might err a month, from neglecting small quantities in the computation. It passed the perihelion on March 13, within 33 days of the time computed. Now if we suppose the time stated by Dr. HALLEY to mean the time of its passing the perihelion, then if we add to that 100 days, arising from the action of Saturn which he did not consider, it will bring it very near to the time in which it did pass the perihelion, and prove his computation of the effect of Jupiter to have been very accurate. If he mean the time when it would first appear, his prediction was very accurate, for it was first seen on December 14, 1758, and his computation of the effects of Jupiter will then be more accurate than could have been expected, considering that he made his calculations only by an indirect method, and in a manner professedly not very accurate. Dr. HALLEY therefore had the glory, first to foretell the return of a comet, and the event answered remarkably to his prediction. He further observed, that the action of Jupiter, in the descent of the comet towards its perihelion in 1682, would tend to increase the inclination of its orbit; and accordingly the

inclination in 1682 was found to be 22' greater than in 1607. A learned Professor (Dr. Long's Astronomy, p. 762) in Italy to an English gentleman writes thus. "Though M. de la Lande, and some other French gentlemen, have taken occasion to find fault with the inaccuracies of Halley's calculation, because he himself had said he only touched upon it slightly; nevertheless they can never rob him of the honour,—First, of finding out that it was one and the same comet which appeared in 1682, 1607, 1531, 1456, and 1305. Secondly, of having observed that the planet Jupiter would cause the inclination of the orbit of the comet to be greater, and the period longer. Thirdly, of having foretold that the return thereof might be retarded till the end of 1758, or the beginning of 1759." From the observations of M. Messier upon a comet in 1770, M. Lexell Prosperin, Member of the Royal Academies of Stockholm and Upsal, showed, that a parabolic orbit would not answer to its motions, and he recommended it to Astronomers to seek for the elliptic orbit. This laborious task M. Lexell undertook, and has shown that an ellipse, in which the periodic time is about five years and seven months, agrees very well with the observations. See the *Phil. Trans.* 1779. As the ellipses which the comets describe are all very excentric, Astronomers, for the ease of calculation, suppose them to move in parabolic orbits, for that part which lies within the reach of observation, by which they can very accurately find the place of the perihelion, its distance from the sun, the inclination of the plane of its orbit to the ecliptic, and the place of the node. Previous therefore to the determination of the orbit of a comet from observation, we must premise such particulars respecting the motion of a body in a parabola, as may be necessary for such an investigation. Several of the principles which we are here obliged to make use of, will be proved when we come to treat on the Physical Principles of Astronomy.

On the Motion of a Body in a Parabola.

641. Let APM be a parabola, S its focus, A the vertex, P the place of the body, draw PQ perpendicular to AS, and PD perpendicular to the tangent PT, also SM perpendicular to AD. Now, by the property of the parabola, QD is equal to half the latus rectum; hence, if $AS=1$, then $QD=2$; also, the angle $PSA=2\ PDA$; therefore if QD be radius, PQ will be the tangent of PDA, or $\frac{1}{2}\ PSA$; hence, to the radius AS, PQ will be twice the tangent of $\frac{1}{2}\ PSA$; therefore if $2t=PQ$, t will be the tangent of (a) half the true anomaly PSA, to the radius $AS=1$. Also, by the property of the parabola, $AQ\times 4\ AS=PQ^2$; hence, $AQ=t^2$; also, the area $AQP=\frac{4}{3}t^3$; and as $QS=1-t^2$, the area $QPS=t-t^3$; hence, the area $ASP=\frac{1}{3}t^3+t$; also, the area $ASM=\frac{4}{3}$. Now let a and b be the times in which the comet moves from A to M, and

from A to P, then, as the areas described about S are proportional to the times, $a : b :: \frac{1}{3}t^3 + t : \frac{4}{3}$, therefore $at^3 + 3at = 4b$.

642. Hence, if a, and the true anomaly be given, we have the time $b = \frac{1}{4}at^3 + \frac{3}{4}at$. Also, because $a : b :: \frac{4}{3} : \frac{1}{3}t^3 + t$, if the true anomaly, and consequently t, be given in different parabolas, the times of describing those true anomalies from the perihelions will be in proportion to the times of describing 90° from the perihelion.

643. If the times a and b be given, the true anomaly may be found from resolving the cubic equation $t^3 + 3t = \frac{b}{\frac{1}{4}a}$, which may be done thus. In the right angled triangle CAB, let $AB = 1$, $AC = \frac{b}{\frac{1}{2}a}$, and compute BC; then find two mean proportionals between $BC + AC$ and $BC - AC$, and their difference is the value of t. FIG. 161.

644. Take the fluxion of $t^3 + 3t = \frac{4b}{a}$, and we have $\dot{t} = \frac{4}{3a} \times \frac{\dot{b}}{1 + t^2}$; but $\dot{t} = \overline{1 + t^2} \times \frac{\dot{x}}{2}$, hence we get $\dot{x} = \frac{8}{3a} \times \frac{\dot{b}}{\overline{1+t^2}|^2} = \frac{8}{3a} \times \cos^4 \frac{x}{2} \times \dot{b}$ the variation of the true anomaly corresponding to any small variation $\dot{b}$ of time expressed in decimals of a day, a being expressed in days.

645. Let SA be the mean distance of the earth from the sun; then the area of the circle, described with that radius, will be 3,14159; also the area $AMS = \frac{4}{3}$. Now the velocity in the parabola : velocity in the circle :: $\sqrt{2}$: 1, and the areas described in the same time will be in the same ratio, because at A the motion in each orbit being perpendicular to SA, the areas described will be as the velocities, and it being so in one case, it must be always so, because in each orbit respectively equal areas are described in equal times, as will be afterwards proved. But the times of describing any two areas are as the areas directly, and the areas described in the same time inversely; therefore $\frac{3,14159}{1} : \frac{4}{3\sqrt{2}} \left(= \frac{\sqrt{8}}{3}\right)$ :: the time of the revolution in the circle $= 365d.\ 6h.\ 9'$: the time of describing $AM = 109d.\ 14h.\ 46'\ 20''$. Now as the time of describing AM is in a given ratio to the time in the circle, which (as will be afterwards shown) varies as $AS^{\frac{3}{2}}$; therefore if $r =$ the perihelion distance in any other parabola, we have $1^{\frac{3}{2}} : r^{\frac{3}{2}} :: 109d.\ 14h.\ 46'.\ 20''$: the time of describing 90° in that pa- FIG. 160.

rabola from the perihelion. Hence, knowing the time corresponding to any true anomaly in that parabola whose perihelion distance = 1, we know the time corresponding to the same true anomaly in any other parabola, because the (642) times of describing 90° are as the times corresponding to the same true anomaly; therefore if n be the number of days corresponding to any given anomaly in that parabola whose perihelion distance is unity, then $nr^{\frac{3}{2}}$ will be the time t corresponding to the same anomaly in that whose perihelion distance is r; this may be readily found thus. Multiply the log. r by 3 and divide by 2, and to the quotient add the log. n, and the sum will be the log. of the time required. Hence also, $n = \frac{t}{r^{\frac{3}{2}}}$; therefore if from the log. t we subtract $\frac{3}{2}$ log. r, it gives the log. n of the number of days corresponding to the same anomaly in the parabola, whose perihelion distance = 1; hence, the anomaly will be found from the Table at the end of this Chapter, which contains the times corresponding to the true anomaly for 200000 days from the perihelion, in that parabola whose perihelion distance is unity. This Table may be constructed by Art. 641. by taking $a = 109,6154$, and assuming $b = 1, 2, 3, 4$, &c. and finding the corresponding values of t. Dr. HALLEY first constructed a Table of this kind. M. de la CAILLE changed it into a more convenient form, by putting the areas for the times; that which we have here given was computed by M. de LAMBRE.

646. Draw SY perpendicular to the tangent; then $SP : SY :: SY : SA$, therefore $\sqrt{SP} : \sqrt{SA} :: SP : SY ::$ rad. : cos. PSY, or $\frac{1}{2}PSA$ the true anomaly; or $SP : SA ::$ rad.^{2} : cos.^{2} $\frac{1}{2}$ true anom.^{2} Hence, if $SA = 1$, and $a + x = \frac{1}{2}PSA$, $a - x = \frac{1}{2}pSA$, then $1 : \sqrt{SP} ::$ cos. $\overline{a+x}$: rad. and $\sqrt{Sp} : 1 ::$ rad. : cos. $\overline{a-x}$; hence, $\sqrt{Sp} : \sqrt{SP} ::$ cos. $\overline{a+x}$: cos. $\overline{a-x}$.

647. Hence, $SP = \frac{SA}{\text{cos.}^2\ \frac{1}{2}\ \text{true anom.}^2}$, radius being unity; therefore from log. SA subtract twice the log. cos. $\frac{1}{2}$ true anomaly, and the remainder is the log. of the distance of the comet from the sun.

648. Erect BD perpendicular to AB, take $BC = AB$, produce AC to E, and draw EDF perpendicular to AE, meeting AF parallel to BD in F, join AD, and draw DG, CH parallel to AB. Then, as $EAF = 45°$, $EA = EF$; also $FG = GD = AB$; hence, $AF = BD + BA$, and $GH = BD - BA$; also, by similar triangles, AF or $BD + BA : GD = GH$ or $BD - BA :: EF$ or $EA : ED ::$ rad. : tan. DAE; but $AB : BD ::$ rad. : tan. BAD, from which subtract 45°, and we have $BD + BA : BD - BA ::$ rad. : tan. of that difference. If BD

$= \sqrt{SP}$, and $BA = \sqrt{Sp}$, then $\sqrt{SP} : \sqrt{Sp} ::$ rad. : tan. $BAD = \sqrt{\frac{Sp}{SP}}$, hence, to get that angle, take half the difference of the logarithms of SP and Sp, and add 10 to the index (because in the log. tangents, the index of log. tan. of 45°, or log. of rad. $= 1$, is 10, instead of 0,) and it gives the log. tangent of the angle; from which take 45°, and we have $\sqrt{SP} + \sqrt{Sp} : \sqrt{SP} - \sqrt{Sp} ::$ rad. : tang. of that difference.

649. Hence, if we know two radii SP, Sp, and the angle PSp between, we 110
can find the two anomalies. For let a be ½ of $ASP + ASp$, and x be ½ of ASP 160
$- ASp$, then ½ $ASP = a + x$, and ½ $ASp = a - x$, hence (646), $\sqrt{Sp} : \sqrt{SP} :: \cos.\ \overline{a+x} : \cos.\ \overline{a-x} ::$ (by plane Trig.) $\cos.\ a \times \cos.\ x - \sin.\ a \times \sin.\ x : \cos.\ a \times \cos.\ x + \sin.\ a \times \sin.\ x$, therefore $\sqrt{SP} + \sqrt{Sp} : \sqrt{SP} - \sqrt{Sp} :: \cos.\ a \times \cos.\ x : \sin.\ a \times \sin.\ x :: \frac{\cos.\ a}{\sin.\ a} : \frac{\sin.\ x}{\cos.\ x} :: \cot.\ a : \tan.\ x$. Now the ratio of the two first terms is found from the last Article, and as the angle PSp is given, the value of a will be given, hence we find x, and consequently we know the sum and difference of ASP, ASp, therefore we know the angles themselves. If p lie on the other side of A, then we know x, to find a.

650. Given two distances SP, Sp from the focus to the curve of a parabola, 110.
and the angle between them, to find the parabola. With the centers P and p, 169.
and radii PS, pS, describe two circular arcs *rnt*, *mvn*, to which draw the tangent *avwb*; draw ST perpendicular to ab, and bisect it in A, and it will be the vertex of the parabola; hence we may describe the parabola.

Given the Elements of the Orbit of a Comet, to compute its Place at any Time.

651. The elements of the orbit of a comet are, 1. The time when the comet passes the perihelion.—2. The place of the perihelion.—3. The distance of the perihelion from the sun.—4. The place of the ascending node.—5. The inclination of the orbit to the ecliptic. From these elements, the place at any time may be computed; and, for example, we shall take that given by M. de la CAILLE in his Astronomy. The comet in 1739, which was retrograde, passed its perihelion on June 17, at 10h. 9'. 30" mean time; the place of the perihelion was in 3s. 12°. 38'. 40"; the perihelion distance was 0,67358, the mean distance of the earth from the sun being unity; the ascending node was in 0s. 27°. 25'. 14", and the inclination of the orbit 55°. 42'. 44", to compute the place seen from the earth on August 17, at 14h. 20' mean time.

Let *HAP* be the parabolic orbit of the comet, *A* the ascending node, *P* the place of the comet, *T* the corresponding place of the earth, and draw *Pv* perpendicular to the ecliptic; produce *SN*, *Sv*, *SP*, *ST* to *n*, *u*, *p* and *t* the sphere of the fixed stars, and describe the great circles *np*, *nu* ♈ *t* and *pu*.

I. The interval of time from the perihelion to the given time is 61*d*. 4*h*. 10'. 30" = 61,174, whose log. is 1,786507; also the log. of ,67358 is 9,828388, 3/2 of which log. (from the nature of logarithms) is 9,742522, which subtracted from 1,786507 leaves 2,043985, the log. of 110,6587 days, which, by the Table, answers to 3s. 0°. 21'. 38" the true anomaly *PSA* at the given time.

II. Subtract 3s. 0°. 21'. 38" from 8s. 12°. 38'. 40" the place of the perihelion, because the comet was retrograde and had passed the perihelion, and it leaves 12°. 17'. 1" for the heliocentric place *p* of the comet in its orbit.

III. The longitude of *n* is 27°. 27'. 14", also *pn* = 27°. 27'. 14" − 12°. 17'. 1" = 15°. 8'. 13"; hence, rad. : cos. *pnu* = 55°. 42'. 44" :: tan. *pn* = 15°. 8'. 13" : tan. *un* = 8°. 39'. 53" the distance of the comet from the ascending node, measured upon the ecliptic.

IV. Subtract this value of *un*, from the place of the node, and there remains 18°. 45'. 21" = ♈*u* the true heliocentric place of the comet reduced to the ecliptic.

V. As rad. : sin. *pn* = 15°. 8'. 13" :: sin. *pnu* = 55°. 42'. 44" : sin. *pu* = 12°. 27'. 34" the latitude seen from the sun, which is south.

VI. The true place *T* of the earth at the same time is 10s. 21°. 34'. 36"; hence, *TS*♈ = 39°. 25'. 24"; therefore *TS*♈ + ♈*Su* = *TSu* = 1s. 24°. 10'. 45". Also, *TS* = 1,0115.

VII. By Art. 646. cos. 45°. 10'. 49" : rad. :: ,67358 : *SP* = 1,3557.

VIII. As rad. : cos. *PSv* = 12°. 27'. 34" :: *SP* = 1,3557 : *Sv* = 1,32377.

IX. In the triangle *TSv*, we know *TS*, *Sv* and the included angle *TSv*; hence, by plane Trigonometry, we find the angle *STv* = 77°. 33'. 58"½, which subtracted from 4s. 24°. 34'. 36", the place of the sun, leaves 2s. 7°. 0'. 37"½ for the comet's true *geocentric longitude*.

X. By Art. 278, as sin. 54°. 10'. 46" : sin. 77°. 33'. 58"½ :: tan. *PSv* = 12°. 27'. 34" : tan. *PTv* = 14°. 54'. 4" the comet's true *geocentric latitude*.

To determine the Orbit of a Comet from Observation.

652. Sir I. NEWTON first resolved this problem, which he called *Problema longe difficillimum*. The orbit of a comet may be computed from three observations; but although that data be sufficient, the direct solution of the problem

is impracticable. Astronomers therefore have solved this problem by indirect methods, first finding an orbit very near to the truth by mechanical and graphical operations, and then, by computation, correcting it, until such a parabola was found as would satisfy the observations. We shall therefore begin, by showing the methods by which the orbit may be nearly determined, and then explain the manner in which it may be corrected by calculation.

653. M. de la LANDE proposes the following mechanical method of finding the orbit nearly. Divide the distance of the earth from the sun into ten equal parts, and describe ten parabolas whose perihelion distances are, 1, 2, 3, &c. of these parts, and divide these parabolas into days from the perihelion, answering to the motion of a body in each. Let *S* be the sun, *a*, *b*, *c*, the places of the earth at the times of three observations of the comet. Then take three geocentric latitudes and longitudes of the comet, and set off the elongations *Saα*, *Sbβ*, *Scγ* in longitude. From *a*, *b*, *c*, extend three fine threads *am*, *bn*, *cp*, vertical to *aα*, *bβ*, *cγ*, making angles with them equal to the geocentric latitudes respectively. Then take any one of the parabolas, and placing its focus in *S* apply the edge to the threads, and observe whether you can make it touch them all, and whether the intervals of time cut off by the threads upon the parabola be equal to the respective intervals of the observations, or very nearly so, and if these circumstances take place, you have then gotten the true parabola, or very nearly the true one. But if the parabola do not agree, try others, till you find one which does agree, or very nearly so, and you will then have got very nearly the true parabola, whose inclination, place of the node, and perihelion are to be determined as accurately as possible from mensuration; also, the projection upon the ecliptic. If none of these parabolas should nearly answer, it shows, that the perihelion distance must be greater than the distance of the earth from the sun, in which case, other parabolas must be constructed; but this does not very often happen. This method will determine the elements very nearly, but it would be extremely troublesome to construct and divide so many parabolas, if we only wanted to compute the elements of one comet, for those who purpose to make many computations of this kind, it might be worth while to have a set of parabolas thus divided. To avoid this trouble therefore, we propose to do it in the following manner by means of one parabola, without dividing it. FIG. 165

654. Take a firm board perfectly plane, and fix on paper for the projection, let a groove be cut near the edge, and five perpendiculars be moveable in it, so that they may be fixed at any distances. Let *S* represent the sun, and describe any number of circles about it. Compute five geocentric latitudes and longitudes of the comet, from which you will have the five elongations of the comet at the times of the respective observations. Draw *SA*, *SB*, *SC*, *SD*, *SE*, making the angles *ASB*, *BSC*, *CSD*, *DSE*, equal to the sun's motion in the

intervals of the observations; and on any one of the circles, make the angles $Sa\alpha$, $Sb\beta$, $Sc\gamma$, $Sd\delta$, $Se\epsilon$ equal to the respective elongations in longitude, and fix the five perpendiculars, so that the edge of each may coincide with α, β, γ, δ, ϵ. From the points a, b, c, d, e, extend threads to the respective perpendiculars, making angles* with the plane equal to the geocentric latitudes of the comet; then fix the focus of the parabola in S, and apply its edge to the threads, and if it can be made to touch them all, it will be the parabola required, corresponding to the mean distance Sa of the earth, which we here suppose to revolve in a circle, as it will be sufficiently accurate for our purpose. If the parabola cannot be made to touch all the threads, change the points, a, b, c, d, e, to such of the other circles as you may judge, from your present trial, will be most likely to succeed, and try again; and by a few repetitions you will get such a distance for the earth, that the parabola shall touch all the threads, in which position, find the inclination, observe the place of the node, and measure the perihelion distance, compared with the earth's distance, and you will get very nearly the elements of the orbit.

655. The next method of approximating to the orbit of a comet, which we shall explain, is that given by BOSCOVICH. Let S be the sun, XZ the orbit of the earth, supposed to be a circle; T the place of the earth at the first observation, and t at the third; draw TC, tc to represent the observed longitudes of the comet; and let L, l, λ be the longitudes at the first, second and third observations; m and n the geocentric latitudes of the comet at the first and third observations; and t, T, the intervals of time between the first and second, second and third observations. Assume C for the place of the comet, at the first observation, reduced to the ecliptic; then to determine the point at the third observation, say, $T \times \sin. \lambda - l : t \times \sin. l - L :: TC : tc$, and c will be nearly† the place required; join Cc, and it will represent the path of the comet on the ecliptic, upon this assumption. Perpendicular to the ecliptic draw CK, ck, taking $CK : TC ::$ tan. m : radius, and $ck : tc ::$ tan. n : radius; join Kk, and it will represent the orbit of the comet, if the first assumption be true. Bisect Cc in x, and draw xy parallel to CK, and y will bisect Kk; join yS. Let $SX = 1$; then if v be the mean velocity of the earth in its orbit, the velocity of the comet at $y = \frac{\sqrt{2} \times v}{\sqrt{Sy}}$; taking therefore $v = Tt$, compute $\frac{\sqrt{2} \times v}{\sqrt{Sy}}$, and if this be equal to Kk, measured by the scale, the assumed point C was the true point. But if these quantities be not equal, assume a new point for C, in doing which,

* The easiest and most correct method to set off these angles, is, for instance, to measure Sa, and then compute the perpendicular from the angle; and the same for the rest.

† For the proof of this, see the Author's paper upon the subject in his *Opuscula*, Vol. iii. or Sir H. ENGLEFIELD's very valuable Work upon Comets, page 97.

the error of the first assumption will direct you which way from the first assumed point, it must be taken, and about how far from it, if, for instance, the computed value of Kk be greater than the true value, and the lines CK, ck are diverging from each other, and receding from the sun, the point C must be taken further from T, and how much further we must conjecture from the value of the error, and also from hence, that the velocity of the comet diminishes as it recedes from the sun. These considerations will lead us to make a second assumption near to the truth. Having thus determined the true points C, c, very nearly, produce cC, kK to meet at N, join NS, and it will be the line of the nodes. Draw Cr, c perpendicular to SN, and the angles KrC, ksc will measure the inclination of the orbit. From the two distances SC, Sc, and the angle between, the parabola may be (650) constructed, and applied as in the last method, from which the time of passing the perihelion may be found.

656. Another method by which we may readily get the orbit very nearly, is thus. Let S be the sun, T, t, τ three places of the earth at the times of the three observations; extend three threads Tp, tn, τm in the directions of the comet, as directed in Article 651. Assume a point y for the place of the comet at the second observation, and measure Sy; then if $ST=1$, and the velocity of the earth be v, the velocity of the comet at y will be $\frac{\sqrt{2}\times v}{\sqrt{Sy}}$, let v be represented by Tt, $t\tau$; and upon any straight edge PQ, set off $ce=\frac{\sqrt{2}\times Tt}{\sqrt{Sy}}$ FIG. 168

and $ed=\frac{\sqrt{2}\times t\tau}{\sqrt{Sy}}$; then apply the point e to y, and, by turning about the edge, try whether you can make the point c fall in Tp, and the point d in τm; if you find this can not be done, the error will direct you to assume another distance; and by a very few trials you will find the point y where the points c and d will fall in Tp, τm. This method is very easy in practice, and sufficiently accurate to obtain a distance Sy from which you may begin to compute, in order to find the orbit more correctly, when the comet is not too near to the sun, as I have found by experience.

657. Having determined the parabola nearly, we first assume some one quantity as known at the first and second observations, and thence compute the place of the comet at those times, and also the time between; and if that time agrees with the observed interval, you have got a parabola which agrees with the two first observations; if the times do not agree, alter one of the assumed quantities, and see how it then agrees; and then, by the rule of false, you may correct the supposition which was altered, and get a parabola which will agree with the two first observations. In like manner, by altering the other assumed quantity, you get another parabola agreeing with the two first observations,

Then see how they agree with the third observation, and if they do not, a correction must be made by proportion, and the three observations will be answered. But this will be best explained by an example, we shall therefore take that which is given by M. de la CAILLE in his Astronomy, and explain the method of computation, and the reasons of the whole operation.

658. In the year 1739, M. ZANOTTI, at Bologna, made the following observations on a comet, where the mean time is reduced to the meridian of Paris. Both the comet and its tail were most vivid about the middle of June, and therefore it must have been in its perihelion about that time.

		Longitude	Latitude
May	28, at 8h. 48'	7°. 0 ♋	27°. 9 N.
July	23, 13. 50	16. 1 ♊	2. 27 N.
——	27, — 14. 8	15. 20 ♊	1. 3 N.
August	2, — 13. 4	13. 46 ♊	2. 48 S.
——	4, — 13. 12	12. 33½ ♊	4. 13 S.
——	10, — 12. 40	10. 11½ ♊	8. 31 S.
——	17, — 14. 20	7. 1 ♊	14. 57 S.

659. From these observations, let the orbit of the comet be determined and projected upon the ecliptic, as nearly as possible, by one of the methods already explained, and let XZ be the projection, and S the sun, then the projection shows the comet to be retrograde. The method used by M. de la CAILLE, is that which was first proposed by Sir I. NEWTON in his Algebra, by cutting four right lines by another right line, so that the parts intercepted may be equal; but this problem is unlimited. In consequence of his using this method, he was obliged to interpolate and get the latitude and longitude at four equidistant times, one of which was August 6, at midnight, at which time the comet's geocentric longitude was found to be 2s. 12°. 17', the sun's longitude 4s. 13°. 55', its distance from the earth 10138, and the comet's western elongation from the sun was 61°. 38'. This time is one which he assumes; but by our methods, one of the times of the observations might have been used instead.

660. The comet passing from north to south latitude, or through the descending node, between July 27, and August 2, interpolate the observations on July 23, 27, and August 2, to find the time and place when the comet had no latitude; this is found to be on July 29, at 8h. 48' mean time in 14°. 34' ♊; at which time, the sun's place was 4s. 6°. 7'. 10", which gives the comet's elongation west 51°. 12'. 10"; and its distance from the earth was 10146,3.

661. On May 28, at 8h. 48' mean time, the sun's place was in 2s. 6°. 56'. 10"; and therefore the comet's elongation was to the east of the sun 30°. 3'. 50", and

the comet's distance from the earth was 10142. The interval of time between this and when the comet was in its node is 62 days.

662. Now to find a parabola which answers to these two times on May 28, and July 29, let a be the place of the earth on August 6, at midnight, make the angles aSL, aSK equal to the motion of the earth from May 28, at $8h$ 48′, and July 29, at $8h$. 48, to Aug. 6, at midnight, and take SL = 10148,5, and SK = 10142, then will L and K be the respective places of the earth at the two former times, and make the angles SLN, SKM equal to 51° 13′ 10, and 30° 9′ 50, the respective western and eastern elongations of the comet, and N and M will be the corresponding places of the comet on the ecliptic, and N the node. Measure, upon the scale, the lines SM and SN, and suppose them to be found 5500 and 10,700.

663. *First Supposition.* Let SM = 5500, SN = 10700. Now in the triangle SKM, we have SK = 10142, SM = 5500, and the angle SKM = 30° 9′ 50″, hence, the angle KSM = 81° 54′ 8″, which subtracted from the longitude of K = 8ˢ 6° 56′ 10, leaves 5ˢ 15°. 2′. 2 for the heliocentric longitude of the comet.

664. By Art. 278 sin. ang. SKM = 30° 9′ 50″ : sin. ang. KSM = 81° 54′ 8″ :: tang. geoc. lat. 27° 9 : tang. hel. lat. = 45° 17′. 49″½. And (plane Trig.) cos. hel. lat. 45° 17′ 49″½ : rad. :: curt. dist. SM = 5500 : 7818,84 the true distance of the comet from the sun in the plane of its orbit.

665. For the position on July 29, in the triangle SLN, we have SL = 10148, 5, SN = 10700, and the angle SLN = 51°. 13. 10; hence, the angle LSN = 81° 6′ 7″½, which added to the longitude of L = 10ˢ. 6°. 7′. 10″, gives 0ˢ. 27°. 13′ 17″½ for the heliocentric longitude of the comet. Now as the comet is in its node at N, SN is the true distance of the comet from the sun, and its latitude is nothing.

666. As the difference of the heliocentric longitudes of the comet is 137°. 48′. 44″½ take an arc MN equal to that quantity, and make a spherical triangle, right angled at M, whose perpendicular Mm = 45° 17′ 49″½, the comet's heliocentric latitude on May 28; and Nm will measure the angle described by the comet about the sun in the interval of these two times, now rad : cos. MN = 137° 48′ 44″½ :: cos. Mm = 45°. 17′. 49″½ : cos. Nm = 121°. 24′. 48″ *. FIG. 170

667. Hence, if WAZ be the true parabola which the comet describes about FIG. the sun S, we have found SN = 10700, Sm = 7818,84, and the angle mSN, = 171

* If the comet had not been in its node N at one of the times, but at m', and $m'M'$ be perpendicular to NM, the heliocentric latitude $M'm'$ must have been calculated, and then from knowing two heliocentric latitudes Mm, $M'm'$, and difference MM' of longitudes we can (281, or 282) find the place of the node N, and the inclination of the orbit, and then calculating Nm, Nm', we get mm' the angle which the comet has described about the sun; and from the curtate distance at M', and the heliocentric latitude, we can find the distance of the comet from the sun, as at the other time.

121°. 21'. 48", to determine the angles ASm, ASN, and the perihelion distance AS.

668. By Art. 648. take half the difference of the logarithms of SN and Sm, which is 0,068121, add 10 to the index, and it becomes 10,068121, which is the log. tangent of 49°. 28'. 31", from which subtract 45°, and there remains 4°. 28'. 31"; hence (649), rad. : tang. 4°. 28'. 31" :: cot. $\frac{1}{2}$ $NSA + mSA$ = 30°. 21'. 12" : tan. 7°. 36'. 45$\frac{1}{2}$ which is $\frac{1}{2}$ of $NSA - mSA$; hence, knowing the sum and difference of ASN, ASm, we find ASN = 74°. 55'. 55", and mSA = 45°. 28'. 53", the true anomalies.

669. To find SA, we have (646), rad.^{2} : cos.^{2} $\frac{1}{2}$ ASm = 22°. 44'. 26"$\frac{1}{2}$ :: 7518,81 : AS = 6650,7, the mean distance of the earth from the sun being 10000, but if we call that distance unity, then AS = 0,66508.

670. Now (Table III) the number of days corresponding to the anomalies 45°. 28'. 53" and 74°. 55'. 55", are 36,4781 and 77,1725, whose sum is 113,6506; hence (645), if to $\frac{3}{2}$ log. of 0,66508 we add the log. of 113,6506, we have the log. of 61,638 days for the time from m to N, which should have been 62 days. Hence we must make a new supposition.

671. *Second Supposition.* Let SM = 5600, and SN = 10700 as before. Then, proceeding as before, the heliocentric longitudes will be found to be 8°. 12°. 38'. and 0°. 27°. 13'. 17"$\frac{1}{2}$; the latitude on May 28, = 43°. 26'. 32"$\frac{1}{2}$; the log. of Sm = 3,992083; the angle mSA = 45°. 32'. 40"$\frac{1}{2}$, NSA = 74°. 26'. 1"$\frac{1}{2}$, the days (Table III.) corresponding to which are 36,541 and 74,418; the logarithm of SA is 9,831391, calling the mean distance of the earth unity; hence we have the time from m to N = 62,039 days.

672. Hence, by increasing SM 100 parts, the time has been increased 0,401 days; therefore, by the rule of false, 0,401 : 100 :: 0,362 of a day (the number wanting in the first supposition) : 90,3; increase therefore, SM in the first supposition by 90,3, instead of 100.

673. *Third Supposition.* Let SM = 5590,3, SN = 10700. Then, by a like proceeding, the heliocentric longitudes will be found to be 8°. 12°. 30'. 44" and 0°. 27°. 13'. 17"$\frac{1}{2}$; the latitude on May 28, = 41°. 25'. 14"; the log. of Sm = 3,991264; the angle mSA = 45°. 32'. 8", NSA = 74°. 34'. 16", the days (Table III.) corresponding to which are 36,531 and 74,693; the log. of SA is 9,830802; hence we have the time from m to N = 62,001 days, which answers very accurately to the observed time. A parabola therefore being found which agrees with two observations, we must see how it will agree with some third observation; for instance, that on August 17. To do this, we must first find all the other elements of the parabola.

674. Now the descending node N is in 0°. 27°. 13'. 17"; add to this the angle NSA = 74°. 34'. 16", and it gives 3°. 21°. 47'. 33", for the longitude of the perihelion on its orbit. The time in the Table corresponding to the angle NSA

is 74,693; hence (645), to the log. of SA add the log. of 74,693, and it gives the log. of 41,6374 days, the time of describing the angle ASN, subtract this from July 29, 8*h* 18', the time when the comet was in its node, and it gives June 17, 17*h* 30' the time when the comet was in its perihelion. Also, as $NM = 120°\ 6'\ 24''$, and $Mm = 45°.\ 25'\ 54''$, the angle $MNm = 55°\ 20'\ 16''$, the inclination of the orbit.

675. From these elements, calculate (651) the geocentric longitude of the comet at some other time at which it was observed, for instance, on August 17, at 14*h* 20', and it will be found in ♊ 6° 55' 31'', differing 5' 23'' from observation; the parabola therefore does not satisfy this third observation. We must therefore make another supposition.

676. *Fourth Supposition.* Let $SM = 5500$ and $SN = 10800$. Then, proceeding as before, the heliocentric longitudes are $5^s.\ 15°.\ 2'.\ 2''$ and $0^s.\ 27°.\ 47'.\ 4''$ for May 28 and July 29, the latitude on May 28, $= 47°.\ 17'.\ 49''\frac{1}{2}$, the logarithm of $Sm = 3,898148$, the angle $mSA = 44°.\ 49'.\ 57''$, $NSA = 76°\ 16'\ 5''$; the corresponding days (Table III.) are 35,8965 and 77,8022, and the logarithm of $SA = 9,824898$; hence, the time from m to $N = 62,068$ days.

677. This supposition compared with the first, shows, that by increasing SN by 100, the time has been increased 0,43 days; hence, 0,43 : 100 :: 0,362 (the defect of the time from the first supposition) : 84 the quantity by which SN should have been increased in the first supposition, to have made the time 62 days. Hence,

678. *Fifth Supposition.* Let $SM = 5500$ and $SN = 10784$. Then, as before, the respective heliocentric longitudes will be found to be $5^s.\ 15°.\ 2'.\ 2''$ and $0^s.\ 27°.\ 42'.\ 25''$; the latitude $= 45°\ 17'\ 49''\frac{1}{2}$; the logarithm of $Sm = 3,898142$; the angle $mSA = 44°.\ 55'\ 34''$, $NSA = 76°.\ 12'.\ 48''$; the corresponding days (Table III.) are 34,944 and 76,697; the logarithm of $SA = 9,824588$, hence, the time from m to $N = 61,999$ days, answering extremely near.

679. Determine (671) the other elements of the orbit, and we shall find the descending node in $0^s.\ 27°.\ 42'.\ 25''$; the perihelion in $3^s.\ 13°.\ 55'.\ 8''$; the time of the passage through the perihelion June 16, at 23*h*. 23'; and the inclination $56°.\ 8'.\ 44''$.

680. With these elements, calculate (651) the geocentric longitude of the comet on August 17, at 14*h*. 20', and it will be found in ♊ 7°. 8'. 42'', which exceeds that by observation by 7'. 42''.

681. Now as the corrections made to the distances SM, SN are very nearly in proportion to the differences between the calculations and the observations, we have, as 5'. 23'' + 7'. 42'' (the sum of the errors, or differences between the calculations and observations) : 5'. 23'' (the error from the third supposition) :: 90,5 and 84 (the corrections made to SM, SN) : 36,75 and 34 the corrections necessary to be made to satisfy all the conditions. If the

two errors had been of the same kind, the first term must have been their difference.

682. To apply these corrections, it is manifest, from comparing the computations with the observations, that SM is too much in the third supposition, and too little in the fifth; therefore subtract 36,75 from 5590,5 and it gives 5553,75 for SM; and, that the time may remain the same, add 34 to SN in the third supposition, and it gives SN = 10734. With these values make a

683. *Sixth Supposition.* Let SM = 5553,75, SN = 10734. Compute as before, and the heliocentric longitudes will be found to be 3s. 11°. 42'. 13", and 0s. 27°. 25'. 14"; the latitude = 45°. 23'. 4" on May 28; the logarithm of Sm = 9,898045; the angle mSA = 45°. 16'. 54", NSA = 73°. 13'. 26", the days (Table III.) corresponding to which are 36,28 and 75,872; the logarithm of SA = 9,828388, and the time from m to N is 62 days. Hence, from these correct elements, as before explained, the place of the descending node is found to be in 0s. 27°. 54'. 14"; the place of the perihelion in 3s. 13°. 38'. 40"; the passage through the perihelion June 17, at 10h. 9'. 30" mean time; the inclination of the orbit = 85°. 49'. 44"; and the true place of the comet (651) on August 17, at 14h. 30' is in 7s. 0°. 57'½ u, with 14°. 54'. 4". south latitude, agreeing, very nearly, with the observations.

684. When we compute to see how the assumptions agree with the observations on August 17, we might have compared the latitudes instead of the longitudes; and it will be best to do so, when the latitudes vary faster than the longitudes, which will happen when the inclination of the orbit is very considerable, and the comet near the node.

685. If it so happen that the node does not lie so near the observations that its place can be found by interpolation, after finding the orbit, compute two heliocentric latitudes and longitudes near to the node, and then (281, or 282) the place of the node, and the inclination of the orbit may be found at the same time. In this case, one of the first assumed times does not give the projected point N that of the node; but this makes no difference in the operation, except that we must then compute the heliocentric latitude (as at the other time), in order to get the distance of the comet from the sun, and the angle described about the sun, as explained in the *first* supposition.

686. As the comets do not move in parabolas, but in very excentric ellipses, it is impossible to find a parabola agreeing accurately to all the *data*; it will be sufficient therefore when it agrees very nearly. If after assuming SM, SN, you find the result to differ very considerably from observation, the proportion will not give you their values sufficiently near; in that case, you must compute again the error, and repeat the operation till the result from the computation does agree with the observations.

687 In Art. 266, it is proved, that if you increase any one quantity of the *data* from which a calculation is to be made, by a very small quantity, the result will vary in proportion to that increase. Hence, the reason of the whole operation will be manifest from this Table.

	1st *Sup*	2d *Sup*	3d *Sup*	4th *Sup*	7th *Sup*	3d *Sup*	5th *Sup*	6th *Sup*
$SM =$	A	$A+a$	$A+\alpha$	A	A	$A+\alpha$	A	$A+\alpha+\alpha'$
SN	B	B	B	$B+b$	$B+\beta$	B	$B+\beta$	$B+\beta'$
	m	n	0	r	0	x	y	0
	Error in the interval of *Time*					Error in *Long.* or *Lat.*		

688 The suppositions A and B produce an error m, also, $A+a$ and B produce an error n, hence, from what we have just now explained, as the difference of the results $m \pm n$ (according as the errors are of different or the same affections) must vary as a varies, $m \pm n : m :: a : \alpha$ the quantity by which we must alter A, in order to destroy the error m, or to make the error $=0$. In like manner, the suppositions A and $B+b$ produce an error r, therefore $m \pm r : m :: b : \beta$ the quantity by which B must be altered in order to destroy the error m. Hence, we have got two suppositions, or two parabolas, which will answer the first condition, that is, of the time. Now for the second condition, the third and fifth suppositions will produce an error of x and y respectively; one of these suppositions therefore must be corrected, so that no error may remain in either condition; let therefore $A+\alpha+\alpha'$ and $B+\beta'$ be the values of SM and SN to satisfy both. Now A altered by α produces an effect m, for it corrects the whole error; hence, $\alpha : \alpha' :: m : \frac{m\alpha'}{\alpha}$ the error that would be made by altering A by α'; and as B altered by β produces an effect m, we have $\beta : \beta' :: m : \frac{m\beta'}{\beta}$ the effect that would arise by altering B by β'; hence, that no error may be produced in the time in the third supposition, by adding α' to $A+\alpha$, and β' to B, these two effects thus produced must destroy each other, or $\frac{m\alpha'}{\alpha}+\frac{m\beta'}{\beta}=0$, or $\frac{\alpha'}{\alpha}=-\frac{\beta'}{\beta}$. Hence, that no error may be made in the time in the third supposition, by altering the values SM and SN, the increments or decrements must be in the ratio of $\alpha : \beta$; this was done in Art. 684, and therefore the first condition will remain fulfilled. Now the changing of $A+\alpha$ to A, and of B to $B+\beta$, together produce an effect $x \pm y$, according as they are of different, or the

same affections; hence, to produce the effect x, or to destroy that error, as the effect is in proportion to the variations of A and B, $x \pm y : x :: v$ and $\beta : a$ and β', the corrections to be applied to $A + a$ and B to fulfil the second condition, or make the error $= 0$; and this also took place in Art. 681. Hence, by assuming $SM = A + a + v$, and $SN = B + b'$, both errors are destroyed. Although a, b, a, β, v, β', are annexed to A and B by the sign $+$, yet it must be understood that the sign must be $+$ or $-$, according to circumstances.

689. When great accuracy is required, we must take into consideration, the effect of aberration and parallax; the former may be computed by Art. 532. and the latter, by taking the horizontal parallax : that of the sun $= 8'',73$ :: the distance of the sun : the distance of the comet, and then finding the parallax in latitude and longitude, as for the planets.

Ex. On August 21, 1769, the diurnal motion of a comet was 64' in longitude, and 25' in latitude, and its distance from the earth 0,667. Hence (532) the aberration in longitude $= 14''$, and in latitude $= 6''$, both to be added. Now the apparent longitude was 47°. 1'. 31", and latitude 5°. 53'. 48"; hence the apparent longitude corrected for aberration was 47°. 1'. 45", and latitude 5°. 53'. 54". Also, 0,667 : 1 :: 8",73 : 13" the horizontal parallax. Hence, the parallax in longitude is found to be 4", to be added to the true, to give the apparent longitude, and as the true longitude (by computation) was 47°. 2'. 3", the apparent ought to have been 47°. 2'. 7"; hence the error in longitude was 22". Also, the parallax in latitude was 10", to be added to the true, to give the apparent latitude, and as the true latitude (by computation) was 5°. 54'. 16", the apparent ought to have been, 5°. 54'. 26; hence the error in latitude was 32".

690. It is extremely difficult to determine, from computation, the elliptic orbit of a comet, to any degree of accuracy; for when the orbit is very excentric, a very small error in the observation will change the computed orbit into a parabola, or hyperbola. Now, from the thickness and inequality of the atmosphere with which the comet is surrounded, it is impossible to determine, with any great precision, when either the limb or center of the comet pass the wire at the time of observation. And this uncertainty in the observations will subject the computed orbit to a great error. Hence it happened, that M. Bouguer determined the orbit of the comet in 1729 to be an hyperbola. M. Euler first determined the same for the comet in 1744; but having received more accurate observations, he found it to be an ellipse. The period of the comet in 1680 appears, from observation, to be 575 years, which M. Euler, by his computation, determined to be 166½ years. The only safe way to get the period of comets, is to compare the elements of all those which have been computed, and where you find they agree very well, you may conclude that they

are elements of the *same* comet, it being so extremely improbable that the orbits of two different comets should have the same inclination, the same perihelion distance, and the places of the perihelion and node the same. Thus, knowing the periodic time, we get the major axis of the ellipse, and the perihelion distance being known, the minor axis will be known. When the elements of the orbits agree, the comets may be the same, although the periodic times should vary a little, as that may arise from the attraction of the bodies in our system, and which may also alter all the other elements a little. We have already observed, that the comet which appeared in 1759, had its periodic time increased considerably by the attraction of *Jupiter* and *Saturn*. This comet was seen in 1682, 1607 and 1531, all the elements agreeing, except a little variation of the periodic time. Dr. HALLEY suspected the comet in 1680, to have been the same which appeared in 1106, 531, and 44 years before CHRIST. He also conjectured, that the comet observed by APIAN in 1532, was the same as that observed by HEVELIUS in 1661, if so, it ought to have returned in 1790, but it has never been observed. But M. MECHAIN having collected all the observations in 1532, and calculated the orbit again, found it to be sensibly different from that determined by Dr. HALLEY, which renders it very doubtful whether this was the comet which appeared in 1661; and this doubt is increased, by its not appearing in 1790. The comet in 1770, whose periodic time M. LEXELL computed to be 5 years and 7 months, has not been observed since. There can be no doubt but that the path of this comet, for the time it was observed, belonged to an orbit whose periodic time was that found by M. LEXELL, as the computations for such an orbit agreed so very well with the observations. But the revolution was probably longer before 1770; for as the comet passed very near to Jupiter in 1767, its periodic time might be sensibly increased by the action of that planet; and as it has not been observed since, we may conjecture, with M. LEXELL, that having passed in 1779 again into the sphere of sensible attraction of Jupiter, a new disturbing force might probably take place and destroy the effect of the other. According to the above elements, the comet would be in conjunction with Jupiter on August 23, 1779, and its distance from Jupiter would be only $\frac{1}{491}$ of its distance from the sun, consequently the sun's action would be only $\frac{1}{224}$ times that of Jupiter. What a change must this make in the orbit! If the comet returned to its perihelion in March 1776, it would then not be visible. See M. LEXELL's account in the *Phil. Trans.* 1779. The elements of the orbits of the comets in 1264 and 1556 were so nearly the same, that it is very probable it was the same comet; if so, it ought to appear again about the year 1848.

On the Nature and Tails of Comets.

691. Comets are not visible till they come into the planetary regions. They are surrounded with a very dense atmosphere, and from the side opposite to the sun they send forth a tail, which increases as the comet approaches its perihelion, immediately after which it is longest and most luminous, and then it is generally a little bent and convex towards those parts to which the comet is moving; the tail then decreases, and at last it vanishes. Sometimes the tail is observed to put on this figure towards its extremity ~, as that did in 1769. The smallest stars are seen through the tail, notwithstanding its immense thickness, which proves that its matter must be extremely rare. The opinion of the ancient philosophers, and of ARISTOTLE himself, was, that the tail is a very thin fiery vapour arising from the comet. APIAN, CARDAN, TYCHO, and others, believed that the sun's rays being propagated through the transparent head of the comet, were refracted, as in a lens. But the figure of the tail does not answer to this; and, moreover, there should be some reflecting substance to render the rays visible, in like manner as there must be dust or smoke flying about in a dark room, in order that a ray of light entering it may be seen by a spectator standing side-ways from it. KEPLER supposed, that the rays of the sun carry away some of the gross parts of the comet which reflects the sun's rays, and gives the appearance of a tail. HEVELIUS thought, that the thinnest parts of the atmosphere of a comet are rarefied by the force of the heat, and driven from the fore part and each side of the comet towards the parts turned from the sun. Sir I. NEWTON thinks, that the tail of a comet is a very thin vapour, which the head, or nucleus of the comet, sends out by reason of its heat. He supposes, that when a comet is descending to its perihelion, the vapours behind the comet in respect to the sun, being rarefied by the sun's heat, ascend, and take up with them the reflecting particles with which the tail is composed, as air rarefied by heat carries up the particles of smoke in a chimney. But as beyond the atmosphere of the comet, the ætherial air (*auram ætheream*) is extremely rare, he attributes something to the sun's rays carrying with them the particles of the atmosphere of the comet. And when the tail is thus formed, it, like the nucleus, gravitates towards the sun, and by the projectile force it received from the comet, it describes an ellipse about the sun, and accompanies the comet. It conduces also to the ascent of these vapours, that they revolve about the sun, and therefore endeavour to recede from it; whilst the atmosphere of the sun is either at rest or moves with such a slow motion as it can acquire from the rotation of the sun about its axis. These are the causes of the ascent of the tails in the neighbourhood of the sun, where the orbit has a greater cur-

vature, and the comet moves in a denser atmosphere of the sun. The tail of the comet therefore being formed from the heat of the sun, will increase till it comes to its perihelion, and decrease afterwards. The atmosphere of the comet is diminished as the tail increases, and is least immediately after the comet has passed its perihelion, where it sometimes appears covered with a thick black smoke. As the vapour receives two motions when it leaves the comet, it goes on with the compound motion, and therefore the tail will not be turned directly from the sun, but decline from it towards those parts which are left by the comet; and meeting with a small resistance from the æther, will be a little curved. When the spectator therefore is in the plane of the comet's orbit, the curvature will not appear. The vapour thus rarefied and dilated, may be at last scattered through the heavens, and be gathered up by the planets, to supply the place of those fluids which are spent in vegetation and converted into earth. This is the substance of Sir I. NEWTON's account of the tails of comets. Against this opinion, Dr HAMILTON, in his *Philosophical Essays*, observes, that we have no proof of the existence of a solar atmosphere; and if we had, that when the comet is moving in its perihelion in a direction at right angles to the direction of its tail, the vapours which then arise, partaking of the great velocity of the comet, and being also specifically lighter than the medium in which they move, must suffer a much greater resistance than the dense body of the comet does, and therefore ought to be left behind, and would not appear opposite to the sun; and afterwards they ought to appear towards the sun. Also, if the splendor of the tails be owing to the reflection and refraction of the sun's rays, it ought to diminish the lustre of the stars seen through it, which would have their light reflected and refracted in like manner, and consequently their brightness would be diminished. Dr HALLEY, in his description of the *Aurora Borealis* in 1716, says, "the streams of light so much resembled the long tails of comets, that at first sight they might well be taken for such." And afterwards, "this light seems to have a great affinity to that which the effluvia of electric bodies emit in the dark." *Phil. Trans.* N° 347. D. de MAIRAN also calls the tail of a comet, the aurora borealis of the comet. This opinion Dr HAMILTON supports by the following arguments. A spectator, at a distance from the earth, would see the aurora borealis in the form of a tail opposite to the sun, as the tail of a comet lies. The aurora borealis has no effect upon the stars seen through it, nor has the tail of a comet. The atmosphere is known to abound with electric matter, and the appearance of the electric matter in vacuo is exactly like the appearance of the aurora borealis, which, from its great altitude, may be considered to be in as perfect a vacuum as we can make. The electric matter in vacuo suffers the rays of light to pass through, without being affected by them. The tail of a comet does not expand itself sideways, nor does the electric matter. Hence, he sup-

poses the tails of comets, the aurora borealis, and the electric fluid, to be matter of the same kind. We may add, as a further confirmation of this opinion, that the comet in 1607 appeared to shoot out the end of its tail. Le P. CYSAT remarked the undulations of the tail of the comet in 1618. HEVELIUS observed the same in the tails of the comets in 1652 and 1661. M. PINGRE took notice of the same appearance in the comet of 1769. These are circumstances exactly similar to the aurora borealis. Dr. HAMILTON conjectures, that the use of the comets may be to bring the electric matter, which continually escapes from the planets, back into the planetary regions. The arguments are certainly strongly in favour of this hypothesis; and if this be true, we may further add, that the tails are hollow; for if the electric fluid only proceed in its first direction, and do not diverge sideways, the parts directly behind the comet will not be filled with it; and this thinness of the tails will account for the appearance of the stars through them. From Dr. HERSCHEL's observations on the comet in 1811, he concluded that comets are luminous bodies; for this comet did not appear gibbous, when, as an opaque body, it ought; and its brilliancy, when at an immense distance, was such as could be expected only from a luminous body.

692. The length of a comet's tail may be thus found. Let S be the sun, E the earth, C the comet, CL the tail when directed from the sun; then knowing the place of the comet, we know the angle ECL, EC, and the angle CEL, the angle under which the tail appears; hence we find CL the length of the tail. If the tail deviate by any angle LCM, found from observation, then we shall know the angle ECM, with CE, and the angle CEM, to find CM. The tail of the comet in 1680 appeared under an angle of 70°, according to Sir I. NEWTON, and very brilliant; that of 1618, under an angle of 104°, according to LONGOMONTANUS; that of 1769, under an angle of 90°, according to M. PINGRE, but the light was very faint.

693. The limit of a comet's distance may be very easily ascertained from its tail, it being supposed to be directed from the sun. For let S be the sun, E the earth, ET the line in which the head of the comet appears, EW the line in which the extremity of the tail is observed, and draw ST parallel to EW; then the comet is within the distance ET; for if the comet were at T, the tail would be directed in a line parallel to EW, and therefore it never could appear in that line. Now we know TEW by observation, and consequently its equal ETS, together with TES the angular distance of the comet from the sun, and ES, to find ST the limit. For example; on December 21, 1680, the distance of the comet from the sun was 32°. 24′, and length of the tail 70°; hence, $ST : SE$ :: sin. 32°. 24′ : sin. 70° :: 4 : 7 nearly, therefore the comet's distance from the sun was less than $\frac{4}{7}$ of the earth's distance from the sun. Hence Sir I. NEWTON deduced this conclusion, that all comets, whilst they are visible, are

not further distant from the sun than three times the earth's distance from the sun This however must depend upon the goodness of the telescope, and magnitude of the comet

694 In respect to the nature of comets, Sir I NEWTON observes, that they must be solid bodies like the planets. For if they were nothing but vapours, they must be dissipated when they come near the sun For the comet in 1680, when it was in its perihelion, was less distant from the sun than one sixth of the sun's diameter, consequently the heat of the comet at that time was to the heat of the summer sun as 28000 to 1 But the heat of boiling water is about three times greater than the heat which dry earth acquires from the summer sun, and the heat of red hot iron about three or four times greater than the heat of boiling water Therefore the heat of dry earth at the comet, when in its perihelion, was about 2000 times greater than red hot iron. By such heat, all vapours would be immediately dissipated.

695 This heat of the comet must be retained a very long time For a red hot globe of iron of an inch diameter, exposed to the open air, scarce loses all its heat in an hour, but a greater globe would retain its heat longer, in proportion to its diameter, because the surface, at which it grows cold, varies in that proportion less than the quantity of hot matter Therefore a globe of red hot iron, as big as our earth, would scarcely cool in 50000 years.

696. The comet in 1680 coming so near to the sun, must have been considerably retarded by the sun's atmosphere, and therefore being attracted nearer at every revolution, it will at last fall into the sun, and be a fresh supply of fuel for what the sun loses by its constant emission of light. In like manner, fixed stars which have been gradually wasted, may be supplied with fresh fuel, and acquire new splendor, and pass for new stars. Of this kind are those fixed stars which appear on a sudden, and shine with a wonderful brightness at first, and afterwards vanish by degrees. Such is the conjecture of Sir I NEWTON

697 From the beginning of our æra to this time, it is probable, according to the best accounts, that there have appeared about 500 comets. Before that time about 100 others are recorded to have been seen, but it is probable that not above half of them were comets And when we consider, that many others may not have been perceived, from being too near the sun—from appearing in moon light—from being in the other hemisphere—from being too small to be perceived, or which may not have been recorded, we might imagine the whole number to be considerably greater; but it is likely, that of the comets which are recorded to have been seen, the same may have appeared several times, and therefore the number may be less than is here stated The comet in 1786, which first appeared on August 1, was discovered by MISS CAROLINE HERSCHEL, a sister of DR. HERSCHEL; since that time, she has discovered three

others. As the plan of this Work does not permit us to give all the different methods by which the orbits of comets may be computed, and all the various opinions respecting them, if the reader wish to see any thing further upon the subject, I refer him to a Treatise entitled, *Cometographie ou Traité Historique et Théorique des Cometes, par* M. PINGRÉ, *II. Tom. quarto, Paris,* 1784, or Sir H. ENGLEFIELD'S *Determination of the Orbits of Comets,* a very valuable work, in which the ingenious Author has explained, with great clearness and accuracy, the manner of computing the orbits of comets, according to the methods of BOSCOVICH, and M. de la PLACE.

The following Tables are for converting time into Decimals of a Day; and for the Parabolic Motion of Comets.

TABLE I.

FOR CONVERTING TIME INTO DECIMALS OF A DAY.

HOURS	DECIMALS	MINUTES	DECIMALS	MINUTES.	DECIMALS.
1	0,04166	1	0,000694	31	0,021527
2	0,08333	2	0,001388	32	0,022222
3	0,12500	3	0,002083	33	0,022916
4	0,16666	4	0,002777	34	0,023611
5	0,20833	5	0,003472	35	0,024305
6	0,25000	6	0,004166	36	0,025000
7	0,29166	7	0,004861	37	0,025694
8	0,33333	8	0,005555	38	0,026388
9	0,37500	9	0,006250	39	0,027083
10	0,41666	10	0,006944	40	0,027777
11	0,45833	11	0,007638	41	0,028472
12	0,50000	12	0,008333	42	0,029166
13	0,54166	13	0,009027	43	0,029861
14	0,58333	14	0,009722	44	0,030555
15	0,62500	15	0,010416	45	0,031250
16	0,66666	16	0,011111	46	0,031944
17	0,70833	17	0,011805	47	0,032638
18	0,75000	18	0,012500	48	0,033333
19	0,79166	19	0,013194	49	0,034027
20	0,83333	20	0,013888	50	0,034722
21	0,87500	21	0,014583	51	0,035416
22	0,91666	22	0,015277	52	0,036111
23	0,95833	23	0,015972	53	0,036805
24	1,00000	24	0,016666	54	0,037500
		25	0,017361	55	0,038194
		26	0,018055	56	0,038888
		27	0,018750	57	0,039583
		28	0,019444	58	0,040277
		29	0,020138	59	0,040972
		30	0,020833	60	0,041666

The First TABLE continued.

SEC.	DECIMALS.	SEC.	DECIMALS.
1"	0,00001157	31	0,00035880
2	0,00002315	32	0,00037037
3	0,00003472	33	0,00038194
4	0,00004630	34	0,00039351
5	0,00005787	35	0,00040509
6	0,00006944	36	0,00041666
7	0,00008102	37	0,00042824
8	0,00009259	38	0,00043981
9	0,00010416	39	0,00045138
10	0,00011574	40	0,00046296
11	0,00012731	41	0,00047454
12	0,00013888	42	0,00048611
13	0,00015046	43	0,00049769
14	0,00016204	44	0,00050926
15	0,00017361	45	0,00052083
16	0,00018518	46	0,00053241
17	0,00019676	47	0,00054398
18	0,00020833	48	0,00055555
19	0,00021991	49	0,00056713
20	0,00023148	50	0,00057870
21	0,00024305	51	0,00059027
22	0,00025463	52	0,00060185
23	0,00026620	53	0,00061342
24	0,00027777	54	0,00062500
25	0,00028935	55	0,00063657
26	0,00030092	56	0,00064813
27	0,00031250	57	0,00065972
28	0,00032407	58	0,00067130
29	0,00033565	59	0,00068287
30	0,00034722	60	0,00069444

TABLE II, for converting decimals of a day into time.

[illegible]

TABLE III.

GENERAL TABLE OF THE PARABOLA. By M. DE LAMBERT

Days	Anomaly D	M	S	Differ. M	S	Days	Anomaly D	M	S	Differ. M	S	Days	Anomaly D	M	S	Differ. M	S
0,00	0	0	0,0	20	54,5	8,00	11	4	53,0	20	30,1	16,00	21	45	53,4	19	25,4
25	0	20	54,5		54,4	25	11	25	23,1		29,0	25	22	5	18,8		22,8
50	0	41	48,9		54,3	50	11	45	52,1		27,5	50	22	24	41,6		20,2
0,75	1	2	43,2	20	54,2	8,75	12	6	19,9	20	26,0	16,75	22	44	1,8	19	17,6
1,00	1	23	37,4		54,0	9,00	12	26	45,9		24,4	17,00	23	3	19,4		14,9
25	1	44	31,4		53,8	25	12	47	10,3		22,7	25	23	22	34,3		12,3
50	2	5	25,2		53,5	50	13	7	33,0		21,1	50	23	41	46,6		9,6
1,75	2	26	18,7	20	53,2	9,75	13	27	54,1	20	19,3	17,75	24	0	56,2	19	6,9
2,00	2	47	11,9		52,8	10,00	13	48	13,4		17,7	18,00	24	20	3,1		4,1
25	3	8	4,7		52,4	25	14	8	31,1		15,8	25	24	39	7,2	19	1,3
50	3	28	57,1		51,9	50	14	28	46,9		14,0	50	24	58	8,5	18	58,6
2,75	3	49	49,0	20	51,4	10,75	14	49	0,9	20	12,2	18,75	25	17	7,1		55,8
3,00	4	10	40,4		50,8	11,00	15	9	13,1		10,3	19,00	25	36	2,9		52,9
25	4	31	31,2		50,3	25	15	29	23,4		8,4	25	25	54	55,8		50,0
50	4	52	21,5		49,6	50	15	49	31,8		6,4	50	26	13	45,8		47,2
3,75	5	13	11,1	20	49,0	11,75	16	9	38,2	20	4,3	19,75	26	32	33,0	18	44,3
4,00	5	34	0,1		48,2	12,00	16	29	42,5		2,4	20,00	26	51	17,3		41,4
25	5	54	48,3		47,4	25	16	49	44,9	20	0,3	25	27	9	58,7		38,4
50	6	15	35,7		46,6	50	17	9	45,2	19	58,2	50	27	28	37,1		35,5
4,75	6	36	22,3	20	45,7	12,75	17	29	43,4		56,0	20,75	27	47	12,6	18	32,5
5,00	6	57	8,0		44,8	13,00	17	49	39,4		53,8	21,00	28	5	45,1		29,5
25	7	17	52,8		43,9	25	18	9	33,2		51,7	25	28	24	14,6		26,5
50	7	38	36,7		42,8	50	18	29	24,9		49,4	50	28	42	41,1		23,5
5,75	7	59	19,5	20	41,8	13,75	18	49	14,3	19	47,2	21,75	29	1	4,6	18	20,4
6,00	8	20	1,3		40,7	14,00	19	9	1,5		44,9	22,00	29	19	25,0		17,3
25	8	40	42,0		39,6	25	19	28	46,4		42,5	25	29	37	42,3		14,3
50	9	1	21,6		38,4	50	19	48	28,9		40,1	50	29	55	56,6		11,2
6,75	9	22	0,0	20	37,2	14,75	20	8	9,0	19	37,8	22,75	30	14	7,8	18	8,0
7,00	9	42	37,2		35,9	15,00	20	27	46,8		35,4	23,00	30	32	15,8		5,0
25	10	3	13,1		34,7	25	20	47	22,2		32,9	25	30	50	20,8	18	1,8
50	10	23	47,8		33,3	50	21	6	55,1		30,4	50	31	8	22,6	17	58,6
7,75	10	44	21,1	20	31,9	15,75	21	26	25,5	19	27,9	23,75	31	26	21,2		55,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly.	Differ	Days.	Anomaly.	Differ.	Days.	Anomaly.	Differ.
	D. M. S.	M. S.		D. M. S.	M. S.		D. M. S.	M. S.
24,00	31. 44. 16,7	17. 52,9	32,00	40. 49. 2,9	16. 6,3	40,00	48. 56. 34,5	14. 19,8
25	32. 2. 9,0	49,1	25	41. 5. 9,2	2,8	25	49. 10. 53,8	16,0
50	32. 19. 58,1	45,9	50	41. 21. 12,0	15. 59,5	50	49. 25. 9,8	12,7
24,75	32. 37. 44,0	17. 42,7	32,75	41. 37. 11,5	56,1	40,75	49. 39. 22,5	14. 9,5
25,00	32. 55. 26,7	39,5	33,00	41. 53. 7,6	52,8	41,00	49. 53. 32,0	6,3
25	33. 13. 6,2	36,2	25	42. 9. 0,4	49,3	25	50. 7. 38,3	14. 3,0
50	33. 30. 42,4	33,0	50	42. 24. 49,7	45,9	50	50. 21. 41,3	13. 59,8
25,75	33. 48. 15,4	17. 29,8	33,75	42. 40. 35,0	15. 42,6	41,75	50. 35. 41,1	13. 56,5
26,00	34. 5. 45,2	26,4	34,00	42. 56. 18,2	39,2	42,00	50. 49. 37,6	53,4
25	34. 23. 11,6	23,2	25	43. 11. 57,4	35,9	25	51. 3. 31,0	50,1
50	34. 40. 34,8	19,9	50	43. 27. 33,3	32,4	50	51. 17. 21,1	47,0
26,75	34. 57. 54,7	17. 16,7	34,75	43. 43. 5,7	15. 29,1	42,75	51. 31. 8,1	13. 43,7
27,00	35. 15. 11,4	13,3	35,00	43. 58. 34,8	25,7	43,00	51. 44. 51,8	40,6
25	35. 32. 24,7	10,0	25	44. 14. 0,5	22,4	25	51. 58. 32,4	37,4
50	35. 49. 34,7	6,7	50	44. 29. 22,9	19,0	50	52. 12. 9,8	34,3
27,75	36. 6. 41,4	17. 3,4	35,75	44. 44. 41,9	15. 15,6	43,75	52. 25. 44,1	13. 31,1
28,00	36. 23. 44,8	17. 0,0	36,00	44. 59. 57,5	12,3	44,00	52. 39. 15,2	27,9
25	36. 40. 44,8	16. 56,7	25	45. 15. 9,8	9,0	25	52. 52. 43,1	24,9
50	36. 57. 41,5	53,4	50	45. 30. 18,8	5,6	50	53. 6. 8,0	21,7
28,75	37. 14. 34,9	16. 50,0	36,75	45. 45. 24,4	15. 2,2	44,75	53. 19. 29,7	13. 18,5
29,00	37. 31. 24,9	46,7	37,00	46. 0. 26,6	14. 58,9	45,00	53. 32. 48,2	15,5
25	37. 48. 11,0	43,3	25	46. 15. 25,5	55,6	25	53. 46. 3,7	12,4
50	38. 4. 54,9	40,0	50	46. 30. 21,1	52,2	50	53. 59. 16,1	9,3
29,75	38. 21. 34,9	16. 36,6	37,75	46. 45. 13,3	14. 48,9	45,75	54. 12. 25,4	13. 6,2
30,00	38. 38. 11,5	33,2	38,00	47. 0. 2,2	45,6	46,00	54. 25. 31,6	3,1
25	38. 54. 44,7	29,9	25	47. 14. 47,8	42,3	25	54. 38. 34,7	13. 0,1
50	39. 11. 14,6	26,5	50	47. 29. 30,1	39,0	50	54. 51. 34,8	12. 57,0
30,75	39. 27. 41,1	16. 23,1	38,75	47. 44. 9,1	14. 35,7	46,75	55. 4. 31,8	54,0
31,00	39. 44. 4,2	19,7	39,00	47. 58. 44,8	32,3	47,00	55. 17. 25,8	51,0
25	40. 0. 23,9	16,4	25	48. 13. 17,1	29,1	25	55. 30. 16,8	47,9
50	40. 16. 40,3	13,0	50	48. 27. 46,2	25,8	50	55. 43. 4,7	44,9
31,75	40. 32. 53,3	16. 9,0	39,75	48. 42. 12,0	14. 22,5	47,75	55. 55. 49,6	12. 41,9

THE THIRD TABLE CONTINUED.

Days	Anomaly D	Anomaly M	Anomaly S	Differ M	Differ S
48,00	56	8	31,5	12	39,0
25	56	21	10,5		35,9
50	56	33	46,4		33,0
48,75	56	46	19,1		
				12	30,0
49,00	56	58	19,4		27,1
25	57	11	7,5		24,1
50	57	21	40,6		21,9
49,75	57	36	1,8		
				12	18,3
50,00	57	48	20,1		15,4
25	58	0	35,5		12,5
50	58	12	48,0		9,6
50,75	58	24	57,6		
				12	6,7
51,00	58	37	4,3		3,9
25	58	49	8,2	12	1,0
50	59	1	9,2	11	58,2
51,75	59	13	7,4		
					55,8
52,00	59	25	2,7		52,5
25	59	36	55,2		49,7
50	59	48	44,9		46,9
52,75	60	0	31,8		
				11	44,1
53,00	60	12	15,9		41,4
25	60	23	57,1		38,6
50	60	35	35,9		35,8
53,75	60	47	11,7		
				11	33,1
54,00	60	58	44,8		30,4
25	61	10	15,2		27,6
50	61	21	42,8		24,9
54,75	61	33	7,7		
				11	22,2
55,00	61	44	29,9		19,6
25	61	55	49,5		16,8
50	62	7	6,3		14,5
55,75	62	18	20,5		
				11	11,6

Days	Anomaly D	Anomaly M	Anomaly S	Differ M	Differ S
56,00	62	29	32,1	11	8,9
25	62	40	41,0		6,3
50	62	51	47,3		3,6
56,75	63	2	50,9		
				11	1,1
57,00	63	13	52,0	10	58,4
25	63	24	50,4		55,9
50	63	35	46,3		53,8
57,75	63	46	39,6		
				10	50,7
58,00	63	57	30,3		48,1
25	64	8	18,4		45,7
50	64	19	4,1		43,1
58,75	64	29	47,2		
				10	40,5
59,00	64	40	27,7		38,1
25	64	51	5,8		35,5
50	65	1	41,3		33,1
59,75	65	12	14,4		
				10	30,6
60,00	65	22	45,0		28,1
25	65	33	13,1		25,7
50	65	43	38,8		23,2
60,75	65	54	2,0		
				10	20,8
61,00	66	4	22,8		18,4
25	66	14	41,2		16,0
50	66	24	57,2		13,6
61,75	66	35	10,8		
				10	11,2
62,00	66	45	22,0		8,8
25	66	55	30,8		6,4
50	67	5	37,2		4,1
62,75	67	15	41,3		
				10	1,8
63,00	67	25	43,1	9	59,4
25	67	35	42,5		57,1
50	67	45	39,6		54,8
63,75	67	55	34,4		
				9	52,5

Days	Anomaly D	Anomaly M	Anomaly S	Differ M	Differ S
64,00	68	5	26,9	9	50,2
25	68	15	17,1		47,9
50	68	25	5,0		45,6
64,75	68	34	50,6		
				9	43,4
65,00	68	44	34,0		41,1
25	68	54	15,1		38,9
50	69	3	54,0		36,7
65,75	69	13	30,7		
				9	34,4
66,00	69	23	5,1		32,3
25	69	32	37,4		30,0
50	69	42	7,4		27,9
66,75	69	51	35,3		
				9	25,7
67,00	70	1	1,0		23,5
25	70	10	24,5		21,3
50	70	19	45,8		19,2
67,75	70	29	5,0		
				9	17,1
68,00	70	38	22,1		15,0
25	70	47	37,1		12,9
50	70	56	50,0		10,7
68,75	71	6	0,7		
				9	8,6
69,00	71	15	9,3		6,6
25	71	24	15,9		4,5
50	71	33	20,4		2,4
69,75	71	42	22,8		
				9	0,4
70,00	71	51	23,2	8	58,3
25	72	0	21,5		56,3
50	72	9	17,8		54,3
70,75	72	18	12,1		
				8	52,3
71,00	72	27	4,4		50,2
25	72	35	54,6		48,2
50	72	44	42,8		46,3
71,75	72	53	29,1		
				8	44,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
120,00	93.	24.	13,6			128,00	95.	45.	45,4			136,00	97.	55.	28,6		
				4.	37,1					4.	13,5					3.	52,8
25	93.	28.	50,7			25	95.	49.	58,9			25	97.	59.	21,4		
					36,3						12,8						52,2
50	93.	33.	27,0			50	95.	54.	11,7			50	98.	3.	13,6		
					35,5						12,0						51,6
120,75	93.	38.	2,5			128,75	95.	58.	23,7			136,75	98.	7.	5,2		
				4.	34,7					4.	11,3					3.	51,0
121,00	93.	42.	37,2			129,00	96.	2.	35,2			137,00	98.	10.	56,2		
					33,9						10,7						50,4
25	93.	47.	11,1			25	96.	6.	45,9			25	98.	14.	46,6		
					33,2						10,0						49,8
50	93.	51.	44,3			50	96.	10.	55,9			50	98.	18.	36,4		
					32,4						9,4						49,3
121,75	93.	56.	16,7			129,75	96.	15.	5,3			137,75	98.	22.	25,7		
				4.	31,7					4.	8,7					3.	48,6
122,00	94.	0.	48,4			130,00	96.	19.	14,0			138,00	98.	26.	14,3		
					30,8						8,0						48,1
25	94.	5.	19,2			25	96.	23.	22,0			25	98.	30.	2,4		
					30,1						7,4						47,4
50	94.	9.	49,3			50	96.	27.	29,4			50	98.	33.	49,8		
					29,4						6,7						46,9
122,75	94.	14.	18,7			130,75	96.	31.	36,1			138,75	98.	37.	36,7		
				4.	28,6					4.	6,1					3.	46,3
123,00	94.	18.	47,3			131,00	96.	35.	42,2			139,00	98.	41.	23,0		
					27,8						5,3						45,8
25	94.	23.	15,1			25	96.	39.	47,5			25	98.	45.	8,8		
					27,1						4,8						45,1
50	94.	27.	42,2			50	96.	43.	52,3			50	98.	48.	53,9		
					26,4						4,1						44,6
123,75	94.	32.	8,6			131,75	96.	47.	56,4			139,75	98.	52.	38,5		
				4.	25,6					4.	3,4					3.	44,0
124,00	94.	36.	34,2			132,00	96.	51.	59,8			140,00	98.	56.	22,5		
					24,8						2,8						43,4
25	94.	40.	59,0			25	96.	56.	2,6			25	99.	0.	5,9		
					24,1						2,1						42,9
50	94.	45.	23,1			50	97.	0.	4,7			50	99.	3.	48,8		
					23,4						1,5						42,3
124,75	94.	49.	46,5			132,75	97.	4.	6,2			140,75	99.	7.	31,1		
				4.	22,7					4.	0,9					3.	41,8
125,00	94.	54.	9,2			133,00	97.	8.	7,1			141,00	99.	11.	12,9		
					22,0					4.	0,2						41,2
25	94.	58.	31,2			25	97.	12.	7,3			25	99.	14.	54,1		
					21,2					3.	59,6						40,6
50	95.	2.	52,4			50	97.	16.	6,9			50	99.	18.	34,7		
					20,5						59,0						40,1
125,75	95.	7.	12,9			133,75	97.	20.	5,9			141,75	99.	22.	14,8		
				4.	19,8					3.	58,3					3.	39,5
126,00	95.	11.	32,7			134,00	97.	24.	4,2			142,00	99.	25.	54,3		
					19,0						57,7						39,0
25	95.	15.	51,7			25	97.	28.	1,9			25	99.	29.	33,3		
					18,4						57,1						38,4
50	95.	20.	10,1			50	97.	31.	59,0			50	99.	33.	11,7		
					17,6						56,5						37,9
126,75	95.	24.	27,7			134,75	97.	35.	55,5			142,75	99.	36.	49,6		
				4.	17,0					3.	55,8					3.	37,3
127,00	95.	28.	44,7			135,00	97.	39.	51,3			143,00	99.	40.	26,9		
					16,2						55,3						36,8
25	95.	33.	0,9			25	97.	43.	46,6			25	99.	44.	3,7		
					15,5						54,6						36,3
50	95.	37.	16,4			50	97.	47.	41,2			50	99.	47.	40,0		
					14,9						54,0						35,7
127,75	95.	41.	31,3			135,75	97.	51.	35,2			143,75	99.	51.	15,7		
				4.	14,1					3.	53,4					3.	35,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
144,00	99.	4.	[illegible]	1.	[illegible]	152,00	101.	47.	6,7	3.	18,6	160,00	103.	27.	18,5	3.	4,5
25	99.	[illegible]	[illegible]		[illegible]	25	101.	[illegible]	25,1		18,2	25	103.	30.	23,0		4,0
50	[illegible]	[illegible]	[illegible]		[illegible]	50	101.	[illegible]	43,3		17,7	50	103.	33.	27,0		3,6
144,75	[illegible]	[illegible]	[illegible]			152,75	101.	55.	1,0			160,75	103.	36.	30,6		
				1.	[illegible]					3.	17,2					3.	3,2
145,00	100.	[illegible]	[illegible]		[illegible]	153,00	101.	58.	18,2		16,8	161,00	103.	39.	33,8		2,8
25	[illegible]	[illegible]	[illegible]		[illegible]	25	102.	1.	35,0		16,3	25	103.	42.	36,6		2,3
50	[illegible]	[illegible]	[illegible]		[illegible]	50	102.	4.	51,3		15,9	50	103.	45.	38,9		2,0
145,75	[illegible]	[illegible]	[illegible]			153,75	102.	8.	7,2			161,75	103.	48.	40,9		
				1.	[illegible]					3.	15,4					3.	1,5
146,00	100.	[illegible]	[illegible]		[illegible]	154,00	102.	11.	[illegible]		14,9	162,00	103.	51.	42,4		1,2
25	100.	20.	[illegible]		[illegible]	25	102.	14.	[illegible]		14,5	25	103.	54.	43,6		0,7
50	100.	[illegible]	[illegible]		[illegible]	50	102.	17.	52,0		14,0	50	103.	57.	44,3		0,3
146,75	100.	[illegible]	[illegible]			154,75	102.	21.	6,0			162,75	104.	0.	44,6		
				1.	[illegible]					3.	13,6					3.	0,0
147,00	100.	[illegible]	12,2		[illegible]	155,00	102.	24.	[illegible]		13,1	163,00	104.	3.	44,6	2.	59,5
25	100.	41.	40,5		27,9	25	102.	27.	32,7		12,7	25	104.	6.	44,1		59,1
50	100.	44.	8,5		[illegible]	50	102.	30.	45,4		12,2	50	104.	9.	43,2		58,8
147,75	100.	47.	[illegible]			155,75	102.	33.	57,6			163,75	104.	12.	42,0		
				1.	[illegible]					3.	11,8					2.	[illegible]
148,00	100.	51.	[illegible]		[illegible]	156,00	102.	37.	9,4		11,4	164,00	104.	15.	40,3		57,9
25	100.	[illegible]	[illegible]		[illegible]	25	102.	40.	20,7		10,9	25	104.	18.	38,2		57,6
50	100.	57.	55,1		[illegible]	50	102.	43.	31,6		10,5	50	104.	21.	35,8		57,1
148,75	101.	1.	[illegible]			156,75	102.	46.	42,1			164,75	104.	24.	32,9		
				1.	24,9					3.	10,0					2.	56,8
149,00	101.	[illegible]	[illegible]		[illegible]	157,00	102.	49.	52,1		9,6	165,00	104.	27.	29,7		[illegible]
25	101.	[illegible]	[illegible]		[illegible]	25	102.	53.	1,7		9,1	25	104.	30.	26,0		56,0
50	101.	11.	[illegible]		[illegible]	50	102.	56.	10,8		8,7	50	104.	33.	22,0		55,6
149,75	101.	[illegible]	[illegible]			157,75	102.	59.	19,5			165,75	104.	36.	17,6		
				1.	22,8					3.	8,3					2.	55,2
150,00	101.	18.	20,1		[illegible]	158,00	103.	2.	27,8		7,8	166,00	104.	39.	12,8		54,8
25	101.	21.	42,6		[illegible]	25	103.	5.	35,6		7,4	25	104.	42.	7,6		54,5
50	101.	[illegible]	[illegible]		[illegible]	50	103.	8.	43,0		7,0	50	104.	45.	2,1		54,0
150,75	101.	28.	26,1			158,75	103.	11.	[illegible]			166,75	104.	47.	56,1		
				1.	21,4					3.	6,6					2.	53,7
151,00	101.	[illegible]	47,1		20,6	159,00	103.	14.	[illegible]		6,1	167,00	104.	50.	49,8		53,3
25	101.	35.	7,7		[illegible]	25	103.	[illegible]	2,7		[illegible]	25	104.	53.	43,1		52,9
50	101.	38.	27,8		19,[illegible]	50	103.	21.	[illegible]		[illegible]	50	104.	56.	36,0		52,6
151,75	101.	41.	[illegible]			159,75	103.	[illegible]	[illegible]			167,75	104.	59.	28,6		
				1.	19,5					3.	4,8					2.	52,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
168,00	105.	2.	20,7	2.	51,8	176,00	106.	30.	59,4	2.	40,5	184,00	107.	53.	54,6	2.	30,3
25	105.	5.	12,5		51,4	25	106.	33.	39,9		40,2	25	107.	56.	24,9		30,0
50	105.	8.	3,9		51,1	50	106.	36.	20,1		39,8	50	107.	58.	54,9		29,8
168,75	105.	10.	55,0			176,75	106.	38.	59,9			184,75	108.	1.	24,7		
				2.	50,6					2.	39,5					2.	29,4
169,00	105.	13.	45,6		50,3	177,00	106.	41.	39,4		39,2	185,00	108.	3.	54,1		29,1
25	105.	16.	35,9		50,0	25	106.	44.	18,6		38,8	25	108.	6.	23,2		28,8
50	105.	19.	25,9		49,6	50	106.	46.	57,4		38,5	50	108.	8.	52,0		28,5
169,75	105.	22.	15,5			177,75	106.	49.	35,9			185,75	108.	11.	20,5		
				2.	49,2					2.	38,2					2.	28,2
170,00	105.	25.	4,7		48,8	178,00	106.	52.	14,1		37,8	186,00	108.	13.	48,8		27,9
25	105.	27.	53,5		48,5	25	106.	54.	51,9		37,5	25	108.	16.	16,7		27,7
50	105.	30.	42,0		48,1	50	106.	57.	29,4		37,2	50	108.	18.	44,4		27,3
170,75	105.	33.	30,1			178,75	107.	0.	6,6			186,75	108.	21.	11,7		
				2.	47,8					2.	36,9					2.	27,1
171,00	105.	36.	17,9		47,4	179,00	107.	2.	43,5		36,5	187,00	108.	23.	38,8		26,8
25	105.	39.	5,3		47,0	25	107.	5.	20,0		36,2	25	108.	26.	5,6		26,5
50	105.	41.	52,3		46,7	50	107.	7.	56,2		35,9	50	108.	28.	32,1		26,2
171,75	105.	44.	39,0			179,75	107.	10.	32,1			187,75	108.	30.	58,3		
				2.	46,4					2.	35,6					2.	25,9
172,00	105.	47.	25,4		46,0	180,00	107.	13.	7,7		35,3	188,00	108.	33.	24,2		25,6
25	105.	50.	11,4		45,6	25	107.	15.	43,0		35,0	25	108.	35.	49,8		25,3
50	105.	52.	57,0		45,3	50	107.	18.	18,0		34,6	50	108.	38.	15,1		25,1
172,75	105.	55.	42,3			180,75	107.	20.	52,6			188,75	108.	40.	40,2		
				2.	44,9					2.	34,3					2.	24,8
173,00	105.	58.	27,2		44,6	181,00	107.	23.	26,9		34,0	189,00	108.	43.	5,0		24,5
25	106.	1.	11,8		44,2	25	107.	26.	0,9		33,7	25	108.	45.	29,5		24,2
50	106.	3.	56,0		43,9	50	107.	28.	34,6		33,4	50	108.	47.	53,7		23,9
173,75	106.	6.	39,9			181,75	107.	31.	8,0			189,75	108.	50.	17,6		
				2.	43,5					2.	33,1					2.	23,7
174,00	106.	9.	23,4		43,2	182,00	107.	33.	41,1		32,8	190,00	108.	52.	41,3		23,3
25	106.	12.	6,6		42,9	25	107.	36.	13,9		32,4	25	108.	55.	4,6		23,1
50	106.	14.	49,5		42,5	50	107.	38.	46,3		32,1	50	108.	57.	27,7		22,8
174,75	106.	17.	32,0			182,75	107.	41.	18,4			190,75	108.	59.	50,5		
				2.	42,2					2.	31,8					2.	22,6
175,00	106.	20.	14,2		41,8	183,00	107.	43.	50,2		31,5	191,00	109.	2.	13,1		22,3
25	106.	22.	56,0		41,5	25	107.	46.	21,8		31,2	25	109.	4.	35,4		22,0
50	106.	25.	37,5		41,1	50	107.	48.	53,0		31,0	50	109.	6.	57,4		21,7
175,75	106.	28.	18,6			183,75	107.	51.	24,0			191,75	109.	9.	19,0		
				2.	40,8					2.	30,6					2.	21,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
192,00	109.	11.	40,4		
				2.	21,2
25	109.	14.	1,6		
					20,9
50	109.	16.	22,5		
					20,6
192,75	109.	18.	43,1		
				2.	20,4
193,00	109.	21.	3,5		
					20,0
25	109.	23.	23,5		
					19,8
50	109.	25.	43,3		
					19,6
193,75	109.	28.	2,9		
				2.	19,3
194,00	109.	30.	22,2		
					19,0
25	109.	32.	41,2		
					18,7
50	109.	34.	59,9		
					18,5
194,75	109.	37.	18,4		
				2.	18,3
195,00	109.	39.	36,7		
					17,9
25	109.	41.	54,6		
					17,7
50	109.	44.	12,3		
					17,5
195,75	109.	46.	29,8		
				2.	17,1
196,00	109.	48.	46,9		
					17,0
25	109.	51.	3,9		
					16,7
50	109.	53.	20,6		
					16,3
196,75	109.	55.	36,9		
				2.	16,2
197,00	109.	57.	53,1		
					15,9
25	110.	0.	9,0		
					15,6
50	110.	2.	24,6		
					15,4
197,75	110.	4.	40,0		
				2.	15,1
198,00	110.	6.	55,1		
					14,9
25	110.	9.	10,0		
					14,6
50	110.	11.	24,6		
					14,4
198,75	110.	13.	39,0		
				2.	14,1
199,00	110.	15.	53,1		
					13,9
25	110.	18.	7,0		
					13,6
50	110.	20.	20,6		
					13,4
199,75	110.	22.	34,0		
				2.	13,1

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
200,00	110.	24.	47,1		
				4.	25,5
5	110.	29.	12,6		
					24,5
201,0	110.	33.	37,1		
					23,6
5	110.	38.	0,7		
				4.	22,5
202,0	110.	42.	23,2		
					21,6
5	110.	46.	44,8		
					20,7
203,0	110.	51.	5,5		
					19,7
5	110.	55.	25,2		
				4.	18,7
204,0	110.	59.	43,9		
					17,8
5	111.	4.	1,7		
					16,9
205,0	111.	8.	18,6		
					16,0
5	111.	12.	34,6		
				4.	15,0
206,0	111.	16.	49,6		
					14,1
5	111.	21.	3,7		
					13,2
207,0	111.	25.	16,9		
					12,3
5	111.	29.	29,2		
				4.	11,3
208,0	111.	33.	40,5		
					10,5
5	111.	37.	51,0		
					9,6
209,0	111.	42.	0,6		
					8,7
5	111.	46.	9,3		
				4.	7,8
210,0	111.	50.	17,1		
					7,0
5	111.	54.	24,1		
					6,1
211,0	111.	58.	30,2		
					5,2
5	112.	2.	35,4		
				4.	4,3
212,0	112.	6.	39,7		
					3,5
5	112.	10.	43,2		
					2,6
213,0	112.	14.	43,8		
					1,8
5	112.	18.	47,6		
				4.	0,9
214,0	112.	22.	48,5		
				4.	0,1
5	112.	26.	48,6		
				3.	59,3
215,0	112.	30.	47,9		
					58,5
5	112.	34.	46,4		
				3.	57,6

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
216,0	112.	40.	44,0		
				3.	56,8
5	112.	42.	40,8		
					56,0
217,0	112.	46.	36,8		
					55,1
5	112.	50.	31,9		
				3.	54,4
218,0	112.	54.	26,3		
					53,6
5	112.	58.	19,9		
					52 8
219,0	113.	2.	12,7		
					52,0
5	113.	6.	4,7		
				3.	51,2
220,0	113.	9.	55,9		
					50 4
5	113.	13.	46,3		
					49 7
221,0	113.	17.	36,0		
					48,8
5	113.	21.	24,8		
				3.	48,1
222,0	113.	25.	12,9		
					47,3
5	113.	29.	0,2		
					46,6
223,0	113.	32.	46,8		
					45,9
5	113.	36.	32,7		
				3.	45,0
224,0	113.	40.	17,7		
					44,3
5	113.	44.	2,0		
					43,5
225,0	113.	47.	45,5		
					42,8
5	113.	51.	28,3		
				3.	42,1
226,0	113.	55.	10,4		
					41,4
5	113.	58.	51,8		
					40,6
227,0	114.	2.	32,4		
					39,9
5	114.	6.	12,3		
				3.	39,2
228,0	114.	9.	51,5		
					38,5
5	114.	13.	30,0		
					37,7
229,0	114.	17.	7,7		
					37,0
5	114.	20.	44,7		
				3.	36,3
230,0	114.	24.	21,0		
					35,7
5	114.	27.	56,7		
					34,9
231,0	114.	31.	31,6		
					34,3
5	114.	35.	5,9		
				3.	33,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
232,0	114.	38.	29,4	3.	32,9	248,0	116.	26.	48,8	3.	12,6	264,0	118.	5.	0,0	2.	55,4
,5	114.	42.	12,3		32,1	,5	116.	30.	1,3		12,1	,5	118.	7.	55,4		55,0
233,0	114.	45.	44,4		31,5	249,0	116.	33.	13,6		11,5	265,0	118.	10.	50,4		54,4
,5	114.	49.	15,9	3.	30,8	,5	116.	36.	25,1	3.	10,9	,5	118.	13.	44,8	2.	54,0
234,0	114.	52.	46,7		30,2	250,0	116.	39.	36,0		10,4	266,0	118.	16.	38,8		53,5
,5	114.	56.	16,9		29,5	,5	116.	42.	46,4		9,8	,5	118.	19.	32,3		53,0
235,0	114.	59.	46,4		28,9	251,0	116.	45.	56,2		9,2	267,0	118.	22.	25,3		52,5
,5	115.	3.	15,3	3.	28,1	,5	116.	49.	5,4	3.	8,6	,5	118.	25.	17,8	2.	52,0
236,0	115.	6.	43,3		27,5	252,0	116.	52.	14,0		8,1	268,0	118.	28.	9,8		51,5
,5	115.	10.	10,8		26,9	,5	116.	55.	22,1		7,6	,5	118.	31.	1,3		51,1
237,0	115.	13.	37,7		26,2	253,0	116.	58.	29,7		7,0	269,0	118.	33.	52,4		50,5
,5	115.	17.	3,9	3.	25,5	,5	117.	1.	36,7	3.	6,4	,5	118.	36.	42,9	2.	50,1
238,0	115.	20.	29,4		24,9	254,0	117.	4.	43,1		5,9	270,0	118.	39.	33,0		49,7
,5	115.	23.	54,3		24,3	,5	117.	7.	49,0		5,3	,5	118.	42.	22,7		49,1
239,0	115.	27.	18,7		23,6	255,0	117.	10.	54,3		4,8	271,0	118.	45.	11,8		48,7
,5	115.	30.	42,1	3.	23,0	,5	117.	13.	59,1	3.	4,2	,5	118.	48.	0,5	2.	48,3
240,0	115.	34.	5,1		22,4	256,0	117.	17.	3,3		3,7	272,0	118.	50.	48,8		47,8
,5	115.	37.	27,5		21,7	,5	117.	20.	7,0		3,2	,5	118.	53.	36,6		47,2
241,0	115.	40.	49,2		21,1	257,0	117.	23.	10,2		2,7	273,0	118.	56.	23,8		46,9
,5	115.	44.	10,3	3.	20,4	,5	117.	26.	12,9	3.	2,1	,5	118.	59.	10,7	2.	46,3
242,0	115.	47.	30,7		19,9	258,0	117.	29.	15,0		1,6	274,0	119.	1.	57,0		46,0
,5	115.	50.	50,6		19,2	,5	117.	32.	16,6		1,1	,5	119.	4.	43,0		45,5
243,0	115.	54.	9,8		18,6	259,0	117.	35.	17,7		0,5	275,0	119.	7.	28,5		45,0
,5	115.	57.	28,4	3.	18,0	,5	117.	38.	18,2	3.	0,1	,5	119.	10.	13,5	2.	44,6
244,0	116.	0.	46,4		17,4	260,0	117.	41.	18,3	2.	59,5	276,0	119.	12.	58,1		44,1
,5	116.	4.	3,8		16,8	,5	117.	44.	17,8		58,9	,5	119.	15.	42,2		43,7
245,0	116.	7.	20,6		16,2	261,0	117.	47.	16,7		58,5	277,0	119.	18.	25,9		43,2
,5	116.	10.	36,8	3.	15,6	,5	117.	50.	15,2	2.	58,0	,5	119.	21.	9,1	2.	42,9
246,0	116.	13.	52,4		15,0	262,0	117.	53.	13,2		57,4	278,0	119.	23.	52,0		42,4
,5	116.	17.	7,4		14,4	,5	117.	56.	10,6		57,0	,5	119.	26.	34,4		41,9
247,0	116.	20.	21,8		13,8	263,0	117.	59.	7,6		56,4	279,0	119.	29.	16,3		41,5
,5	116.	23.	35,6	3.	13,2	,5	118.	2.	4,0	2.	56,0	,5	119.	31.	57,8	2.	41,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
[illegible]	119.	31.	48,9	2.	40,7	296,0	120.	56.	33,5	2.	27,8	312,0	122.	12.	47,3	2.	16,6
[illegible]	119.	[illegible]	[illegible]		40,3	5	120.	58.	23,3		27,4	5	122.	15.	3,9		16,3
[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	297,0	121.	1.	50,7		27,0	313,0	122.	17.	20,2		15,9
[illegible]	119.	[illegible]	[illegible]			5	121.	4.	17,7			5	122.	19.	36,1		
				2.	39,1					2.	26,7					2.	15,6
[illegible]	119.	44.	[illegible]		38,9	298,0	121.	6.	44,4		26,3	314,0	122.	21.	51,7		15,3
[illegible]	119.	47.	[illegible]		38,5	5	121.	9.	10,7		26,0	5	122.	24.	7,0		15,0
[illegible]	119.	50.	26,2		38,1	299,0	121.	11.	36,7		25,6	315,0	122.	26.	22,0		14,6
[illegible]	119.	[illegible]	11,3			5	121.	14.	2,3			5	122.	28.	36,6		
				2.	37,7					2.	25,2					2.	14,3
[illegible]	119.	[illegible]	[illegible]		[illegible]	300,0	121.	16.	27,5		24,8	316,0	122.	30.	50,9		14,0
[illegible]	119.	[illegible]	[illegible]		[illegible]	5	121.	18.	52,3		24,5	5	122.	33.	4,9		13,7
[illegible]	120.	1.	[illegible]		36,1	301,0	121.	21.	16,8		24,2	317,0	122.	35.	18,6		13,4
[illegible]	120.	[illegible]	[illegible]			5	121.	23.	41,0			5	122.	37.	32,0		
				[illegible]	36,0					2.	23,7					2.	13,1
[illegible]	120.	[illegible]	[illegible]		35,6	302,0	121.	26.	1,7		23,3	318,0	122.	39.	45,1		12,7
[illegible]	120.	[illegible]	[illegible]		35,3	5	121.	28.	28,2		23,0	5	122.	41.	57,8		12,5
[illegible]	120.	11.	[illegible]		34,7	303,0	121.	30.	51,2		22,7	319,0	122.	44.	10,3		12,1
[illegible]	120.	14.	[illegible]			5	121.	33.	13,9			5	122.	46.	22,4		
				2.	34,4					2.	22,5					2.	11,9
[illegible]	120.	[illegible]	[illegible]		34,0	304,0	121.	35.	36,4		22,0	320,0	122.	48.	34,3		11,5
[illegible]	120.	[illegible]	[illegible]		33,6	5	121.	37.	58,4		21,6	5	122.	50.	45,8		11,2
[illegible]	120.	21.	[illegible]		33,2	305,0	121.	40.	20,0		21,3	321,0	122.	52.	57,0		10,9
[illegible]	120.	[illegible]	[illegible]			5	121.	42.	41,3			5	122.	55.	7,9		
				2.	[illegible]					2.	20,9					2.	10,5
[illegible]	120.	[illegible]	[illegible]		[illegible]	306,0	121.	45.	2,2		20,6	322,0	122.	57.	18,4		10,3
[illegible]	120.	[illegible]	[illegible]		[illegible]	5	121.	47.	22,8		20,3	5	122.	59.	28,7		10,0
[illegible]	120.	31.	[illegible]		31,6	307,0	121.	49.	43,1		20,0	323,0	123.	1.	38,7		9,7
[illegible]	120.	[illegible]	[illegible]			5	121.	52.	3,1			5	123.	3.	48,4		
				2.	31,2					2.	19,6					2.	9,4
[illegible]	120.	[illegible]	[illegible]		30,9	308,0	121.	54.	22,7		19,2	324,0	123.	5.	57,8		9,1
[illegible]	120.	[illegible]	[illegible]		[illegible]	5	121.	56.	41,9		18,9	5	123.	8.	6,9		8,7
[illegible]	120.	42.	0,7		[illegible]	309,0	121.	59.	0,8		[illegible]	325,0	123.	10.	15,6		8,5
[illegible]	120.	44.	[illegible]			5	122.	1.	19,4			5	123.	12.	24,1		
				2.	[illegible]					2.	18,3					2.	8,2
[illegible]	120.	47.	[illegible]		[illegible]	310,0	122.	3.	37,7		17,9	326,0	123.	14.	32,3		7,9
[illegible]	120.	49.	29,8		[illegible]	5	122.	5.	55,6		17,5	5	123.	16.	40,2		7,6
[illegible]	120.	51.	[illegible]		[illegible]	311,0	122.	8.	13,1		17,3	327,0	123.	18.	47,8		7,3
[illegible]	120.	54.	[illegible]			5	122.	10.	30,4			5	123.	20.	55,1		
				2.	[illegible]					2.	16,9					2.	7,0

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
328,0	123.	23.	2,1			344,0	124.	28.	19,5			360,0	125.	29.	13,3		
				2.	6,8					1.	58,0					1.	50,3
5	123.	25.	8,9			5	124.	30.	17,5			5	125.	31.	3,6		
					6,4						57,8						50,0
329,0	123.	27.	15,3			345,0	124.	32.	15,3			361,0	125.	32.	53,6		
					6,2						57,4						49,8
5	123.	29.	21,5			5	124.	34.	12,7			5	125.	34.	43,4		
				2.	5,8					1.	57,3					1.	49,6
330,0	123.	31.	27,3			346,0	124.	36.	10,0			362,0	125.	36.	33,0		
					5,6						57,0						49,3
5	123.	33.	32,9			5	124.	38.	7,0			5	125.	38.	22,3		
					5,3						56,7						49,2
331,0	123.	35.	38,2			347,0	124.	40.	3,7			363,0	125.	40.	11,5		
					5,0						56,5						48,9
5	123.	37.	43,2			5	124.	42.	0,2			5	125.	42.	0,4		
				2.	4,8					1.	56,2					1.	48,6
332,0	123.	39.	48,0			348,0	124.	43.	56,4			364,0	125.	43.	49,0		
					4,4						56,0						48,5
5	123.	41.	52,4			5	124.	45.	52,4			5	125.	45.	37,5		
					4,2						55,7						48,2
333,0	123.	43.	56,6			349,0	124.	47.	48,1			365,0	125.	47.	25,7		
					3,9						55,5						48,1
5	123.	46.	0,5			5	124.	49.	43,6			5	125.	49.	13,8		
				2.	3,6					1.	55,3					1.	47,8
334,0	123.	48.	4,1			350,0	124.	51.	38,9			366,0	125.	51.	1,6		
					3,3						55,0						47,5
5	123.	50.	7,4			5	124.	53.	33,9			5	125.	52.	49,1		
					3,1						54,7						47,4
335,0	123.	52.	10,5			351,0	124.	55.	28,7			367,0	125.	54.	36,5		
					2,8						54,4						47,1
5	123.	54.	13,3			5	124.	57.	23,2			5	125.	56.	23,6		
				2.	2,5					1.	54,2					1.	46,9
336,0	123.	56.	15,8			352,0	124.	59.	17,4			368,0	125.	58.	10,5		
					2,2						54,1						46,7
5	123.	58.	18,0			5	125.	1.	11,5			5	125.	59.	57,2		
					2,0						53,8						46,5
337,0	124.	0.	20,0			353,0	125.	3.	5,3			369,0	126.	1.	43,7		
					1,7						53,5						46,3
5	124.	2.	21,7			5	125.	4.	58,8			5	126.	3.	30,0		
				2.	1,4					1.	53,3					1.	46,0
338,0	124.	4.	23,1			354,0	125.	6.	52,1			370,0	126.	5.	16,0		
					1,2						53,1						45,8
5	124.	6.	24,3			5	125.	8.	45,2			5	126.	7.	1,8		
					0,9						52,8						45,7
339,0	124.	8.	25,2			355,0	125.	10.	38,0			371,0	126.	8.	47,5		
					0,6						52,6						45,4
5	124.	10.	25,8			5	125.	12.	30,6			5	126.	10.	32,9		
				2.	0,3					1.	52,3					1.	45,1
340,0	124.	12.	26,1			356,0	125.	14.	22,9			372,0	126.	12.	18,0		
				2.	0,1						52,1						45,0
5	124.	14.	26,2			5	125.	16.	15,0			5	126.	14.	3,0		
				1.	59,8						51,9						44,8
341,0	124.	16.	26,0			357,0	125.	18.	6,9			373,0	126.	15.	47,8		
					59,6						51,7						44,6
5	124.	18.	25,6			5	125.	19.	58,6			5	126.	17.	32,4		
				1.	59,3					1.	51,4					1.	44,4
342,0	124.	20.	24,9			358,0	125.	21.	50,0			374,0	126.	19.	16,8		
					59,0						51,2						44,1
5	124.	22.	23,9			5	125.	23.	41,2			5	126.	21.	0,9		
					58,8						50,9						43,9
343,0	124.	24.	22,7			359,0	125.	25.	32,1			375,0	126.	22.	44,8		
					58,5						50,7						43,8
5	124.	26.	21,2			5	125.	27.	22,8			5	126.	24.	28,6		
				1.	58,3					1.	50,5					1.	43,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
376,0	126.	26.	12,4	1.	43,3
5	126.	27.	55,4		43,1
377,0	126.	29.	58,7		43,0
5	126.	31.	31,7		
				1.	42,7
378,0	126.	[illegible]	[illegible]		42,5
5	126.	34.	46,7		42,3
379,0	126.	36.	29,0		42,1
5	126.	38.	11,1		
				1.	41,9
380,0	126.	39.	53,0		41,7
5	126.	41.	34,7		41,5
381,0	126.	43.	16,2		41,3
5	126.	44	57,5		
				1.	41,1
382,0	126.	46.	38,6		40,9
5	126.	48	19,5		40,7
383,0	126.	50.	0,2		40,5
5	126.	51	40,7		
				1.	40,4
384,0	126.	53.	21,1		40,1
5	126.	55.	1,2		39,9
385,0	126.	56.	41,1		39,8
5	126.	58.	20,9		
				1.	39,5
386,0	127	0	0,4		39,4
5	127.	1.	39,8		39,1
387,0	127	3.	18,9		39,0
5	127.	4.	57,9		
				1.	38,8
388,0	127.	6.	36,7		38,6
5	127.	8.	15,3		38,4
389,0	127.	9.	53,7		38,2
5	127.	11.	31,9		
				1.	38,0
390,0	127.	13.	9,9		37,8
5	127.	14.	47,7		37,7
391,0	127.	16.	25,4		37,4
5	127.	18.	2,8		
				1.	37,3

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
392,0	127.	19.	40,1	1.	37,1
5	127.	21.	17,2		36,9
393,0	127.	22.	54,1		36,7
5	127.	24.	30,8		
				1.	36,6
394,0	127.	26.	7,4		36,3
5	127.	27.	43,7		36,2
395,0	127.	29.	19,9		36,0
5	127.	30.	55,9		
				1.	35,8
396,0	127.	32.	31,7		35,7
5	127.	34.	7,4		35,4
397,0	127.	35.	42,8		35,3
5	127.	37.	18,1		
				1.	35,1
398,0	127.	38.	53,2		34,9
5	127.	40.	28,1		34,7
399,0	127.	42.	2,8		34,6
5	127.	43.	37,4		
				1.	34,4
400	127.	45.	11,8	2.	8,2
401	127.	48.	20,0	3.	7,6
402	127.	51.	27,6		6,8
403	127.	54.	34,4		
				3.	6,2
404	127.	57.	40,6		5,5
405	128.	0.	46,1		4,7
406	128.	3.	50,8		4,2
407	128.	6.	55,0		
				3.	3,4
408	128.	9.	58,4		2,8
409	128.	13.	1,2		2,1
410	128.	16.	3,3		1,4
411	128.	19.	4,7		
				3.	0,8
412	128.	22.	5,5	3.	0,2
413	128.	25.	5,7	2.	59,5
414	128.	28.	5,2		58,8
415	128.	31.	4,0		
				2.	58,2

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
416	128.	34.	2,2	2.	57,6
417	128.	36.	59,8		56,9
418	128.	39.	56,7		56,3
419	128.	42.	53,0		
				2.	55,7
420	128.	45.	48,7		55,1
421	128.	48.	43,8		54,4
422	128.	51.	38,2		53,8
423	128.	54.	32,0		
				2.	53,3
424	128.	57.	25,3		52,6
425	128.	0.	17,9		52,0
426	128.	3.	9,9		51,4
427	128.	6.	1,3		
				2.	50,8
428	129.	8.	52,1		50,2
429	129.	11.	42,3		49,6
430	129.	14.	31,9		49,0
431	129.	17.	20,9		
				2.	48,5
432	129.	20.	9,4		47,9
433	129.	22.	57,3		47,3
434	129.	25.	44,6		46,7
435	129.	28.	31,3		
				2.	46,2
436	129.	31.	17,5		45,6
437	129.	34.	3,1		45,0
438	129.	36.	48,1		44,5
439	129.	39.	32,6		
				2.	43,9
440	129.	42.	16,5		43,4
441	129.	44.	59,9		42,8
442	129.	47.	42,7		42,3
443	129.	50.	25,0		
				2.	41,7
444	129.	53.	6,7		41,2
445	129.	55.	47,9		40,6
446	129.	58.	28,5		40,1
447	~~129.~~	1.	8,6		
				2.	39,6

130

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Diffs.		Days.	Anomaly.			Diffs.		Days.	Anomaly.			Diffs.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
448	130.	3.	48,2			480	131.	24.	31,0			512	132.	37.	37,3		
449	130.	6.	27,3	2.	39,1	481	131.	26.	54,6	2.	23,6	513	132.	39.	47,8	2.	10,5
450	130.	9.	5,8		38,5	482	131.	29.	17,8		23,2	514	132.	41.	58,0		10,2
451	130.	11.	43,8		38,0	483	131.	31.	40,5		22,7	515	132.	44.	7,7		9,7
				2.	37,5					2.	22,2					2.	9,4
452	130.	14.	21,3			484	131.	34.	2,7			516	132.	46.	17,1		
453	130.	16.	58,3		37,0	485	131.	36.	24,5		21,8	517	132.	48.	26,1		9,0
454	130.	19.	34,7		36,4	486	131.	38.	45,9		21,4	518	132.	50.	34,8		8,7
455	130.	22.	10,7		36,0	487	131.	41.	6,9		21,0	519	132.	52.	43,1		8,3
				2.	35,4					2.	20,5					2.	7,9
456	130.	24.	46,1			488	131.	43.	27,4			520	132.	54.	51,0		
457	130.	27.	21,1		35,0	489	131.	45.	47,5		20,1	521	132.	56.	58,6		7,6
458	130.	29.	55,5		34,4	490	131.	48.	7,2		19,7	522	132.	59.	5,8		7,2
459	130.	32.	29,4		33,9	491	131.	50.	26,5		19,3	523	133.	1.	12,6		6,8
				2.	33,3					2.	18,8					2.	6,5
460	130.	35.	2,9			492	131.	52.	45,3			524	133.	3.	19,1		
461	130.	37.	35,[illegible]		32,8	493	131.	55.	3,7		18,4	525	133.	5.	25,2		6,1
462	130.	40.	8,[illegible]		32,[illegible]	494	131.	57.	21,7		18,0	526	133.	7.	31,0		5,8
463	130.	42.	40,[illegible]		31,[illegible]	495	131.	59.	39,3		17,6	527	133.	9.	36,4		5,4
				2.	31,[illegible]					2.	17,2					2.	5,0
464	130.	45.	11,7			496	132.	1.	56,5			528	133.	11.	41,4		
465	130.	47.	42,7		31,0	497	132.	4.	13,4		16,9	529	133.	13.	46,1		4,7
466	130.	50.	13,2		30,5	498	132.	6.	29,7		16,5	530	133.	15.	50,5		4,4
467	130.	52.	43,2		30,0	499	132.	8.	45,7		16,0	531	133.	17.	54,5		4,0
				2.	29,6					2.	15,6					2.	3,7
468	130.	55.	12,8			500	132.	11.	1,3			532	133.	19.	58,2		
469	130.	57.	41,8		29,0	501	132.	13.	16,4		15,1	533	133.	22.	1,5		3,3
470	131.	0.	10,4		28,6	502	132.	15.	31,1		14,7	534	133.	24.	4,5		3,0
471	131.	2.	38,5		28,1	503	132.	17.	45,5		14,4	535	133.	26.	7,2		2,7
				2.	27,7					2.	14,0					2.	2,3
472	131.	5.	6,2			504	132.	19.	59,5			536	133.	28.	9,5		
473	131.	7.	33,4		27,2	505	132.	22.	13,1		13,6	537	133.	30.	11,4		1,9
474	131.	10.	0,1		26,7	506	132.	24.	26,3		13,2	538	133.	32.	13,1		1,7
475	131.	12.	26,4		26,3	507	132.	26.	39,1		12,8	539	133.	34.	14,4		1,3
				2.	25,8					2.	12,4					2.	1,0
476	131.	14.	52,2			508	132.	28.	51,5			540	133.	36.	15,4		
477	131.	17.	17,6		25,4	509	132.	31.	3,5		12,0	541	133.	38.	16,0		0,6
478	131.	19.	42,5		24,9	510	132.	33.	15,2		11,7	542	133.	40.	16,3		0,3
479	131.	22.	7,0		24,5	511	132.	35.	26,4		11,2	543	133.	42.	16,3	2.	0,0
				2.	24,0					2.	10,9					1.	59,7

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
544	133. 44. 15,9	1. 59,3	576	134. 45. 21,7	1. 49,7	608	135. 41. 39,4	1. 41,4
545	133. 46. 15,2	59,0	577	134. 47. 11,4	49,5	609	135. 43. 20,8	41,1
546	133. 48. 14,2	58,7	578	134. 49. 0,9	49,1	610	135. 45. 1,9	40,8
547	133. 50. 12,9	1. 58,4	579	134. 50. 50,0	1. 48,8	611	135. 46. 42,7	1. 40,6
548	133. 52. 11,3	58,1	580	134. 52. 38,8	48,6	612	135. 48. 23,3	40,3
549	133. 54. 9,4	57,7	581	134. 54. 27,4	48,3	613	135. 50. 3,6	40,1
550	133. 56. 7,1	57,5	582	134. 56. 15,7	48,1	614	135. 51. 43,7	39,9
551	133. 58. 4,6	1. 57,1	583	134. 58. 3,8	1. 47,8	615	135. 53. 23,6	1. 39,6
552	134. 0. 1,7	56,8	584	134. 59. 51,6	47,5	616	135. 55. 3,2	39,4
553	134. 1. 58,5	56,4	585	135. 1. 39,1	47,3	617	135. 56. 42,6	39,2
554	134. 3. 54,9	56,2	586	135. 3. 26,4	47,0	618	135. 58. 21,8	38,9
555	134. 5. 51,1	1. 55,9	587	135. 5. 13,4	1. 46,7	619	136. 0. 0,7	1. 38,7
556	134. 7. 47,0	55,5	588	135. 7. 0,1	46,4	620	136. 1. 39,4	38,5
557	134. 9. 42,5	55,3	589	135. 8. 46,5	46,1	621	136. 3. 17,9	38,2
558	134. 11. 37,8	54,9	590	135. 10. 32,6	45,9	622	136. 4. 56,1	38,0
559	134. 13. 32,7	1. 54,7	591	135. 12. 18,5	1. 45,6	623	136. 6. 34,1	1. 37,8
560	134. 15. 27,4	54,3	592	135. 14. 4,1	45,4	624	136. 8. 11,9	37,5
561	134. 17. 21,7	54,1	593	135. 15. 49,5	45,1	625	136. 9. 49,4	37,3
562	134. 19. 15,8	53,7	594	135. 17. 34,6	44,9	626	136. 11. 26,7	37,1
563	134. 21. 9,5	1. 53,4	595	135. 19. 19,5	1. 44,6	627	136. 13. 3,8	1. 36,9
564	134. 23. 2,9	53,2	596	135. 21. 4,1	44,3	628	136. 14. 40,7	36,6
565	134. 24. 56,1	52,9	597	135. 22. 48,4	44,1	629	136. 16. 17,3	36,4
566	134. 26. 49,0	52,5	598	135. 24. 32,5	43,8	630	136. 17. 53,7	36,2
567	134. 28. 41,5	1. 52,3	599	135. 26. 16,3	1. 43,6	631	136. 19. 29,9	1. 36,0
568	134. 30. 33,8	52,0	600	135. 27. 59,9	43,3	632	136. 21. 5,9	35,7
569	134. 32. 25,8	51,7	601	135. 29. 43,2	43,1	633	136. 22. 41,6	35,5
570	134. 34. 17,5	51,4	602	135. 31. 26,3	42,8	634	136. 24. 17,1	35,3
571	134. 36. 8,9	1. 51,2	603	135. 33. 9,1	1. 42,6	635	136. 25. 52,4	1. 35,1
572	134. 38. 0,1	50,8	604	135. 34. 51,7	42,3	636	136. 27. 27,5	34,8
573	134. 39. 50,9	50,5	605	135. 36. 34,0	42,0	637	136. 29. 2,3	34,7
574	134. 41. 41,4	50,3	606	135. 38. 16,0	41,8	638	136. 30. 37,0	34,4
575	134. 43. 31,7	1. 50,0	607	135. 39. 57,8	1. 41,6	639	136. 32. 11,4	1. 34,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.
640	136.	33.	45,6	1.	34,0
641	136.	35.	19,6		33,8
642	136.	36.	53,4		33,5
643	136.	38.	26,9	1.	33,4
644	136.	40.	0,3		33,1
645	136.	41.	33,4		32,9
646	136.	43.	6,3		32,7
647	136.	44.	39,0	1.	32,5
648	136.	46.	11,5		32,3
649	136.	47.	43,8		32,1
650	136.	49.	15,9		31,9
651	136.	50.	47,8	1.	31,7
652	136.	52.	19,5		31,4
653	136.	53.	50,9		31,3
654	136.	55.	22,2		31,0
655	136.	56.	53,2	1.	30,9
656	136.	58.	24,1		30,6
657	136.	59.	54,7		30,5
658	137.	1.	25,2		30,2
659	137.	2.	55,4	1.	30,1
660	137.	4.	25,5		29,8
661	137.	5.	55,3		29,7
662	137.	7.	25,0		29,4
663	137.	8.	54,4	1.	29,3
664	137.	10.	23,7		29,0
665	137.	11.	52,7		28,9
666	137.	13.	21,6		28,6
667	137.	14.	50,2	1.	28,5
668	137.	16.	18,7		28,3
669	137.	17.	47,0		28,1
670	137.	19.	15,1		27,9
671	137.	20.	43,0	1.	27,7

Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.
672	137.	22.	10,7	1.	27,5
673	137.	23.	38,2		27,3
674	137.	25.	5,5		27,1
675	137.	26.	32,6	1.	27,0
676	137.	27.	59,6		26,7
677	137.	29.	26,3		26,6
678	137.	30.	52,9		26,4
679	137.	32.	19,3	1.	26,2
680	137.	33.	45,5		26,0
681	137.	35.	11,5		25,8
682	137.	36.	37,3		25,6
683	137.	38.	2,9	1.	25,5
684	137.	39.	28,4		25,3
685	137.	40.	53,7		25,1
686	137.	42.	18,8		24,9
687	137.	43.	43,7	1.	24,7
688	137.	45.	8,4		24,6
689	137.	46.	33,0		24,3
690	137.	47.	57,3		24,2
691	137.	49.	21,5	1.	24,1
692	137.	50.	45,6		23,8
693	137.	52.	9,4		23,7
694	137.	53.	33,1		23,5
695	137.	54.	56,6	1.	23,3
696	137.	56.	19,9		23,1
697	137.	57.	43,0		23,0
698	137.	59.	6,0		22,8
699	138.	0.	28,8	1.	22,6
700	138.	1.	51,4	3.	25,8
702,5	138.	5.	17,2		24,7
705,0	138.	8.	41,9		23,7
707,5	138.	12.	5,6	3.	22,6

Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.
710,0	138.	15.	28,2	3.	21,6
712,5	138.	18.	49,8		20,5
715,0	138.	22.	10,3		19,5
717,5	138.	25.	29,8	3.	18,6
720,0	138.	28.	48,4		17,5
722,5	138.	32.	5,9		16,5
725,0	138.	35.	22,4		15,5
727,5	138.	38.	37,9	3.	14,6
730,0	138.	41.	52,5		13,6
732,5	138.	45.	6,1		12,6
735,0	138.	48.	18,7		11,6
737,5	138.	51.	30,3	3.	10,8
740,0	138.	54.	41,1		9,8
742,5	138.	57.	50,9		8,8
745,0	139.	0.	59,7		8,0
747,5	139.	4.	7,7	3.	7,0
750,0	139.	7.	14,7		6,2
752,5	139.	10.	20,9		5,2
755,0	139.	13.	26,1		4,3
757,5	139.	16.	30,4	3.	3,5
760,0	139.	19	33,9		2,5
762,5	139.	22.	36,4		1,7
765,0	139.	25.	38,1		0,9
767,5	139.	28.	39,0	2.	0,0
770,0	139.	31.	39,0	2.	59,1
772,5	139.	34.	38,1		58,3
775,0	139.	37.	36,4		57,5
777,5	139.	40.	33,9	2.	56,6
780,0	139.	43.	30,5		55,9
782,5	139.	46.	26,4		55,0
785,0	139.	49.	21,4		54,2
787,5	139.	52.	15,6	2.	53,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
790,0	139.	55.	9,0	2.	52,6
792,5	139.	58.	1,6		51,8
795,0	140.	0.	53,4		51,1
797,5	140.	3.	44,4		
				2.	50,3
800,0	140.	6.	34,7		49,4
802,5	140.	9.	24,1		48,7
805,0	140.	12.	12,8		48,0
807,5	140.	15.	0,8		
				2.	47,2
810,0	140.	17.	48,0		46,4
812,5	140.	20.	34,4		45,7
815,0	140.	23.	20,1		45,0
817,5	140.	26.	5,1		
				2.	44,3
820,0	140.	28.	49,4		43,5
822,5	140.	31.	32,9		42,8
825,0	140.	34.	15,7		42,1
827,5	140.	36.	57,8		
				2.	41,4
830,0	140.	39.	39,2		40,6
832,5	140.	42.	19,8		40,0
835,0	140.	44.	59,8		39,3
837,5	140.	47.	39,1		
				2.	38,6
840,0	140.	50.	17,7		37,9
842,5	140.	52.	55,6		37,2
845,0	140.	55.	32,8		36,6
847,5	140.	58.	9,4		
				2.	35,9
850,0	141.	0.	45,3		35,2
852,5	141.	3.	20,5		34,6
855,0	141.	5.	55,1		33,9
857,5	141.	8.	29,0		
				2.	33,3
860,0	141.	11.	2,3		32,6
862,5	141.	13.	34,9		32,0
865,0	141.	16.	6,9		31,4
867,5	141.	18.	38,3		
				2.	30,7

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
870,0	141.	21.	9,0	2.	30,1
872,5	141.	23.	39,1		29,5
875,0	141.	26.	8,6		28,8
877,5	141.	28.	37,4		
				2.	28,3
880,0	141.	31.	5,7		27,6
882,5	141.	33.	33,3		27,1
885,0	141.	36.	0,4		26,4
887,5	141.	38.	26,8		
				2.	25,9
890,0	141.	40.	52,7		25,2
892,5	141.	43.	17,9		24,7
895,0	141.	45.	42,6		24,0
897,5	141.	48.	6,6		
				2.	23,6
900,0	141.	50.	30,2		22,9
902,5	141.	52.	53,1		22,3
905,0	141.	55.	15,4		21,7
907,5	141.	57.	37,1		
				2.	21,3
910,0	141.	59.	58,4		20,7
912,5	142.	2.	19,1		20,1
915,0	142.	4.	39,2		19,5
917,5	142.	6.	58,7		
				2.	19,0
920,0	142.	9.	17,7		18,5
922,5	142.	11.	36,2		17,9
925,0	142.	13.	54,1		17,4
927,5	142.	16.	11,5		
				2.	16,9
930,0	142.	18.	28,4		16,3
932,5	142.	20.	44,7		15,8
935,0	142.	23.	0,5		15,3
937,5	142.	25.	15,8		
				2.	14,8
940,0	142.	27.	30,6		14,2
942,5	142.	29.	44,8		13,7
945,0	142.	31.	58,5		13,2
947,5	142.	34.	11,7		
				2.	12,7

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
950,0	142.	36.	24,4	2.	12,2
952,5	142.	38.	36,6		11,7
955,0	142.	40.	48,3		11,3
957,5	142.	42.	59,6		
				2.	10,7
960,0	142.	45.	10,3		10,2
962,5	142.	47.	20,5		9,8
965,0	142.	49.	30,3		9,3
967,5	142.	51.	39,6		
				2.	8,7
970,0	142.	53.	48,3		8,3
972,5	142.	55.	56,6		7,9
975,0	142.	58.	4,5		7,3
977,5	143.	0.	11,8		
				2.	6,9
980,0	143.	2.	18,7		6,5
982,5	143.	4.	25,2		5,9
985,0	143.	6.	31,1		5,5
987,5	143.	8.	36,6		
				2.	5,1
990,0	143.	10.	41,7		4,6
992,5	143.	12.	46,3		4,1
995,0	143.	14.	50,4		3,7
997,5	143.	16.	54,1		
				2.	3,3
1000,0	143.	18.	57,4		2,8
1002,5	143.	21.	0,2		2,3
1005,0	143.	23.	2,5		2,0
1007,5	143.	25.	4,5		
				2.	1,5
1010,0	143.	27.	6,0		1,0
1012,5	143.	29.	7,0		0,6
1015,0	143.	31.	7,6	2.	0,2
1017,5	143.	33.	7,8		
				1.	59,8
1020,5	143.	35.	7,6		59,4
1022,0	143.	37.	7,0		58,9
1025,0	143.	39.	5,9		58,5
1027,5	143.	41.	4,4		
				1.	58,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.	Days.	Anomaly. D. M. S.	Differ. M. S.
1030,0	143. 43. 2,5		1110,0	144. 42. 37,4		1190,0	145. 36. 20,9	
1032,5	143. 45. 0,2	1. 57,7	1112,5	144. 44. 23,1	1. 45,7	1192,5	145. 37. 56,6	1. 35,7
1035,0	143. 46. 57,5	57,3	1115,0	144. 46. 8,5	45,4	1195,0	145. 39. 32,0	35,4
1037,5	143. 48. 54,4	56,9	1117,5	144. 47. 53,5	45,0	1197,5	145. 41. 7,2	35,2
		1. 56,5			1. 44,6			1. 34,9
1040,0	143. 50. 50,9		1120,0	144. 49. 38,1		1200	145. 42. 42,1	
1042,5	143. 52. 47,0	56,1	1122,5	144. 51. 22,6	44,5	1205	145. 45. 50,9	3. 8,8
1045,0	143. 54. 42,6	55,6	1125,0	144. 53. 6,7	44,1	1210	145. 48. 58,7	7,8
1047,5	143. 56. 37,9	55,3	1127,5	144. 54. 50,4	43,7	1215	145. 52. 5,3	6,6
		1. 54,9			1. 43,4			3. 5,6
1050,0	143. 58. 32,8		1130,0	144. 56. 33,8		1220	145. 55. 10,9	
1052,5	144. 0. 27,3	54,5	1132,5	144. 58. 16,9	43,1	1225	145. 58. 15,4	4,5
1055,0	144. 2. 21,4	54,1	1135,0	144. 59. 59,6	42,7	1230	146. 1. 18,6	3,2
1057,5	144. 4. 15,1	53,7	1137,5	145. 1. 42,0	42,4	1235	146. 4. 21,1	2,5
		1. 53,4			1. 42,1			3. 1,5
1060,0	144. 6. 8,5		1140,0	145. 3. 24,1		1240	146. 7. 22,6	
1062,5	144. 8. 1,4	52,9	1142,5	145. 5. 5,9	41,8	1245	146. 10. 22,7	0,8
1065,0	144. 9. 54,0	52,6	1145,0	145. 6. 47,3	41,4	1250	146. 13. 21,9	2. 59,2
1067,5	144. 11. 46,2	52,2	1147,5	145. 8. 28,4	41,1	1255	146. 16. 20,1	58,2
		1. 51,8			1. 40,9			2. 57,2
1070,0	144. 13. 38,0		1150,0	145. 10. 9,3		1260	146. 19. 17,3	
1072,5	144. 15. 29,4	51,4	1152,5	145. 11. 49,8	40,5	1265	146. 22. 13,5	56,2
1075,0	144. 17. 20,5	51,1	1155,0	145. 13. 30,0	40,2	1270	146. 25. 8,7	55,2
1077,5	144. 19. 11,2	50,7	1157,5	145. 15. 9,8	39,8	1275	146. 28. 2,9	54,2
		1. 50,3			1. 39,6			2. 53,2
1080,0	144. 21. 1,5		1160,0	145. 16. 49,4		1280	146. 30. 56,1	
1082,5	144. 22. 51,5	50,0	1162,5	145. 18. 28,7	39,3	1285	146. 33. 48,4	52,3
1085,0	144. 24. 41,0	49,5	1165,0	145. 20. 7,7	39,0	1290	146. 36. 39,8	51,4
1087,5	144. 26. 30,2	49,2	1167,5	145. 21. 46,3	38,6	1295	146. 39. 30,2	50,4
		1. 48,9			1. 38,4			2. 49,4
1090,0	144. 28. 19,1		1170,0	145. 23. 24,7		1300	146. 42. 19,6	
1092,5	144. 30. 7,6	48,5	1172,5	145. 25. 2,8	38,1	1305	146. 45. 8,1	48,5
1095,0	144. 31. 55,8	48,2	1175,0	145. 26. 40,5	37,7	1310	146. 47. 55,7	47,6
1097,5	144. 33. 43,6	47,8	1177,5	145. 28. 18,0	37,5	1315	146. 50. 42,4	46,7
		1. 47,4			1. 37,1			2. 45,8
1100,0	144. 35. 31,0		1180,0	145. 29. 55,1		1320	146. 53. 28,2	
1102,5	144. 37. 18,1	47,1	1182,5	145. 31. 32,0	36,9	1325	146. 56. 13,1	44,9
1105,0	144. 39. 4,9	46,8	1185,0	145. 33. 8,6	36,6	1330	146. 58. 57,2	44,1
1107,5	144. 40. 51,3	46,4	1187,5	145. 34. 44,9	36,3	1335	147. 1. 40,3	43,1
		1. 46,1			1. 36,0			2. 42,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
1340	147.	4.	22,6	2.	41,4
1345	147.	7.	4,0		40,9
1350	147.	9.	44,9		39,7
1355	147.	12.	24,3	2.	38,9
1360	147.	15.	3,2		38,1
1365	147.	17.	41,3		37,2
1370	147.	20.	18,5		36,5
1375	147.	22.	55,0	2.	35,6
1380	147.	25.	30,6		34,8
1385	147.	28.	5,4		34,1
1390	147.	30.	39,5		33,2
1395	147.	33.	12,7	2.	32,5
1400	147.	35.	44,2		31,7
1405	147.	38.	16,0		30,9
1410	147.	40.	47,8		30,1
1415	147.	43.	17,9	2.	29,3
1420	147.	45.	47,4		28,6
1425	147.	48.	16,0		28,0
1430	147.	50.	44,0		27,2
1435	147.	53.	11,2	2.	26,5
1440	147.	55.	37,7		25,7
1445	147.	58.	3,4		25,0
1450	148.	0.	28,4		24,4
1455	148.	2.	52,8	2.	23,6
1460	148.	5.	16,4		22,9
1465	148.	7.	39,3		22,3
1470	148.	10.	1,6		21,7
1475	148.	12.	23,1	2.	20,9
1480	148.	14.	44,0		20,2
1485	148.	17.	4,2		19,6
1490	148.	19.	23,8		18,8
1495	148.	21.	42,6	2.	18,2

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
1500	148.	24.	0,8	2.	17,6
1505	148.	26.	18,4		16,9
1510	148.	28.	35,3		16,3
1515	148.	30.	51,6	2.	15,7
1520	148.	33.	7,3		15,0
1525	148.	35.	22,3		14,4
1530	148.	37.	36,7		13,8
1535	148.	39.	50,5	2.	13,1
1540	148.	42.	3,6		12,6
1545	148.	44.	16,2		11,9
1550	148.	46.	28,1		11,3
1555	148.	48.	39,4	2.	10,8
1560	148.	50.	50,2		10,1
1565	148.	53.	0,3		9,6
1570	148.	55.	9,9		8,9
1575	148.	57.	18,8	2.	8,5
1580	148.	59.	27,3		7,8
1585	149.	1.	35,1		7,2
1590	149.	3.	42,3		6,7
1595	149.	5.	49,0	2.	6,2
1600	149.	7.	55,2		5,5
1605	149.	10.	0,7		5,0
1610	149.	12.	5,7		4,5
1615	149.	14.	10,2	2.	4,0
1620	149.	16.	14,2		3,4
1625	149.	18.	17,6		2,8
1630	149.	20.	20,4		2,3
1635	149.	22.	22,7	2.	1,8
1640	149.	24.	24,5		1,3
1645	149.	26.	25,8		0,7
1650	149.	28.	26,5		0,3
1655	149.	30.	26,8	1.	59,7

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
1660	149.	32.	26,5	1.	59,2
1665	149.	34.	25,7		58,7
1670	149.	36.	24,4		58,2
1675	149.	38.	22,6	1.	57,7
1680	149.	40.	20,3		57,3
1685	149.	42.	17,6		56,7
1690	149.	44.	14,3		56,2
1695	149.	46.	10,5	1.	55,8
1700	149.	48.	6,3		55,3
1705	149.	50.	1,6		54,8
1710	149.	51.	56,4		54,3
1715	149.	53.	50,7	1.	53,9
1720	149.	55.	44,6		53,4
1725	149.	57.	38,0		52,9
1730	149.	59.	30,9		52,5
1735	150.	1.	23,4	1.	52,0
1740	150.	3.	15,4		51,6
1745	150.	5.	7,0		51,1
1750	150.	6.	58,1		50,6
1755	150.	8.	48,7	1.	50,2
1760	150.	10.	38,9		49,8
1765	150.	12.	28,7		49,4
1770	150.	14.	18,1		48,9
1775	150.	16.	7,0	1.	48,5
1780	150.	17.	55,5		48,0
1785	150.	19.	43,5		47,7
1790	150.	21.	31,2		47,2
1795	150.	23.	18,4	1.	46,8
1800	150.	25.	5,2	3.	32,3
1810	150.	28.	37,5		30,6
1820	150.	32.	8,1		29,1
1830	150.	35.	37,2	3.	27,4

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
1840	150.	39.	4,6			2160	152.	17.	27,3			2480	153.	37.	17,3		
				3.	25,8					2.	44,4					2.	15,6
1850	150.	42.	30,5			2170	152.	20.	11,9			2490	153.	39.	32,9		
					24,3						43,4						14,9
1860	150.	45.	54,8			2180	152.	22.	55,3			2500	153.	41.	47,8		
					22,8						42,4						14,1
1870	150.	49.	17,6			2190	152.	25.	37,7			2510	153.	44.	1,9		
				3.	21,2					2.	41,3					2.	13,4
1880	150.	52.	38,8			2200	152.	28.	19,0			2520	153.	46.	15,3		
					19,8						40,3						12,6
1890	150.	55.	58,6			2210	152.	30.	59,3			2530	153.	48.	27,9		
					18,2						39,3						11,9
1900	150.	59.	16,8			2220	152.	33.	38,5			2540	153.	50.	39,8		
					16,8						38,3						11,2
1910	151.	2.	33,6			2230	152.	36.	16,8			2550	153.	52.	51,0		
				3.	15,4					2.	37,3					2.	10,4
1920	151.	5.	49,0			2240	152.	38.	54,1			2560	153.	55.	1,4		
					13,9						36,3						9,7
1930	151.	9.	2,9			2250	152.	41.	30,4			2570	153.	57.	11,1		
					12,5						35,3						9,1
1940	151.	12.	15,4			2260	152.	44.	5,7			2580	153.	59.	20,2		
					11,1						34,4						8,4
1950	151.	15.	26,5			2270	152.	46.	40,1			2590	154.	1.	28,6		
				3.	9,8					2.	33,4					2.	7,7
1960	151.	18.	36,3			2280	152.	49.	13,5			2600	154.	3.	36,3		
					8,4						32,5						7,0
1970	151.	21.	44,7			2290	152.	51.	46,0			2610	154.	5.	43,3		
					7,1						31,5						6,3
1980	151.	24.	51,8			2300	152.	54.	17,5			2620	154.	7.	49,6		
					5,7						30,6						5,6
1990	151.	27.	57,5			2310	152.	56.	48,1			2630	154.	9.	55,2		
				3.	4,5					2.	29,7					2.	5,0
2000	151.	31.	2,0			2320	152.	59.	17,8			2640	154.	12.	0,2		
					3,1						28,9						4,3
2010	151.	34.	5,1			2330	153.	1.	46,7			2650	154.	14.	4,5		
					1,9						28,0						3,7
2020	151.	37.	7,0			2340	153.	4.	14,7			2660	154.	16.	8,2		
					0,6						27,1						3,0
2030	151.	40.	7,6			2350	153.	6.	41,8			2670	154.	18.	11,2		
				2.	59,3					2.	26,2					2.	2,4
2040	151.	43.	6,9			2360	153.	9.	8,0			2680	154.	20.	13,6		
					58,2						25,3						1,7
2050	151.	46.	5,1			2370	153.	11.	33,3			2690	154.	22.	15,3		
					56,9						24,4						1,1
2060	151.	49.	2,0			2380	153.	13.	57,7			2700	154.	24.	16,4		
					55,8						23,6						0,5
2070	151.	51.	57,8			2390	153.	16.	21,3			2710	154.	26.	16,9		
				2.	54,5					2.	22,8					1.	59,9
2080	151.	54.	52,3			2400	153.	18.	44,1			2720	154.	28.	16,8		
					53,4						22,0						59,3
2090	151.	57.	45,7			2410	153.	21.	6,1			2730	154.	30.	16,1		
					52,2						21,2						58,6
2100	152.	0.	37,9			2420	153.	23.	27,3			2740	154.	32.	14,7		
					51,0						20,3						58,1
2110	152.	3.	28,9			2430	153.	25.	47,6			2750	154.	34.	12,8		
				2.	49,9					2.	19,5					1.	57,5
2120	152.	6.	18,8			2440	153.	28.	7,1			2760	154.	36.	10,3		
					48,9						18,7						56,8
2130	152.	9.	7,7			2450	153.	30.	25,8			2770	154.	38.	7,1		
					47,7						17,9						56,3
2140	152.	11.	55,4			2460	153.	32.	43,7			2780	154.	40.	3,4		
					46,6						17,2						55,7
2150	152.	14.	42,0			2470	153.	35.	0,9			2790	154.	41.	59,1		
				2.	45,5					2.	16,4					1.	55,1

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.	Days.	Anomaly. D.	M.	S.	Differ. M.	S.
2800	154.	41.	54,2	1.	54,6	3150,0	155.	45.	36,2	2.	1,5	3550,0	156.	45.	24,9	1.	43,0
2810	154.	45.	48,8		54,0	3162,5	155.	47.	37,7		0,9	3562,5	156.	47.	7,9		42,5
2820	154.	47.	42,8		53,4	3175,0	155.	49.	38,6		0,3	3575,0	156.	48.	50,4		42,1
2830	154.	49.	36,2			3187,5	155.	51.	38,9			3587,5	156.	50.	32,5		
				1.	52,9					1.	59,6					1.	41,5
2840	154.	51.	29,1		52,3	3200,0	155.	53.	38,5		58,9	3600,0	156.	52.	14,0		41,1
2850	154.	53.	21,4		51,8	3212,5	155.	55.	37,4		58,3	3612,5	156.	53.	55,1		40,6
2860	154.	55.	13,2		51,3	3225,0	155.	57.	35,7		57,7	3625,0	156.	55.	35,7		40,1
2870	154.	57.	4,5			3237,5	155.	59.	33,4			3637,5	156.	57.	15,8		
				1.	50,7					1.	57,0					1.	39,6
2880	154.	58.	55,2		50,1	3250,0	156.	1.	30,4		56,4	3650,0	156.	58.	55,4		39,2
2890	155.	0.	45,3		49,7	3262,5	156.	3.	26,8		55,8	3662,5	157.	0.	34,6		38,7
2900	155.	2.	35,0		49,1	3275,0	156.	5.	22,6		55,2	3675,0	157.	2.	13,3		38,2
2910	155.	4.	24,1			3287,5	156.	7.	17,8			3687,5	157.	3.	51,5		
				1.	48,6					1.	54,6					1.	37,8
2920	155.	6.	12,7		48,0	3300,0	156.	9.	12,4		54,0	3700,0	157.	5.	29,3		37,3
2930	155.	8.	0,7		47,6	3312,5	156.	11.	6,4		53,3	3712,5	157.	7.	6,6		36,9
2940	155.	9.	48,3		47,1	3325,0	156.	12.	59,7		52,8	3725,0	157.	8.	43,5		36,4
2950	155.	11.	35,4			3337,5	156.	14.	52,5			3737,5	157.	10.	19,9		
				1.	46,6					1.	52,2					1.	36,0
2960	155.	13.	22,0		46,0	3350,0	156.	16.	44,7		51,7	3750,0	157.	11.	55,9		35,6
2970	155.	15.	8,0		45,6	3362,5	156.	18.	36,4		51,0	3762,5	157.	13.	31,5		35,1
2980	155.	16.	53,6		45,1	3375,0	156.	20.	27,4		50,5	3775,0	157.	15.	6,6		34,6
2990	155.	18.	38,7			3387,5	156.	22.	17,9			3787,5	157.	16.	41,2		
				1.	44,5					1.	50,0					1.	34,2
3000,	155.	20.	23,2	2.	10,1	3400,0	156.	24.	7,9		49,4	3800,0	157.	18.	15,4		33,8
3012,5	155.	22.	33,3		9,3	3412,5	156.	25.	57,3		48,8	3812,5	157.	19.	49,2		33,4
3025,0	155.	24.	42,6		8,6	3425,0	156.	27.	46,1		48,2	3825,0	157.	21.	22,6		33,0
3037,5	155.	26.	51,2			3437,5	156.	29.	34,3			3837,5	157.	22.	55,6		
				2.	7,8					1.	47,7					1.	32,6
3050,0	155.	28.	59,0		7,1	3450,0	156.	31.	22,0		47,2	3850,0	157.	24.	28,2		32,1
3062,5	155.	31.	6,1		6,4	3462,5	156.	33.	9,2		46,7	3862,5	157.	26.	0,3		31,7
3075,0	155.	33.	12,5		5,7	3475,0	156.	34.	55,9		46,1	3875,0	157.	27.	32,0		31,3
3087,5	155.	35.	18,2			3487,5	156.	36.	42,0			3887,5	157.	29.	3,3		
				2.	5,0					1.	45,6					1.	31,0
3100,0	155.	37.	23,2		4,3	3500,0	156.	38.	27,6		45,1	3900,0	157.	30.	34,3		30,5
3112,5	155.	39.	27,5		3,6	3512,5	156.	40.	12,7		44,6	3912,5	157.	32.	4,8		30,1
3125,0	155.	41.	31,1		2,9	3525,0	156.	41.	57,3		44,0	3925,0	157.	33.	34,9		29,7
3137,5	155.	43.	34,0			3537,5	156.	43.	41,3			3937,5	157.	35.	4,6		
				2.	2,2					1.	43,6					1.	29,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
3950,0	157.	36.	33,9			4700	158.	55.	32,7			5500	160.	2.	39,3		
3962,5	157.	38.	2,8	1.	28,9	4725	158.	57.	52,5	2.	19,8	5525	160.	4.	32,0	1.	52,7
3975,0	157.	39.	31,4		28,6	4750	159.	0.	11,3		18,8	5550	160.	6.	24,0		52,0
3987,5	157.	40.	59,5		28,1	4775	159.	2.	29,1		17,8	5575	160.	8.	15,3		51,3
				1.	27,8					2.	16,8					1.	50,7
4000	157.	42.	27,3			4800	159.	4.	45,9			5600	160.	10.	6,0		
4025	157.	45.	21,8	2.	54,5	4825	159.	7.	1,7		15,8	5625	160.	11.	56,0		50,1
4050	157.	48.	14,7		52,9	4850	159.	9.	16,5		14,8	5650	160.	13.	45,3		49,4
4075	157.	51.	6,0		51,3	4875	159.	11.	30,4		13,9	5675	160.	15.	33,9		48,6
				2.	50,1					2.	13,0					1.	48,0
4100	157.	53.	56,3			4900	159.	13.	43,4			5700	160.	17.	21,9		
4125	157.	56.	44,9		48,6	4925	159.	15.	55,4		12,0	5725	160.	19.	9,3		47,4
4150	157.	59.	32,1		47,2	4950	159.	18.	6,5		11,1	5750	160.	20.	56,0		46,7
4175	158.	2.	17,9		45,8	4975	159.	20.	16,7		10,2	5775	160.	22.	42,0		46,0
				2.	44,5					2.	9,3					1.	45,3
4200	158.	5.	2,4			5000	159.	22.	26,0			5800	160.	24.	27,3		
4225	158.	7.	45,5		43,1	5025	159.	24.	34,4		8,4	5825	160.	26.	11,9		44,6
4250	158.	10.	27,3		41,8	5050	159.	26.	42,0		7,6	5850	160.	27.	56,1		44,2
4275	158.	13.	7,8		40,5	5075	159.	28.	48,7		6,7	5875	160.	29.	40,1		44,0
				2.	39,2					2.	5,8					1.	43,0
4300	158.	15.	47,0			5100	159.	30.	54,5			5900	160.	31.	23,1		
4325	158.	18.	24,9		37,9	5125	159.	32.	59,5		5,0	5925	160.	33.	5,5		42,4
4350	158.	21.	1,6		36,7	5150	159.	35.	3,7		4,2	5950	160.	34.	47,3		41,8
4375	158.	23.	37,1		35,5	5175	159.	37.	7,0		3,3	5975	160.	36.	28,6		41,3
				2.	34,2					2.	2,5					1.	40,6
4400	158.	26.	11,3			5200	159.	39.	9,5			6000	160.	38.	9,2		
4425	158.	28.	44,3		33,0	5225	159.	41.	11,2		1,7	6025	160.	39.	49,3		40,1
4450	158.	31.	16,2		31,9	5250	159.	43.	12,1		0,9	6050	160.	41.	28,8		39,5
4475	158.	33.	46,9		30,7	5275	159.	45.	12,2		0,1	6075	160.	43.	7,8		39,0
				2.	29,5					1.	59,4					1.	38,4
4500	158.	36.	16,4			5300	159.	47.	11,6			6100	160.	44.	46,2		
4525	158.	38.	44,7		28,3	5325	159.	49.	10,2		58,6	6125	160.	46.	24,0		37,8
4550	158.	41.	12,0		27,3	5350	159.	51.	8,0		57,8	6150	160.	48.	1,3		37,3
4575	158.	43.	38,1		26,1	5375	159.	53.	5,0		57,0	6175	160.	49.	38,1		36,8
				2.	25,1					1.	56,3					1.	36,2
4600	158.	46.	3,2			5400	159.	55.	1,3			6200	160.	51.	14,3		
4625	158.	48.	27,2		24,0	5425	159.	56.	56,9		55,6	6225	160.	52.	50,0		35,7
4650	158.	50.	50,1		22,9	5450	159.	58.	51,8		54,9	6250	160.	54.	25,2		35,2
4675	158.	53.	11,9		21,8	5475	160.	0.	45,9		54,1	6275	160.	55.	59,8		34,6
				2.	20,8					1.	53,4					1.	34,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
6300	160.	57.	34,0		
				1.	33,6
6325	160.	59.	7,6		
					33,1
6350	161.	0.	40,7		
					32,6
6375	161.	2.	13,3		
				1.	32,2
6400	161.	3.	45,5		
					31,6
6425	161.	5.	17,1		
					31,2
6450	161.	6.	48,3		
					30,6
6475	161.	8.	18,9		
				1.	30,2
6500	161.	9.	49,1		
					29,7
6525	161.	11.	18,8		
					29,3
6550	161.	12.	48,1		
					28,8
6575	161.	14.	16,9		
				1.	28,3
6600	161.	15.	45,2		
					27,9
6625	161.	17.	13,1		
					27,4
6650	161.	18.	40,5		
					27,0
6675	161.	20.	7,5		
				1.	26,5
6700	161.	21.	34,0		
					26,1
6725	161.	23.	0,1		
					25,6
6750	161.	24.	25,7		
					25,2
6775	161.	25.	50,9		
				1.	24,8
6800	161.	27.	15,7		
					24,4
6825	161.	28.	40,1		
					23,9
6850	161.	30.	4,0		
					23,6
6875	161.	31.	27,6		
				1.	23,1
6900	161.	32.	50,7		
					22,7
6925	161.	34.	13,4		
					22,3
6950	161.	35.	35,7		
					21,9
6975	161.	36.	57,6		
				1.	21,4
7000	161.	38.	19,0		
				2.	41,8
7050	161.	41.	0,8		
					40,3
7100	161.	43.	41,1		
					38,7
7150	161.	46.	19,8		
				2.	37,1

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
7200	161.	48.	56,9		
				2.	35,7
7250	161.	51.	32,6		
					34,3
7300	161.	54.	6,9		
					32,8
7350	161.	56.	39,7		
				2.	31,4
7400	161.	59.	11,1		
					30,0
7450	162.	1.	41,1		
					28,7
7500	162.	4.	9,8		
					27,3
7550	162.	6.	37,1		
				2.	25,9
7600	162.	9.	3,0		
					24,7
7650	162.	11.	27,7		
					23,4
7700	162.	13.	51,1		
					22,1
7750	162.	16.	13,2		
				2.	20,9
7800	162.	18.	34,1		
					19,6
7850	162.	20.	53,7		
					18,5
7900	162.	23.	12,2		
					17,2
7950	162.	25.	29,4		
				2.	16,1
8000	162.	27.	45,5		
					14,9
8050	162.	30.	0,4		
					13,8
8100	162.	32.	14,2		
					12,7
8150	162.	34.	26,9		
				2.	11,6
8200	162.	36.	38,5		
					10,5
8250	162.	38.	49,0		
					9,4
8300	162.	40.	58,4		
					8,3
8350	162.	43.	6,7		
				2.	7,3
8400	162.	45.	14,0		
					6,3
8450	162.	47.	20,3		
					5,3
8500	162.	49.	25,6		
					4,2
8550	162.	51.	29,8		
				2.	3,3
8600	162.	53.	33,1		
					2,3
8650	162.	55.	35,4		
					1,4
8700	162.	57.	36,8		
					0,4
8750	162.	59.	37,2		
				1.	59,4

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
8800	163.	1.	36,6		
				1.	58,6
8850	163.	3.	35,2		
					57,6
8900	163.	5.	32,8		
					56,8
8950	163.	7.	29,6		
				1.	55,8
9000	163.	9.	25,4		
					55,0
9050	163.	11.	20,4		
					54,1
9100	163.	13.	14,5		
					53,3
9150	163.	15.	7,8		
				1.	52,5
9200	163.	17.	0,3		
					51,6
9250	163.	18.	51,9		
					50,8
9300	163.	20.	42,7		
					50,0
9350	163.	22.	32,7		
				1.	49,1
9400	163.	24.	21,8		
					48,4
9450	163.	26.	10,2		
					47,7
9500	163.	27.	57,9		
					46,8
9550	163.	29.	44,7		
				1.	46,1
9600	163.	31.	30,8		
					45,4
9650	163.	33.	16,2		
					44,6
9700	163.	35.	0,8		
					43,8
9750	163.	36.	44,6		
				1.	43,2
9800	163.	38.	27,8		
					42,4
9850	163.	40.	10,2		
					41,8
9900	163.	41.	52,0		
					41,0
9950	163.	43.	33,0		
				1.	40,4
10000	163.	45.	13,4		
					39,6
10050	163.	46.	53,0		
					39,0
10100	163.	48.	32,0		
					38,4
10150	163.	50.	10,4		
				1.	37,7
10200	163.	51.	48,1		
					37,0
10250	163.	53.	25,1		
					36,4
10300	163.	55.	1,5		
					35,7
10350	163.	56.	37,2		
				1.	35,2

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
10400	163.	58.	12,2		
				1.	34,3
10450	163.	59.	46,5		
					33,9
10500	164.	1.	20,8		
					33,3
10550	164.	2.	54,1		
				1.	32,7
10600	164.	2.	26,8		
					32,1
10650	164.	3.	58,9		
					31,5
10700	164.	5.	30,4		
					31,0
10750	164.	9.	1,4		
				1.	30,5
10800	164.	10.	31,7		
					29,8
10850	164.	12.	1,5		
					29,3
10900	164.	13.	30,8		
					28,7
10950	164.	14.	59,5		
				1.	28,1
11000	164.	16.	27,6		
					27,6
11050	164.	17.	55,2		
					27,1
11100	164.	19.	22,3		
					26,5
11150	164.	20.	48,8		
				1.	26,0
11200	164.	22.	14,8		
					25,5
11250	164.	23.	40,3		
					25,0
11300	164.	25.	5,3		
					24,4
11350	164.	26.	29,7		
				1.	24,0
11400	164.	27.	53,7		
					23,5
11450	164.	29.	17,2		
					22,9
11500	164.	30.	40,1		
					22,5
11550	164.	32.	2,6		
				1.	22,0
11600	164.	33.	24,6		
					21,5
11650	164.	34.	46,1		
					21,1
11700	164.	36.	7,2		
					20,6
11750	164.	37.	27,8		
				1.	20,1
11800	164.	38.	47,9		
					19,6
11850	164.	40.	7,5		
					19,2
11900	164.	41.	26,7		
					18,8
11950	164.	42.	45,5		
				1.	18,3

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
12000	164.	44.	3,8		
				2.	35,3
12100	164.	46.	39,1		
					33,6
12200	164.	49.	12,7		
					31,9
12300	164.	51.	44,6		
				2.	30,2
12400	164.	54.	14,8		
					28,6
12500	164.	56.	43,4		
					26,9
12600	164.	59.	10,3		
					25,4
12700	165.	1.	35,7		
				2.	23,9
12800	165.	3.	59,6		
					22,3
12900	165.	6.	21,8		
					20,9
13000	165.	8.	42,8		
					19,4
13100	165.	11.	2,2		
				2.	18,0
13200	165.	13.	20,2		
					16,5
13300	165.	15.	36,7		
					15,1
13400	165.	17.	51,8		
					13,5
13500	165.	20.	5,3		
				2.	12,1
13600	165.	22.	18,2		
					11,2
13700	165.	24.	29,4		
					9,8
13800	165.	26.	39,3		
					8,5
13900	165.	28.	47,8		
				2.	7,1
14000	165.	30.	55,2		
					6,1
14100	165.	33.	1,3		
					5,0
14200	165.	35.	6,3		
					3,7
14300	165.	37.	10,0		
				2.	2,6
14400	165.	39.	12,6		
					1,4
14500	165.	41.	14,0		
					0,3
14600	165.	43.	14,3		
				1.	59,2
14700	165.	45.	13,5		
				1.	58,1
14800	165.	47.	11,6		
					57,0
14900	165.	49.	8,6		
					56,0
15000	165.	51.	4,6		
					54,9
15100	165.	52.	59,5		
				1.	53,9

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
15200	165.	54.	53,4		
				1.	52,9
15300	165.	56.	46,3		
					51,9
15400	165.	58.	38,2		
					51,0
15500	166.	0.	29,2		
				1.	49,9
15600	166.	2.	19,1		
					49,0
15700	166.	4.	8,1		
					48,1
15800	166.	5.	56,2		
					47,2
15900	166.	7.	43,4		
				1.	46,2
16000	166.	9.	29,6		
					44,4
16100	166.	11.	14,0		
					44,5
16200	166.	12.	58,5		
					43,6
16300	166.	14.	42,1		
				1.	42,7
16400	166.	16.	24,8		
					41,9
16500	166.	18.	5,7		
					41,1
16600	166.	19.	45,8		
					40,2
16700	166.	21.	25,0		
				1.	39,5
16800	166.	23.	3,5		
					38,6
16900	166.	24.	42,1		
					37,9
17000	166.	26.	20,0		
					37,1
17100	166.	28.	[illegible]		
				1.	36,3
17200	166.	29.	33,4		
					35,6
17300	166.	31.	14,0		
					34,8
17400	166.	32.	48,5		
					34,1
17500	166.	34.	22,9		
				1.	33,4
17600	166.	35.	56,3		
					32,7
17700	166.	37.	29,0		
					31,9
17800	166.	39.	0,9		
					31,3
17900	166.	40.	32,2		
				1.	30,6
18000	166.	42.	2,8		
					29,8
18100	166.	43.	32,6		
					29,2
18200	166.	45.	1,8		
					28,6
18300	166.	46.	30,4		
				1.	27,9

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
18400	166.	47.	58,3		
				1.	27,3
18500	166.	49.	25,6		
					26,6
18600	166.	50.	52,2		
					26,0
18700	166.	52.	18,2		
				1.	25,4
18800	166.	53.	43,6		
					24,7
18900	166.	55.	8,3		
					24,2
19000	166.	56.	32,5		
					23,5
19100	166.	57.	56,0		
				1.	23,0
19200	166.	59.	19,0		
					22,4
19300	167.	0.	41,4		
					21,8
19400	167.	2.	3,2		
					21,3
19500	167.	3.	24,5		
				1.	20,7
19600	167.	4.	45,2		
					20,1
19700	167.	6.	5,3		
					19,6
19800	167.	7.	24,9		
					19,0
19900	167.	8.	43,9		
				1.	18,5
20000	167.	10.	2,4		
					18,0
20100	167.	11.	20,4		
					17,5
20200	167.	12.	37,9		
					17,0
20300	167.	13.	54,9		
				1.	16,4
20400	167.	15.	11,3		
					16,0
20500	167.	16.	27,3		
					15,5
20600	167.	17.	42,8		
					14,9
20700	167.	18.	57,7		
				1.	14,5
20800	167.	20.	12,2		
					14,0
20900	167.	21.	26,2		
					13,5
21000	167.	22.	39,7		
					13,0
21100	167.	23.	52,7		
				1.	12,6
21200	167.	25.	5,3		
					12,1
21300	167.	26.	17,4		
					11,7
21400	167.	27.	29,1		
					11,2
21500	167.	28.	40,3		
				1.	10,8

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
21600	167.	29.	51,1		
				1.	10,3
21700	167.	31.	1,4		
					9,9
21800	167.	32.	11,3		
					9,4
21900	167.	33.	20,7		
				1.	9,0
22000	167.	34.	29,7		
					8,6
22100	167.	35.	38,3		
					8,2
22200	167.	36.	46,5		
					7,8
22300	167.	37.	54,3		
				1.	7,4
22400	167.	39.	1,7		
					6,9
22500	167.	40.	8,6		
					6,6
22600	167.	41.	15,2		
					6,2
22700	167.	42.	21,4		
				1.	5,7
22800	167.	43.	27,1		
					5,4
22900	167.	44.	32,5		
					5,0
23000	167.	45.	37,5		
					4,6
23100	167.	46.	42,1		
				1.	4,3
23200	167.	47.	46,4		
					3,9
23300	167.	48.	50,3		
					3,5
23400	167.	49.	53,8		
					3,1
23500	167.	50.	56,9		
				1.	2,8
23600	167.	51.	59,7		
					2,4
23700	167.	53.	2,1		
					2,1
23800	167.	54.	4,2		
					1,7
23900	167.	55.	5,9		
				1.	1,3
24000	167.	56.	7,2		
				2.	1,8
24200	167.	58.	9,0		
					0,4
24400	168.	0.	9,4		
				1.	59,0
24600	168.	2.	8,4		
				1.	57,8
24800	168.	4.	6,2		
					56,5
25000	168.	6.	2,7		
					55,2
25200	168.	7.	57,9		
					54,0
25400	168.	9.	51,9		
				1.	52,8

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
25600	168.	11.	44,7		
				1.	51,7
25800	168.	13.	36,4		
					50,5
26000	168.	15.	26,9		
					49,3
26200	168.	17.	16,2		
				1.	48,2
26400	168.	19.	4,4		
					47,1
26600	168.	20.	51,5		
					46,0
26800	168.	22.	37,5		
					45,0
27000	168.	24.	22,5		
				1.	43,9
27200	168.	26.	6,4		
					42,9
27400	168.	27.	49,3		
					41,9
27600	168.	29.	31,2		
					40,9
27800	168.	31.	12,1		
				1.	40,0
28000	168.	32.	52,1		
					39,0
28200	168.	34.	31,1		
					38,0
28400	168.	36.	9,1		
					37,1
28600	168.	37.	46,2		
				1.	36,2
28800	168.	39.	22,4		
					35,3
29000	168.	40.	57,7		
					34,4
29200	168.	42.	32,1		
					33,5
29400	168.	44.	5,6		
				1.	32,7
29600	168.	45.	38,3		
					31,9
29800	168.	47.	10,2		
					31,1
30000	168.	48.	41,3		
					30,2
30200	168.	50.	11,5		
				1.	29,4
30400	168.	51.	40,9		
					28,6
30600	168.	53.	9,5		
					27,8
30800	168.	54.	37,3		
					27,1
31000	168.	56	4,4		
				1.	26,4
31200	168.	57.	30,8		
					25,6
31400	168.	58.	56,4		
					24,9
31600	169.	0.	21,3		
					24,1
31800	169.	1.	45,4		
				1.	23,5

THE THIRD TABLE CONTINUED.

Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.		Days.	Anomaly.			Differ.	
	D.	M.	S.	M.	S.		D.	M.	S.	M.	S.		D.	M.	S.	M.	S.
32000	169.	3.	8,9			38400	169.	42.	18,2			46000	170.	18.	44,5		
				1.	22,7					1.	4,8					1.	3,5
32200	169.	4.	31,6			38600	169.	43.	23,0			46250	170.	19.	48,0		
					22,1						4,3						3,0
32400	169.	5.	53,7			38800	169.	44.	27,3			46500	170.	20.	51,0		
					21,3						3,9						2,6
32600	169.	7.	15,0			39000	169.	45.	31,2			46750	170.	21.	53,6		
				1.	20,7					1.	3,5					1.	2,2
32800	169.	8.	35,7			39200	169.	46.	34,7			47000	170.	22.	55,8		
					20,1						3,1						1,7
33000	169.	9.	55,8			39400	169.	47.	37,8			47250	170.	23.	57,5		
					19,4						2,6						1,3
33200	169.	11.	15,2			39600	169.	48.	40,4			47500	170.	24.	58,8		
					18,7						2,2						0,9
33400	169.	12.	33,9			39800	169.	49.	42,6			47750	170.	25.	59,7		
				1.	18,2					1.	1,7					1.	0,4
33600	169.	13.	52,1			40000	169.	50.	44,3			48000	170.	27.	0,1		
					17,5						16,6						59,6
33800	169.	15.	9,6			40250	169.	52.	0,9			48500	170.	28.	59,7		
					16,9						16,0						58,0
34000	169.	16.	26,5			40500	169.	53.	16,9			49000	170.	30.	57,7		
					16,2						15,4						56,4
34200	199.	17.	42,7			40750	169.	54.	32,3			49500	170.	32.	54,1		
				1.	15,7					1.	14,8					1.	54,8
34400	169.	18.	58,4			41000	169.	55.	47,1			50000	170.	34.	48,9		
					15,1						14,1						53,3
34600	169.	20.	13,5			41250	169.	57.	1,2			50500	170.	36.	42,2		
					14,5						13,6						51,8
34800	169.	21.	28,0			41500	169.	58.	14,8			51000	170.	38.	34,0		
					14,0						12,9						50,3
35000	169.	22.	42,0			41750	169.	59.	27,7			51500	170.	40.	24,3		
				1.	13,4					1.	12,4					1.	48,9
35200	169.	23.	55,4			42000	170.	0.	40,1			52000	170.	42.	13,2		
					12,8						11,8						47,6
35400	169.	25.	8,2			42250	170.	1.	51,9			52500	170.	44.	0,8		
					12,3						11,2						46,1
35600	169.	26.	20,5			42500	170.	3.	3,1			53000	170.	45.	46,9		
					11,7						10,7						44,8
35800	169.	27.	32,2			42750	170.	4.	13,8			53500	170.	47.	31,7		
				1.	11,1					1.	10,1					1.	43,5
36000	169.	28.	43,3			43000	170.	5.	23,9			54000	170.	49.	15,2		
					10,7						9,6						42,2
36200	169.	29.	54,0			43250	170.	6.	33,5			54500	170.	50.	57,4		
					10,1						9,0						41,0
36400	169.	31.	4,1			43500	170.	7.	42,5			55000	170.	52.	38,4		
					9,7						8,5						39,7
36600	169.	32.	13,8			43750	170.	8.	51,0			55500	170.	54.	18,1		
				1.	9,1					1.	7,9					1.	38,6
36800	169.	33.	22,9			44000	170.	9.	58,9			56000	170.	55.	56,7		
					8,6						7,4						37,4
37000	169.	34.	31,5			44250	170.	11.	6,3			56500	170.	57.	34,1		
					8,1						6,9						36,2
37200	169.	35.	39,6			44500	170.	12.	13,2			57000	170.	59.	10,3		
					7,6						6,5						35,1
37400	169.	36.	47,2			44750	170	13.	19,7			57500	171.	0.	45,4		
				1.	7,1					1.	5,9					1.	34,0
37600	169.	37.	54,3			45000	170	14.	25,6			58000	171.	2.	19,4		
					6,7						5,5						32,9
37800	169.	39.	1,0			45250	170.	15.	31,1			58500	171.	3.	52,3		
					6,2						4,9						31,9
38000	169.	40.	7,2			45500	170.	16.	36,0			59000	171.	5.	24,2		
					5,7						4,5						30,8
38200	169.	41.	12,9			45750	170.	17.	40,5			59500	171.	6.	55,0		
				1.	5,3					1.	4,0					1.	29,8

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
60000	171.	8.	24,8	1.	28,8
60500	171.	9.	53,6		27,8
61000	171.	11.	21,4		26,9
61500	171.	12.	48,3	1.	25,9
62000	171.	14.	14,2		25,0
62500	171.	15.	39,2		24,1
63000	171.	17.	3,3		23,2
63500	171.	18.	26,5	1.	22,3
64000	171.	19.	48,8		21,5
64500	171.	21.	10,3		20,6
65000	171.	22.	30,9		19,8
65500	171.	23.	50,7	1.	19,0
66000	171.	25.	9,7		18,2
66500	171.	26.	27,9		17,4
67000	171.	27.	45,3		16,6
67500	171.	29.	1,9	1.	15,9
68000	171.	30.	17,8		15,1
68500	171.	31.	32,9		14,4
69000	171.	32.	47,3		13,7
69500	171.	34.	1,0	1.	13,0
70000	171.	35.	14,0		12,3
70500	171.	36.	26,3		11,6
71000	171.	37.	37,9		10,9
71500	171.	38.	48,8	1.	10,3
72000	171.	39.	59,1		9,6
72500	171.	41.	8,7		8,9
73000	171.	42.	17,6		8,3
73500	171.	43.	25,9	1.	7,8
74000	171.	44.	33,7		7,1
74500	171.	45.	40,8		6,5
75000	171.	46.	47,3		5,9
75500	171.	47.	53,2	1.	5,3

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
76000	171.	48.	58,5	1.	4,8
76500	171.	50.	3,3		4,2
77000	171.	51.	7,5		3,6
77500	171.	52.	11,1	1.	3,1
78000	171.	53.	14,2		2,5
78500	171.	54.	16,7		2,1
79000	171.	55.	18,8		1,5
79500	171.	56.	20,3	1.	1,0
80000	171.	57.	21,3		0,4
80500	171.	58.	21,7		0,0
81000	171.	59.	21,7	0.	59,5
81500	172.	0.	21,2	0.	59,0
82000	172.	1.	20,2		58,5
82500	172.	2.	18,7		58,0
83000	172.	3.	16,7		57,6
83500	172.	4.	14,3	0.	57,1
84000	172.	5.	11,4		56,7
84500	172.	6.	8,1		56,2
85000	172.	7.	4,3		55,8
85500	172.	8.	0,1	0.	55,3
86000	172.	8.	55,4		54,9
86500	172.	9.	50,3		54,5
87000	172.	10.	44,8		54,1
87500	172.	11.	38,9	0.	53,6
88000	172.	12.	32,5		53,2
88500	172.	13.	25,7		52,9
89000	172.	14.	18,6		52,4
89500	172.	15.	11,0	0.	52,1
90000	172.	16.	3,1		51,6
90500	172.	16.	54,7		51,3
91000	172.	17.	46,0		50,9
91500	172.	18.	36,9	0.	50,6

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
92000	172.	19.	27,5	0.	50,2
92500	172.	20.	17,7		49,8
93000	172.	21.	7,5		49,4
93500	172.	21.	56,9	0.	49,1
94000	172.	22.	46,0		48,7
94500	172.	23.	34,7		48,5
95000	172.	24.	23,2		48,0
95500	172.	25.	11,2	0.	47,7
96000	172.	25.	58,9		47,4
96500	172.	26.	46,3		47,1
97000	172.	27.	33,4		46,8
97500	172.	28.	20,2	0.	46,4
98000	172.	29.	6,6		46,1
98500	172.	29.	52,7		45,8
99000	172.	30.	38,5		45,5
99500	172.	31.	24,0	0.	45,2
100000	172.	32.	9,2	1.	29,5
101000	172.	33.	38,7		29,1
102000	172.	35.	7,8		26,3
103000	172.	36.	34,1	1.	26,0
104000	172.	38.	0,1		25,0
105000	172.	39.	25,1		23,8
106000	172.	40.	48,9		22,3
107000	172.	42.	11,7	1.	21,8
108000	172.	43.	33,5		20,7
109000	172.	44.	54,2		19,8
110000	172.	46.	14,0		18,3
111000	172.	47.	32,8	1.	17,8
112000	172.	48.	50,6		17,0
113000	172.	50.	7,6		16,0
114000	172.	51.	23,6		15,1
115000	172.	52.	38,7	1.	14,3

THE THIRD TABLE CONTINUED.

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
116000	172.	53.	53,0		
				1.	13,4
117000	172.	55.	6,4		
					12,6
118000	172.	56.	19,0		
					11,8
119000	172.	57.	30,8		
				1.	10,9
120000	172.	58.	41,7		
					10,2
121000	172.	59.	51,9		
					9,4
122000	173.	1.	1,3		
					8,6
123000	173.	2.	9,9		
				1.	7,9
124000	173.	3.	17,8		
					7,3
125000	173.	4.	25,1		
					6,4
126000	173.	5.	31,5		
					5,8
127000	173.	6.	37,3		
				1.	5,1
128000	173.	7.	42,4		
					4,3
129000	173.	8.	46,7		
					3,8
130000	173.	9.	50,5		
					3,1
131000	173.	10.	53,6		
				1.	2,4
132000	173.	11.	56,0		
					1,9
133000	173.	12.	57,9		
					1,1
134000	173.	13.	59,0		
					0,5
135000	173.	14.	59,5		
				1.	0,1
136000	173.	15.	59,6		
				0.	59,4
137000	173.	16.	59,0		
					58,8
138000	173.	17.	57,8		
					58,2
139000	173.	18.	56,0		
				0.	57,7
140000	173.	19.	53,7		
					57,1
141000	173.	20.	50,8		
					56,6
142000	173.	21.	47,4		
					56,1
143000	173.	22.	43,5		
				0.	55,5

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
144000	173.	23.	39,0		
				0.	55,1
145000	173.	24.	34,1		
					54,5
146000	173.	25.	28,6		
					54,0
147000	173.	26.	22,6		
				0.	53,6
148000	173.	27.	16,2		
					53,1
149000	173.	28.	9,3		
					52,5
150000	173.	29.	1,8		
					52,2
151000	173.	29.	54,0		
				0.	51,8
152000	173.	30.	45,8		
					51,0
153000	173.	31.	36,8		
					50,8
154000	173.	32.	27,6		
					50,3
155000	173.	33.	17,9		
				0.	49,9
156000	173.	34.	7,8		
					49,5
157000	173.	34.	57,3		
					49,0
158000	173.	35.	46,3		
					48,6
159000	173.	36.	34,9		
				0.	48,3
160000	173.	37.	23,2		
					47,9
161000	173.	38.	11,1		
					47,6
162000	173.	38.	58,7		
					47,2
163000	173.	39.	45,9		
				0.	46,7
164000	173.	40.	32,6		
					46,4
165000	173.	41.	19,0		
					46,0
166000	173.	42.	5,0		
					45,6
167000	173.	42.	50,6		
				0.	45,3
168000	173.	43.	35,9		
					44,9
169000	173.	44.	20,8		
					44,6
170000	173.	45.	5,4		
					44,2
171000	173.	45.	49,6		
				0.	43,9

Days.	Anomaly. D.	M.	S.	Differ. M.	S.
172000	173.	46.	33,5		
				0.	43,5
173000	173.	47.	17,0		
					43,1
174000	173.	48.	0,1		
					42,8
175000	173.	48.	42,9		
				0.	42,4
176000	173.	49.	25,3		
					42,0
177000	173.	50.	7,3		
					41,7
178000	173.	50.	49,0		
					41,3
179000	173.	51.	30,3		
				0.	41,0
180000	173.	52.	11,3		
					40,7
181000	173.	52.	52,0		
					40,4
182000	173.	53.	32,4		
					40,1
183000	173.	54.	12,5		
				0.	39,8
184000	173.	54.	52,3		
					39,6
185000	173.	55.	31,9		
					39,3
186000	173.	56.	11,2		
					39,1
187000	173.	56.	50,3		
				0.	38,9
188000	173.	57.	29,2		
					38,5
189000	173.	58.	7,7		
					38,3
190000	173.	58.	46,0		
					38,1
191000	173.	59.	24,1		
				0.	37,8
192000	174.	0.	1,9		
					37,5
193000	174.	0.	39,4		
					37,2
194000	174.	1.	16,6		
					37,0
195000	174.	1.	53,6		
				0.	36,7
196000	174.	2.	30,3		
					36,5
197000	174.	3.	6,8		
					36,3
198000	174.	3.	43,1		
					36,0
199000	174.	4.	19,1		
				0.	35,7
200000	174.	4.	54,8		

TABLE IV.

CONTAINING THE

ABSCISSAS AND CORRESPONDING ORDINATES OF A PARABOLA;

USEFUL FOR CONSTRUCTING THAT CURVE.

Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
0,125	0,70710	18	8,4853	48	13,8564
0,25	1,00000	19	8,7178	49	14,0000
0,50	1,41421	20	8,9444	50	14,1422
0,75	1,73205	21	9,1651	52	14,4222
1,00	2,00000	22	9,3808	56	14,9666
1,5	2,44950	23	9,5916	60	15,4920
2,0	2,82843	24	9,7979	64	16,0000
2,5	3,16225	25	10,0000	68	16,4924
3,0	3,46410	26	10,1980	72	16,9706
3,5	3,74165	27	10,3923	76	17,4356
4,0	4,00000	28	10,5830	80	17,8888
4,5	4,24265	29	10,7703	81	18,0000
5,0	4,47210	30	10,9545	84	18,3302
5,5	4,69040	31	11,1355	88	18,7616
6,0	4,89900	32	11,3138	92	19,1832
6,5	5,09900	33	11,4891	96	19,5958
7,0	5,29150	34	11,6619	100	20,0000
7,5	5,47725	35	11,8322	104	20,3960
8,0	5,65690	36	12,0000	108	20,7846
8,5	5,83095	37	12,1655	112	21,1660
9,0	6,00000	38	12,3288	116	21,5406
9,5	6,16400	39	12,4900	120	21,9090
10,0	6,3245	40	12,6490	121	22,0000
11	6,6332	41	12,8062	124	22,2710
12	6,9282	42	12,9615	128	22,6276
13	7,2111	43	13,1149	132	22,9782
14	7,4833	44	13,2664	136	23,3238
15	7,7460	45	13,4164	140	23,6644
16	8,0000	46	13,5647	144	24,0000
17	8,2462	47	13,7113	...	......

EXPLANATION AND USE OF THE TABLES.

TABLE I, is for the reduction of hours, minutes and seconds of time, into decimal parts of a day.

RULE. Find, in the column of time, the hours, minutes and seconds given, and opposite to each is the corresponding decimal, the sum of which is the decimal fraction required.

Ex. Required the decimal of a day of 17*h*. 27′. 44″.

17 hours - - - - -	0,708333
27 minutes - - - -	0,018750
44 seconds - - - -	0,000509
Sum. Decimal of 17*h*. 27′. 44″ -	0,727592

TABLE II, is for the reduction of decimal parts of a day, into hours, minutes and seconds.

RULE. Enter the Table with the first figure to the left hand of the given decimal, and take out its value in hours, &c. Repeat the same operation with the second, third, and the rest of the figures; and the sum of the times, so taken from the Table, is the value of the decimal required.

Ex. Required the value in time of 0,727592 of a day.

	H.	M.	S.
0,7 - - - - - - -	16.	48.	0
0,02 - - - - - -	0.	28.	48
0,007 - - - - - -	0.	10.	4,8
0,0005 - - - - - -	0.	0.	43,2
0,00009 - - - - - -	0.	0.	7,776
0,000002 - - - - - -	0.	0.	0,173
Sum. Value of 0,727592 - - -	17.	27.	43,949

TABLE III, of the motion of comets in a parabolic orbit, was first published by Dr. HALLEY, and since augmented by M. de la CAILLE, M. de la LANDE and SCHULZE of Berlin. M. PINGRE' recomputed and extended the whole, so as to make it much more complete than any before published. And lately, M. de LAMBRE, whose abilities as a Calculator are well known, has recomputed the whole Table to decimals of seconds, and still farther enlarged it.

The perihelion distance of any comet, and the time of its passage through the perihelion being given, to find its true anomaly, or angular distance from the perihelion, for any given time before or after the perihelion.

RULE. To the logarithm of the perihelion distance of the comet, add its half; subtract the sum from the logarithm of the time elapsed (expressed in days) between the given time and the arrival of the comet at its perihelion, and the remainder will be the logarithm of a number of days; find this number in the Table of the parabola, and opposite to it is the anomaly sought. If the given number be not in the Table, a simple proportion will give the anomaly.

If the characteristic of the logarithm of the perihelion distance be 9, 8, or 7, in taking its half, it must be supposed 19, 18, or 17.

Ex. The logarithm of the perihelion distance of the comet in 1769 was 9,0886320, according to EULER. What was its anomaly at 50 days before or after its perihelion?

Log. of perihelion distance - -	9,0886320
Its half - - - - - - -	9,5443160
Their sum - - - - - -	8,6329480
Log. of 50 days - - - - -	1,6985700
Remainder - - - - - -	3,0656220

Which is the logarithm of 1164,185 days. Seeking this number in the Table of the parabola, it is not found there; but for 1160 days the anomaly is 145°. 16'. 49", and for 1165 days the anomaly is 145°. 20'. 7"; hence the difference for 5 days is 3'. 18"; therefore, as 5 days : 4,185 days :: 3'. 18" : 2'. 46", which must be added to 145°. 16'. 49", the anomaly for 1160 days, and the sum 145°. 19'. 35" is the true anomaly for 1164,185 days in the Table, or

for 50 actual days before or after the passage of the comet through the perihelion.

If the true anomaly of a comet, for any instant, be given, and the time elapsed between that and the passage through the perihelion be required, it may be found from the same Table.

RULE. Seek in the Table the given anomaly, and find the time corresponding to it, taking, if necessary, proportional parts. To the logarithm of the perihelion distance, add its half, and the logarithm of the days found in the Table; their sum is the logarithm of the time elapsed between the comet's passing the perihelion, and its arrival at the given anomaly.

Ex. Given the anomaly of Dr. HALLEY's comet of 1759, 64°. 36'. 37"; to find the time it took to describe that angle from the perihelion; the logarithm of its perihelion distance being 9,766033, according to M. de la CAILLE.

The given anomaly is not in the Table; the two nearest are 64°. 29'. 47", and 64°. 40'. 28". The first answers to 58,75 days, and the other to 59,0 days. The difference of the given anomaly from the first of these two tabular ones is 6'. 50", or 410"; the difference of the tabular anomalies is 10'. 41", or 601", and the difference of times is 0,25 days; hence, 641 : 410 :: 0,25 : 0,15991, which must therefore be added to the tabular time 58,75 days, answering to the anomaly 64°. 29'. 47", and the sum 58,90991 days, will be the tabular time answering to the given anomaly. Now the

Logarithm of 58,90991 - - -	1,7701883
Logarithm of perihelion distance - -	9,7660330
Half logarithm of perihelion distance -	9,8830165
Sum - - - - - - - -	1,4192378

Whose number is 26,25656, the number of days that the comet will employ in describing the angle of 64°. 36'. 37' on either side of the perihelion.

This general Table will be sufficient in all cases to determine the true anomaly from the time given; but it will not be equally accurate for finding the time from the anomaly; for at considerable distances from the perihelion, errors will arise. The following little Table shows how far the Table may be used without incurring an error greater than 30 seconds of time.

Perihelion distance.	Anomalies. D.
0,25	130
0,50	118
0,80	100
1,00	90
1,20	80
1,50	65
2,00	50

Beyond these anomalies, comets of the respective perihelion distances are seldom visible, and for comets of a less perihelion distance, the limits extend proportionably further. Indeed, except when extreme accuracy is required, this Table may be used far beyond the limits here prescribed; and if the utmost precision be necessary, the following Rule will give the time free from error, in all cases. The demonstration will be found in PINGRÉ, Vol. ii. page 339.

RULE. To the log. tangent of half the given anomaly, add the constant logarithm 1,9149328; and to triple the log. tangent of half the anomaly, add the constant logarithm 1,4378116; find the numbers to these logarithms, and add them together. To the logarithm of the sum, add $\frac{3}{2}$ of the logarithm of the perihelion distance of the comet, and the sum will be the logarithm of the days from the perihelion.

Ex. Required the time from the perihelion, answering to 144°. 38′. 28″ of anomaly for the comet of 1769; its perihelion distance being 9,0886320.

	Logarithms.	Number.
Tang. ½ anomaly - -	0,4965560	
Constant - - - - -	1,9149328	
Sum - - - - -	2,4114888	- - 257,92225
Triple tang. ½ anomaly - -	1,4896680	
Constant - - - - -	1,4378116	
Sum - - - - -	2,9274796	- - 846,21275
Log. of sum of numbers -	3,0430222	- - 1104,13500
Perihelion distance $\frac{3}{2}$ - -	8,6322480	- -
Log. of days from the perihelion	1,6759702	

Hence, the time from the perihelion is 47,421 days.

In the following Table, the numbers, denoted by the *figures* in the first column, show the same comets with those of the same numbers marked with the *numerals*. Thus, 49 which stands against the year 1456, denotes the same comet as that against which XLIX stands.

THE ELEMENTS OF EIGHTY-SIX COMETS,

WHICH HAVE BEEN OBSERVED AND CALCULATED TO 1811

Order of the Comets.	Years of appearance.	Passage through the perihelion, mean time at Greenwich.	Longitude of the Ascending Node	Inclination of the Orbit	Place of the Perihelion	Perihelion distance, that of the Sun being 1.	Motion.	Authors who have calculated the Orbits.
		DAYS. H. M. S	S. D M S.	D M S	S D M. S			
I.	837	1 March -	6 26 33 0	10, or 12°	9 19 3 0	0,58	Retrograde	PINGRE'
II.	1231	30 January - 7. 12. 40	0 13 30. 0	6 5 0	4 14 48 0	0,9478	Direct.	PINGRE'.
III.	1264	6 July - - 7. 50. 40	5. 19 0 0	36 30 0	9. 21. 0 0	0,445	Direct.	DUNTHORN
		17 July - - 6. 0 40	5 28 45 0	30 25 0	9 5. 45. 0	0,41081	Direct.	PINGRE'
IV	1299	31 March - 7 28. 40	3 17 8. 0	68 57 0	0 3. 20. 0	0,3179	Retrograde	PINGRE'
V.	1301	22 Oct. (about)	0 15 (about)	70 (about)	9, or 10°	0,457	Retrograde	PINGRE'
VI.	1337	2 June - - 6. 24. 40	2. 24 21. 0	32 11 0	1. 7 59 0	0,40666	Retrograde	HALLEY.
		1 June - - 0 30 40	2. 6. 22. 0	32 11. 0	0. 20. 0. 0	0,6445		PINGRE'
49.	1456	8 June - - 22. 0 40	1. 18. 30. 0	17 56. 0	10 1. 0 0	0,5855	Retrograde	PINGRE'
VII	1472	28 February 22. 22. 10	9 11. 46. 20	5 20 0	1. 15. 33. 30	0,54273	Retrograde	HALLEY.
49.	1531	24 August - 21. 17. 40	1 19. 25 0	17 56. 0	10 1 39 0	0,56700	Retrograde	HALLEY.
19.	1532	19 October - 22 11. 40	2. 20. 27 0	32 36 0	3 21 7 0	0,50910	Direct	HALLEY
VIII.	1533	16 June - 19. 29. 40	4. 5. 44 0	35 49 0	4 27. 16 0	0,2028	Retrograde	DOUWES.
3.	1556	21 April - 20. 2. 40	5. 25. 42. 0	32 6 30	9. 8 50. 0	0,46390	Direct	HALL.
IX	1577	26 October - 18. 44 40	0. 25 52. 0	74 32 45	4 9. 22 0	0,18342	Retrograde	HALLEY
X.	1580	28 November 13. 44. 10	0. 19. 7. 37	64 51 50	3 19 11 55	0,59553	Direct	PINGRE'
XI.	1582	7 May - -	7 5, or 21	59, or 61	8 5, or 9 11	0,23 or 0,04	Retrograde	PINGRE'
XII.	1585	7 Oct N.S. 19 19 40	1 7. 42 30	6 4 0	0 8 51 0	1,09358	Direct	HALLEY
XIII.	1590	8 Feb. N.S. 3. 44. 40	5. 15. 30 40	29 40 40	7 6 54 30	0,57661	Retrograde	HALLEY
XIV.	1593	18 July N. S. 13. 38. 40	5 14. 15 0	87 58 0	5 26 19. 0	0,08911	Direct	De la CAILLE
XV.	1596	8 August - 15. 33. 40	10 15. 36 50	52 9. 45	7 28. 30. 50	0,519415	Retrograde	PINGRE'
49.	1607	26 October - 3 49 40	1. 20. 21. 0	17 2 0	10. 2. 16. 0	0,58680	Retrograde	HALLEY.
XVI.	1618	17 August - 3 2 40	9 23 25 0	21 28 0	10. 18. 20 0	0,51298	Direct	PINGRE
XVII.	1618	8 November 12 22 40	2. 16 1 0	37. 34 0	0. 2. 14. 0	0,37975	Direct	HALLEY.
XVIII.	1652	12 November 15 39 40	2 28. 10 0	79 28. 0	0. 28. 18. 40	0,84750	Direct	HALLEY
XIX	1661	26 January - 23 40 40	2 22 30 30	32 35. 50	3. 25. 58. 40	0,44851	Direct	HALLEY.
XX.	1664	4 December 11 51 40	2 21 14 0	21. 18 30	4 10 41 25	1,025755	Retrograde	HALLEY
XXI.	1665	24 April - 5 14 40	7 18 2 0	76 5 0	2. 11. 54. 30	0,10649	Retrograde	HALLEY.
XXII.	1672	1 March - 8 36 40	9 27 30 30	83. 22 10	1 16. 59 30	0,69739	Direct	HALLEY
XXIII.	1677	6 May - - 0 36 40	7 26 49 10	79. 3. 15	4 17. 37. 5	0,28059	Retrograde	HALLEY
XXIV.	1678	26 August - 14. 2 40	5 11 40 0	3. 4 20	10. 27 46. 0	1,23801	Direct	DOUWES.
XXV.	1680	18 December 0 1. 2	9 1. 57 13	61. 22 55	8. 22 40. 10	0,006030	Direct	PINGRE'
49.	1682	14 September 7 38 40	1. 21 16 30	17. 56. 0	10 2. 52. 45	0,58328	Retrograde	HALLEY
XXVI.	1683	13 July - - 2. 49 10	5 23. 23 0	83. 11. 0	2 25. 29. 30	0,56020	Retrograde	HALLEY
XXVII	1684	8 June - - 10 15 40	8 28 15 0	65 48. 40	7. 28. 52 0	0,96015	Direct	HALLEY.
XXVIII.	1686	16 September 14 32 40	11 20 34 40	31. 21. 40	2 17 0 30	0,32500	Direct	HALLEY
XXIX.	1689	1 December 14 55 40	10 23 45 20	69 17. 0	8 23 44 45	0,016889	Retrograde	PINGRE'.
XXX.	1698	18 October - 16 56 40	8 27 44 15	11 46 0	9 0. 51 15	0,69129	Retrograde	HALLEY
XXXI	1699	13 January - 8. 22 40	10 21 45. 35	69 20 0	7 2 31. 6	0,75435	Retrograde	De la CAILLE
XXXII.	1702	13 March - 14 12. 40	6 9 25 15	4. 30. 0	4. 18. 41 3	0,64590	Direct	De la CAILLE.
XXXIII.	1706	30 January - 4 55 40	0 13. 11. 23	55. 14 5	2 12 36 25	0,426865	Direct.	STRUYCK.
XXXIV	1707	11 December 23. 43 27	1 22 50 29	88. 37. 40	2 19 58 9	0,85904	Direct	STRUYCK.
XXXV	1718	15 January - 1 15. 16	4 7 55 20	31. 12. 53	4. 1. 26. 36	1,02565	Retrograde	DOUWES
XXXVI.	1723	27 September 16 10 40	0. 14 16 0	49. 59 0	1. 12 52. 20	0,99865	Retrograde	BRADLEY.
XXXVII.	1729	25 June - - 11 6 40	10 10 32 37	79. 58. 4	10. 22. 40. 0	4,26140	Direct	De la CAILLE
		23 June - - 6 36 2	10 10 35 15	77. 1. 58	10 22 16 53	4,0698		DOUWES
XXXVIII.	1737	30 January - 8 20 40	7 16 22 0	18. 20. 45	10. 25. 55. 0	0,22282	Direct	BRADLEY
XXXIX.	1739	17 June - - 9 59 40	6 27 25 14	55 42 44	3. 12 38. 40	0,67358	Retrograde	De la CAILLE
XL.	1742	8 February - 4 38 40	6 5 38 29	66 59. 14	7. 7. 35. 13	0,76568	Retrograde	De la CAILLE.
		8 February - 4 21 10	6 5 34 45	67. 4. 11	7. 7. 33 14	0,765555	Retrograde	STRUYCK
XLI.	1743	10 January - 20 25 40	2 8 21 15	2. 19 33	3 2 41 45	0,83501	Direct	De la CAILLE
		10 January - 21 15 37	2 8 10 48	2. 15 50	3. 2 58. 4	0,838115		STRUYCK.
XLII.	1743	20 September 21 16 40	0. 5 16 25	45 48 20	8 6 33. 52	0,52157	Retrograde	KLINKENBERG
XLIII.	1744	1 March - 8 17 0	1 15 45 20	47. 8. 36	6 17. 12 55	0,22206	Direct.	BLISS

THE ELEMENTS OF EIGHTY-SIX COMETS,

WHICH HAVE BEEN OBSERVED AND CALCULATED TO 1811.

Order of the Comets.	Years of appearance.	Passage through the perihelion, mean time at Greenwich. DAYS. H. M. S.	Longitude of the Ascending Node. S. D. M. S.	Inclination of the Orbit. D. M. S.	Place of the Perihelion. S. D. M. S.	Perihelion distance, that of the Sun being 1.	Motion.	Authors who have calculated the Orbits.
XLIV.	1746	3 Mar. 1747, 7. 10. 40	4. 27. 18. 50	79. 6. 20	9. 7. 2. 0	2,19851	Retrograde.	De la Caille.
XLV.	1748	28 April - 19. 35. 25	7. 22. 52. 16	85. 26. 57	7. 5. 0. 50	0,84067	Retrograde.	Maraldi.
XLVI.	1748	18 June - 1. 23. 40	1. 4. 39. 43	56. 59. 3	9. 6. 9. 21	0,65525	Direct.	Struyck.
XLVII.	1757	21 October - 9. 46. 40	7. 4. 4. 0	12. 48. 0	4. 2 49. 0	0,3380	Direct.	Pingré.
XLVIII.	1758	11 June - 3. 17. 40	7. 20. 50. 0	68. 19. 0	8. 27. 38. 0	0,21535	Direct.	Pingré.
XLIX.	1759	12 March - 13 31. 40	1. 23. 49. 0	17. 39. 0	10. 3. 16. 0	0,58349	Retrograde.	De la Caille.
		12 March - 13. 50. 4	1. 23. 45. 35	17. 40. 14	10. 3. 8. 10	0,58490		De la Lande.
		12 March - 12. 48. 16	1. 23. 49. 21	17. 35. 20	10. 3. 16. 20	0,58360		Maraldi.
L.	1760	27 Nov. 1759, 0. 2 37	4. 19. 39. 41	79. 6. 38	1. 23. 34. 19	0,80139	Direct.	Pingré.
LI.	1760	16 Dec. 1759, 21. 3. 40	2. 19. 50. 45	4. 51. 32	4. 18. 24. 35	0,96599	Retrograde.	De la Caille.
LII.	1762	28 May - 15. 17. 40	11. 19. 20. 0	84. 45. 0	3. 15. 15. 0	1,0124	Direct.	De la Lande.
		28 May - 6. 51. 29	11. 19. 2. 22	85. 3. 2	3. 14. 29. 46	1,009856		Struyck.
		29 May - 0. 18. 28	11. 18. 55. 31	85. 22. 21	3. 15. 22. 23	1,01415		Maraldi.
LIII.	1763	1 November 19. 43. 18	11. 26. 23. 26	72. 40. 40	2. 24. 51. 54	0,49876	Direct.	Pingré.
LIV.	1764	12 February 13. 42. 16	4. 0. 4. 33	52. 53. 31	0. 15. 14. 52	0,55522	Retrograde	Pingré.
LV.	1766	17 February 8. 40. 40	8. 4. 10. 50	40. 50. 20	4. 23. 15. 25	0,50533	Retrograde.	Pingré.
LVI.	1766	22 April - 20. 46. 20	2. 14. 22. 50	11. 8. 4	8. 2. 17. 53	0,33274	Direct.	Pingré.
LVII.	1769	7 October 12. 20. 40	5. 25. 0. 43	40. 37. 33	4. 24. 5. 54	0,12376	Direct.	De la Lande.
		7 October 13. 36 53	5. 25. 6. 33	40. 48. 49	4. 24. 11. 7	0,12272		Prosperin.
LVIII.	1770	14 August - 0. 4. 4	4. 12. 17. 3	1. 34. 30	11. 26. 26 13	0,676893	Direct.	Pingré.
		13 August - 12. 55. 40	4 12. 0. 0	1. 33. 40	11. 26. 16. 26	0,674381	Direct.	Lexell.
LIX.	1771	22 Nov. 1770, 5. 38. 40	3. 18. 42. 10	31. 25. 55	6 28. 22. 44	0,52824	Retrograde	Pingré.
LX.	1771	18 April - 22. 5. 7	0. 27. 51. 0	11. 15. 20	3. 13. 28. 13	0,90576	Direct.	Pingré.
LXI.	1772	18 February 20. 41. 15	8. 12. 43. 5	18. 59. 40	3. 18. 6. 22	1,01815	Direct.	De la Lande.
LXII.	1773	5 September 11. 9. 25	4. 1. 15. 37	61. 25. 21	2. 15. 35. 43	1,1339	Direct.	Pingré.
LXIII.	1774	15 August - 10. 46. 15	6. 0. 49. 48	83. 0. 25	10. 17. 22. 4	1,4286	Direct.	Mechain.
LXIV.	1779	4 January - 2. 2. 40	0. 25. 5. 51	32. 24. 0	2. 27. 13. 11	0,71312	Direct.	Mechain.
		4 January - 2. 15. 10	0. 25. 3. 57	32. 25. 30	2. 27. 13. 40	0,7132		Chev. d'Angos.
LXV.	1780	30 September 18. 3. 30	4. 4. 9. 19	53. 48. 5	8. 6. 21. 18	0,09925	Retrograde.	Mechain.
LXVI.	1781	7 July - - 4. 32. 0	2. 23. 0. 38	81. 43. 26	7. 29. 11. 25	0,775861	Direct.	Mechain.
LXVII.	1781	29 November 12. 32. 26	2. 17. 22. 52	27. 13. 8	0. 16. 3. 28	0,96101	Retrograde.	Mechain.
LXVIII.	1783	15 November 5. 44. 3	1. 24. 13. 50	53. 9. 9	1. 15. 24. 46	1,5653	Direct.	Mechain.
LXIX.	1784	21 January - 4. 47. 40	1. 26 49. 21	51. 9. 12	2. 20. 44. 21	0,70786	Retrograde.	Mechain.
LXX.	1784	9 April - 21. 7. 26	2. 26. 52. 9	47. 55. 8	10 28. 54 57	0,650531	Retrograde.	Chev. d'Angos.
LXXI.	1785	27 January - 7 48. 44	8. 24. 12. 15	70. 14. 12	3. 19. 51. 56	1,143398	Direct.	Mechain.
LXXII.	1785	8 April - 8. 58. 52	2. 4. 33. 36	87. 31. 54	9. 27. 29. 33	0,427300	Retrograde.	Mechain.
LXXIII.	1786	7 July - - 21. 50. 52	6. 14. 22. 40	50. 54. 28	5. 9. 25. 36	0,41010	Direct.	Mechain.
LXXIV.	1787	10 May - - 19. 48. 40	3. 16. 51. 36	48. 15. 51	0. 7. 44. 9	0,34891	Retrograde.	P. de Saron.
LXXV.	1788	10 November 7. 25. 40	5. 7. 10. 38	12. 28. 20	3. 9. 8. 27	1,06301	Retrograde.	Mechain.
LXXVI.	1788	20 November 9. 4. 25	11. 21. 42. 15	64. 52. 32	0. 23. 12. 22	0,766941	Direct.	Mechain.
LXXVII.	1790	17 January -	5. 22. 0. 0	29. 31. 0	1. 28. 0 0	0,75	Retrograde.	De S - - - -
LXXVIII	1790	28 January - 7. 36. 13	8. 27. 8. 37	56. 58. 13	3. 21. 44. 37	1,063286	Direct.	Mechain.
LXXIX.	1790	May 21 at 5. 56. 15 Par.	1. 3. 11 2	63. 52. 27	9. 3. 43. 27	9.9019814	Retrograde.	Mechain.
LXXX.	1792	Jan. 13 13 44 15 Par.	6. 10. 46. 15	39. 46 55	1. 6. 29. 42	0,1116064	Retrograde	Mechain.
LXXXI.	1795	Dec 15 15. 39. 0 Par.	11. 13. 23. 0	20. 3. 0	5. 7. 37. 0	0,258		Bouvard.
LXXXII.	Année 7	Nivose 11 13. 8. 18 Par	8. 9. 30. 44	42. 23. 25	1. 4. 29. 48	0,77968		Burcharth.
LXXXIII.	Année 8	Nivose 4 21. 40 10 Par.	10 26. 49. 11	77. 1. 38	6. 10. 20 12	0,62580	Retrograde.	Mechain.
LXXXIV.	1804	Feb 13 14. 6. 16 Bre	5. 26. 47. 58	56. 28. 40	4. 28. 44. 51	1,07117	Direct.	Bouvard.
LXXXV	1807	Sep 18 20 55. 32 Par	8. 26. 33. 4	63. 11. 18	9. 1. 6 53	0,648769	Direct.	M. Pl. Sierra
LXXXVI.	1811	Sep 12 9. 48. 0 Gr.	4. 20. 13. 0	72. 12. 0	2. 14. 12	1,02241	Retrograde.	Burckhards.

CHAP. XXVII.

ON THE FIXED STARS

698. ALL the heavenly bodies beyond our system are called *Fixed* 's, because they do not appear to have any proper motion of their own, ex- some few, which will be mentioned hereafter. From their immense dis- 'e, as appears by Article 524, they must be bodies of very great magnitude, rwise they could not be visible; and when we consider the weakness of ected light, there can be no doubt but that they shine with their own light. 'y are easily known from the planets, by their twinkling. The number of s visible at once to the naked eye is about 1000; but Dr. HERSCHEL, by his rovements of the reflecting telescope, has discovered that the whole number reat, beyond all conception. In that bright tract of the heavens called the *lky Way*, which, when examined by good telescopes, appears to be an im- nse collection of stars which gives that whitish appearance to the naked eye, has, in a quarter of an hour, seen 116000 stars pass through the field of view telescope of only 15′ aperture. Every improvement of his telescopes has covered stars not seen before, so that there appears to be no bounds to their mber, or to the extent of the universe. These stars, which can be of no use us, are probably suns to other systems of planets.

699. From an attentive examination of the stars with good telescopes, many ich appear only single to the naked eye, are found to consist of two, three, more stars. Dr. MASKELYNE had observed α *Herculis*, to be a double star; . HORNSBY had found π *Bootis* to be double; M. CASSINI, Mr. MAYER, Mr. GOTT, and many other Astronomers have made discoveries of the like kind. it Dr. HERSCHEL, by his improved telescopes, has found about 700, of which, t above 42 had been observed before. We shall here give an account of a w of the most remarkable.

α *Herculis*, FLAM. 64, a beautiful double star; the two stars very unequal, e largest is red, and the smallest blue, inclining to green.

δ *Lyræ*, FLAM. 12, double, very unequal, the largest red, and smallest dusky; ot easily to be seen with a magnifying power of 227.

α *Geminorum*, FLAM. 66, double, a little unequal, both white; with a power 146, their distance appears equal to the diameter of the smallest.

ϵ *Lyræ*, FLAM. 4 and 5, a double-double star; at first sight it appears double a considerable distance, and by a little attention each will appear double; ne set are equal, and both white; the other unequal, the largest white, and the

smallest inclined to red. The interval of the stars, of the unequal set, is one diameter of the largest, with a power of 227.

γ *Andromedæ*, Flam. 57, double, very unequal, the largest reddish white, the smallest a fine bright sky-blue, inclining to green. A very beautiful object.

α *Ursæ minoris*, Flam. 1, double, very unequal, the largest white, the smallest red.

β *Lyræ*, Flam. 10, quadruple, unequal, white, but three of them a little inclined to red.

α *Leonis*, Flam. 32, double, very unequal, largest white, smallest dusky.

ε *Bootis*, Flam. 36, double, very unequal, largest reddish, smallest blue, or rather a faint lilac; very beautiful.

b Draconis, Flam. 39, a very small double star, very unequal, the largest white, smallest inclining to red.

λ *Orionis*, Flam. 39, quadruple, or rather a double star, and has two more at a small distance, the double star considerably unequal, the largest white, smallest pale rose colour.

ξ *Libræ*, Flam. *ultima*, double-double, one set very unequal, the largest a very fine white.

μ *Cygni*, Flam. 78, double, considerably unequal, the largest white, the smallest blueish.

μ *Herculis*, Flam. 86, double, very unequal, the small star is not visible with a power of 278, but is seen very well with one of 460; the largest is inclined to a pale red, smallest duskish.

α *Capricorni*, Flam. 5, double, very unequal, the largest white, smallest dusky.

υ *Lyræ*, Flam. 8, treble, very unequal, the largest white, smallest both dusky.

α *Lyræ*, Flam. 3, double, very unequal, the largest a fine brilliant white, the smallest dusky; it appears with a power of 227. Dr. Herschel measured the diameter of this fine star, and found it to be 0″,3553.

700. These are a few of the principal double, treble, &c. stars mentioned by Dr. Herschel in his catalogues which he has given us in the *Phil. Trans.* 1782 and 1785. The examination of double stars with a telescope is a very excellent and ready method of proving its powers. Dr. Herschel recommends the following method. The telescope and the observer having been some time in the open air, adjust the focus of the telescope to some single star of nearly the same magnitude, altitude and colour of the star to be examined; attend to all the phænomena of the adjusting star as it passes through the field of view—whether it be perfectly round and well defined, or affected with little appendages playing about the edge, or any other circumstances of the like kind. Such deceptions may be detected by turning the object glass a little in its cell, when these appendages will turn the same way. Thus you will detect the im-

perfections of the instrument, and therefore will not be deceived when you come to examine the double star.

701. If $ABCD$ be the earth's orbit, and its diameter AC bear a sensible proportion to the distance As of a near fixed star s, this star will appear in different situations in the heavens when the earth is at A and C; and it will, in the course of a year, appear to describe a circle $abcd$, or an ellipse, according as the plane of $abcd$ is perpendicular or oblique to the axis Ssm, or according as the star is in or out of the pole of the ecliptic. The angle AsC is called the *Annual Parallax* of the star.

702. Dr. HERSCHEL proposes to find the annual parallax of the fixed stars by observing how the angle between two stars, very near to each other, vary in opposite parts of the year. This method was suggested by GALILEO in his *System. Cos. Dial.* 3. The theory is true, if you admit his postulata, which is, that the stars are all of the same magnitude, and that a star of the second magnitude is double the distance of one of the first, and so on. But we have no reason whatever for making the former supposition, and if we reason from the bodies in our own system, analogy will be against it; and in respect to the magnitudes, the arrangement of that is merely arbitrary. We will however explain the method in the most simple case. Let x and y be two stars situated in a line with the earth at A, and perpendicular to the diameter AB of the earth's orbit, and when the earth is at B, observe the angle xBy. Let $P=$ the angle AxB, or the annual parallax of x, $p=$the angle xBY found from observation, M and m the angles under which the diameters of x and y appear, and draw zx perpendicular to Bx. Then $p : P :: xz : AB :: xy : Ay ::$ (because $M : m :: Ay : Ax$) $M-m : M$; hence, $P=\dfrac{p \times M}{M-m}$ the parallax of x. If x be a star of the first magnitude, and y one of the third, and $p=1''$, then $P=1''\frac{1}{2}$ on these suppositions. See the *Phil. Trans.* 1782.

703. Several stars mentioned by ancient Astronomers are not now to be found, and several are now observed, which do not appear in their catalogues. The most ancient observation of a new star is that by HIPPARCHUS, about 120 years before J. C. which occasioned his making a catalogue of the fixed stars, in order that future Astronomers might see what alterations had taken place since his time. We have no account where this new star appeared. A new star is also said to have appeared in the year 130; another in 389; another in the ninth century, in 15° of *Scorpio*; a fifth in 945; and a sixth in 1264; but the accounts we have of all these are very imperfect.

704. The first new star we have any accurate account of, is that which was discovered by CORNELIUS GEMMA, on November 8, 1572, in the Chair of *Cassiopea*. It exceeded *Sirius* in brightness, and was seen at mid-day. It first appeared bigger than *Jupiter*, but it gradually decayed, and after sixteen months it en-

them in 1668. The star θ in the tail of the *Serpent*, reckoned by TYCHO of the third, was found, by him, of the fifth magnitude. The star ρ in *Serpentarius* did not appear, from the time it was observed by him, till 1695. The star ψ in the *Lion*, after disappearing, was seen by him in 1667. He observed also that β in *Medusa's Head* varied in its magnitude.

711. M. CASSINI discovered *one* new star of the fourth, and *two* of the fifth magnitude in *Cassiopea;* also *five* new stars in the same constellation, of which three have disappeared; *two* new ones in the beginning of the constellation *Eridanus*, of the fourth and fifth magnitude; and *four* new ones of the fifth or sixth magnitude, near the north pole. He further observed, that the star, placed by BAYER near ϵ of the *Little Bear*, is no longer visible; that the star A of *Andromeda*, which had disappeared, had come into view again in 1695; that in the same constellation, instead of one in the *Knee*, marked υ, there are two others come more northerly; and that ξ is diminished; that the star placed by TYCHO at the end of the *Chain of Andromeda*, as of the fourth magnitude, could then scarcely be seen; and that the star which, in TYCHO's catalogue, is the twentieth of *Pisces*, was no longer visible.

712. M. MARALDI observed, that the star κ in the left leg of *Sagittarius*, marked by BAYER of the third magnitude, appeared of the sixth, in 1671; in 1676 it was found by Dr. HALLEY to be of the third; in 1692 it could hardly be perceived, but in 1693 and 1694 it was of the fourth magnitude. In 1704 he discovered a star in *Hydra* to be periodical; its position is in a right line with those in the tail marked π and γ. The time between its greatest lustre, of the fourth magnitude, was about two years; in the intermediate time it disappeared. In 1666, HEVELIUS says he could not find a star of the fourth magnitude in the eastern scale of *Libra*, observed by TYCHO and BAYER; but MARALDI, in 1709, says, that it had then been seen for 15 years, smaller than one of the fourth. See *Elem. d'Astron.* page 57.

713. J. GOODRICKE, Esq. has determined the periodic variation of *Algol*, or β *Persei* (observed by MONTANARI to be variable) to be about 2*d.* 21*h.* Its greatest brightness is of the second magnitude, and least of the fourth. It changes from the second to the fourth in about three hours and a half, and back again in the same time, and retains its greatest brightness for the other part of the time. See the *Phil. Trans.* 1783. In the *Connoissance des Temps*, for 1792, M. de la LANDE has given the following Tables to find the time when the brightness is the least. I have reduced the epochs to the meridian of Greenwich.

TABLES OF THE VARIATION OF *ALGOL.*

EPOCHS.

YEARS.	D.	H.	M.
1796 *B*	2.	7.	38
1797	1.	11.	25
1798	0.	15.	12
1799	2.	15.	49
1800 *C*	1.	19.	36
1801	0.	23.	23
1802	0.	3.	10
1803	2.	3.	47
1804 *B*	0.	7.	34

In leap-year, we must add a day to the calculation, in January and February.

MEAN MOTION FOR MONTHS

MONTHS.	D.	H.	M.
January	0.	0.	0
February	0.	12.	59
March	1.	5.	10
April	1.	18.	9
May	0.	10.	19
June	0.	23.	19
July	2.	12.	18
August	0.	4.	28
September	0.	17.	28
October	2.	6.	27
November	2.	19.	27
December	1.	11.	37

MEAN MOTION FOR YEARS.

YEARS.	D.	H.	M.
1	2.	0.	36
2	1.	4.	23
3	0.	8.	11
4 *B*	1.	8.	47
5	0.	12.	34
6	2.	13.	10
7	1.	16.	58
8 *B*	2.	17.	34

REVOLUTIONS

	D.	H.	M.	S.
1	2.	20.	49.	2
2	5.	17.	38.	4
3	8.	14.	27.	6
4	11.	11.	16.	8
5	14.	8.	5.	10
6	17.	4.	54.	12
7	20.	1.	43.	14
8	22.	22.	32.	16
9	25.	19.	21.	18
10	28.	16.	10.	20

-OODRICKE also discovered, that β *Lyræ* was subject to a periodic
ie following is the result of his observations. It completes all its
days 19 hours, during which time, it undergoes the following
It is of the third magnitude for about two days.—2. It diminishes
ays.—3. It is between the fourth and fifth magnitude for less than
increases in about two days.—5. It is of the third magnitude for
ays.—6. It diminishes in about one day.—7. It is something larger
h magnitude for a little less than a day.—8. It increases in about
hree quarters to the first point, and so completes a whole period.
. *Trans.* 1785. He has also found, that δ *Cephei* is subject to a
tion of *5d.* *8h.* $37'\frac{1}{2}$, during which time it undergoes the following
It is at its greatest brightness about 1 day 13 hours.—2. Its dimi-
formed in about 1 day 18 hours.—3. It is at its greatest obscura-
day 12 hours.—4. It increases in about 13 hours. Its greatest and
ess is that between the third and fourth, and between the fourth
nitudes.
IGOTT, Esq. has discovered η *Antinoi* to be a variable star, with a
days 4 hours 38 minutes. The changes happen as follows. 1. It
est brightness 44 ± hours.—2. It decreases 62 ± hours.—3. It is at
tness 30 ± hours.—4. It increases 36 ± hours. When most bright
third or fourth magnitude, and when least, of the fourth or fifth.
. *Trans.* 1785.
ie *Phil. Trans.* 1796, Dr. HERSCHEL has proposed a method of
e changes that may happen to the fixed stars; with a catalogue of
rative brightness, in order to ascertain the permanency of their

HERSCHEL, in a Paper in the *Phil. Trans.* 1783, upon the proper
e solar system, has given a large collection of stars which were for-
but are now lost; also a catalogue of variable stars, and of new
ery justly observes, that it is not easy to prove that a star was never
; for though it should not be contained in any catalogue whatever,
ument for its former non-appearance, which is taken from its not
observed before, is only so far to be regarded, as it can be made
r almost certain, that a star would have been observed, had it been

re have been various conjectures to account for the appearances of
ble stars. M. MAUPERTUIS supposes, that they may have so quick
out their axes, that the centrifugal force may reduce them to flat
oids, not much unlike a mill-stone; that its plane may be inclined
of the orbits of its planets, by whose attraction the position of the
e altered, so that when its plane passes through the earth, it may

be almost or entirely invisible, and then become again visible as its broad side is turned towards us. Others have conjectured, that considerable parts of their surfaces are covered with dark spots, so that when, by the rotation of the star, these spots are presented to us, the stars become almost or entirely invisible. Others have supposed, that these stars have very large opaque bodies revolving about and near to them, so as to obscure them when they come in conjunction with us. The irregularity of the phases of some of them, shows the cause to be variable, and therefore may perhaps be best accounted for, by supposing that a great part of the body of the star is covered with spots, which appear and disappear like those on the sun's surface. The total disappearance of a star may probably be the destruction of its system; and the appearance of a new star, the creation of a new system of planets.

719. The fixed stars are not all evenly spread through the heavens, but the greater part of them are collected into clusters, of which it requires a large magnifying power, with a great quantity of light, to be able to distinguish the stars separately. With a small magnifying power and quantity of light, they only appear small whitish spots, something like a small light cloud, and from thence they were called *Nebulæ*. There are some nebulæ, however, which do not receive their light from stars. In the year 1656, HUYGENS discovered a nebula in the middle of *Orion's Sword;* it contains only seven stars, and the other part is a bright spot upon a dark ground, and appears like an opening into brighter regions beyond. In 1612, SIMON MARIUS discovered a nebula in the *Girdle of Andromeda.* Dr. HALLEY, when he was observing the southern stars, discovered one in the *Centaur*, but this is never visible in England. In 1714, he found another in *Hercules*, nearly in a line with ζ and η of BAYER. This shows itself to the naked eye, when the sky is clear and the moon absent. M. CASSINI discovered one between the *Great Dog* and the *Ship*, which he describes as very full of stars, and very beautiful, when viewed with a good telescope. There are two whitish spots near the south pole, called, by sailors, the *Magellanic Clouds*, which, to the naked eye, resemble the milky way, but through telescopes they appear to be composed of stars. M. de la CAILLE, in his catalogue of fixed stars observed at the Cape of Good Hope, has remarked 42 nebulæ which he observed, and which he divided into three classes; fourteen, in which he could not discover the stars; fourteen, in which he could see a distinct mass of stars; and fourteen, in which the stars appeared of the sixth magnitude, or below, accompanied with white spots, and nebulæ of the first and third kind. In the *Connoissance des Temps*, for 1783, and 1784, there is a catalogue of 103 nebulæ, observed by MESSIER and MECHAIN, some of which they could resolve, and others they could not. But Dr. HERSCHEL has given us a catalogue of 2000 nebulæ and cluster of stars, which he himself has discovered. Some of them form a round compact system, others

are more irregular, of various forms, and some are long and narrow. The globular systems of stars appear thicker in the middle than they would do if the stars were all at equal distances from each other; they are therefore condensed towards the center. That the stars should be thus accidentally disposed, is too improbable a supposition to be admitted; he supposes therefore, that they are thus brought together by their mutual attractions, and that the gradual condensation towards the center, is a proof of a central power of such a kind. He further observes, that there are some additional circumstances in the appearance of extended clusters and nebulæ, that very much favour the idea of a power lodged in the brightest part. For although the form of them be not globular, it is plainly to be seen that there is a tendency towards sphericity, by the swell of the dimensions as they draw near the most luminous place, denoting, as it were, a course, or tide of stars, setting towards a center. As the stars in the same nebulæ must be very nearly all at the same relative distances from us, and they appear nearly of the same size, their real magnitudes must be nearly equal. Granting therefore that these nebulæ and clusters of stars are formed by their mutual attraction, Dr. Herschel concludes that we may judge of their relative age by the disposition of their component parts, those being the oldest which are most compressed. He supposes the milky way to be a nebula, of which our sun is one of its component stars. See the *Phil. Trans.* 1786 and 1789.

720. Dr. Herschel has discovered other phænomena in the heavens which he calls *Nebulous Stars*, that is, stars surrounded with a faint luminous atmosphere, of a considerable extent. Cloudy or nebulous stars, he observes, have been mentioned by several Astronomers; but this name ought not to be applied to the objects which they have pointed out as such; for, on examination, they proved to be either clusters of stars, or such appearances as may reasonably be supposed to be occasioned by a multitude of stars at a vast distance. He has given an account of seventeen of these stars, one of which he has thus described. "November 13, 1790. A most singular phænomenon! A star of the eighth magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ diameter. The star is perfectly in the center, and the atmosphere is so diluted, faint and equal throughout, that there can be no surmise of its consisting of stars; nor can there be a doubt of the evident connection between the atmosphere and the star. Another star not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance." Hence he draws the following consequences. Granting the connection between the star and the surrounding nebulosity, if it consist of stars very remote which gives the nebulous appearance, the central star, which is visible, must be immensely greater than the rest; or if the central star be no bigger than common, how extremely small and compressed must be those other lumi-

nous points which occasion the nebulosity? As, by the former supposition, the luminous central point must far exceed the standard of what we call a star, so, in the latter, the shining matter about the center will be much too small to come under the same denomination; we therefore either have a central body which is not a star, or a star which is involved in a shining fluid, of a nature totally unknown to us. This last opinion Dr HERSCHEL adopts. The existence of this shining matter, he says, does not seem to be so essentially connected with the central points, that it might not exist without them. The great resemblance there is between the chevelure of these stars, and the diffused nebulosity there is about the constellation of *Orion*, which takes up a space of more than 60 square degrees, renders it highly probable that they are of the same nature. If this be admitted, the separate existence of the luminous matter is fully proved. Light reflected from the star could not be seen at this distance. And besides, the outward parts are nearly as bright as those near the star. In further confirmation of this, he observes, that a cluster of stars will not so completely account for the milkiness, or soft tint of the light of these nebulæ, as a self luminous fluid. This luminous matter seems more fit to produce a star by its condensation, than to depend on the star for its existence. There is a telescopic milky way extending in right ascension from $5h.\ 15'.\ 8''$ to $5h.\ 39'.\ 1''$, and in polar distance from $87°.\ 46'$ to $98°.\ 10'$. This, Dr. HERSCHEL thinks, is better accounted for, by a luminous matter, than from a collection of stars. He observes, that perhaps some may account for these nebulous stars, by supposing that the nebulosity may be formed by a collection of stars at an immense distance, and that the central star may be a near star accidentally so placed; the appearance however of the luminous part does not, in his opinion, at all favour the supposition that it is produced by a great number of stars; on the other hand, it must be granted that it is extremely difficult to admit the other supposition, when we know nothing but a solid body that is self-luminous, or, at least, that a fixed luminary must immediately depend upon such, as the flame of a candle upon the candle itself. See the *Phil. Trans.* 1791, for Dr. HERSCHEL's account.

On the Constellations.

721. As soon as Astronomy began to be studied, it must have been found necessary to divide the heavens into separate parts, and to give some representations to them, in order that Astronomers might describe and speak of the stars, so as to be understood. Accordingly we find that these circumstances took place very early. The ancients divided the heavens into *Constellations*, or collections of stars, and represented them by animals, and other figures accord-

ing to the ideas which the dispositions of the stars suggested. We find some of them mentioned by JOB, and although it has been disputed, whether our translation has sometimes given the true meaning to the Hebrew words, yet it is agreed, that they signify the constellations. Some of them are mentioned by HOMER and HESIOD, but ARATUS professedly treats of all the ancient ones, except three which were invented after his time. The number of the ancient constellations was 48, but the present number upon a globe is about 70; by rectifying which (as will be afterwards explained), and setting it to correspond with the stars in the heavens, you may, by comparing them, very easily get a knowledge of the different constellations and stars. Those stars which do not come into any of the constellations, are called *unformed stars*. The stars visible to the naked eye are divided into six classes, according to their magnitudes; the largest are called of the first magnitude, the next of the second, and so on. Those which cannot be seen without telescopes, are called *Telescopic Stars*. The stars are now generally marked upon maps and globes with BAYER's letters; the first letter in the Greek alphabet being put to the greatest star of each constellation; the second letter to the next greatest, and so on, and when any more letters are wanted, the Italic characters are generally used; this serves as a name to the star, by which it may be pointed out. Twelve of these constellations lie upon the ecliptic, including a space about 15° broad, called the *Zodiac*, within which all the planets move. The constellation *Aries*, or the *Ram*, about 2000 years ago, lay in the *first* sign of the ecliptic; but, on account of the precession of the equinox, it now lies in the *second*. The following are the names of the constellations, and the number of the stars observed in them by different Astronomers. *Antinous* was made out of the unformed stars near *Aquila*; and *Coma Berenices* out of the unformed stars near the *Lion's Tail*. They are both mentioned by PTOLEMY, but as unformed stars. The constellations as far as the Triangle, with Coma Berenices, are *northern*; those after Pisces, are *southern*.

THE ANCIENT CONSTELLATIONS.

		PTOLEMY.	TYCHO.	HEVELIUS.	FLAMSTEAD.
Ursa Minor	The Little Bear	8	7	12	24
Ursa Major	The Great Bear	35	29	73	87
Draco	The Dragon	31	32	40	80
Cæpheus	Cæpheus	13	4	51	35
Bootes	Bootes	23	18	52	54
Corona Borealis	The Northern Crown	8	8	8	21
Hercules	Hercules kneeling	29	28	45	113
Lyra	The Harp	10	11	17	21
Cygnus	The Swan	19	18	47	81
Cassiopea	The Lady in her Chair	13	26	37	55
Perseus	Perseus	29	29	46	59
Auriga	The Waggoner	14	9	40	66
Serpentarius	Serpentarius	29	15	40	74
Serpens	The Serpent	18	13	22	64
Sagitta	The Arrow	5	5	5	18
Aquila	The Eagle	15	12	23	71
Antinous	Antinous		3	19	
Delphinus	The Dolphin	10	10	14	18
Equulus	The Horse's Head	4	4	6	10
Pegasus	The Flying Horse	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	The Triangle	4	4	12	16
Aries	The Ram	18	21	27	66
Taurus	The Bull	44	43	51	141
Gemini	The Twins	25	25	38	85
Cancer	The Crab	23	15	29	83
Leo	The Lion	35	30	49	95
Coma Berenices	Berenice's Hair		14	21	43
Virgo	The Virgin	32	33	50	110
Libra	The Scales	17	10	20	51
Scorpius	The Scorpion	24	10	20	44
Sagittarius	The Archer	31	14	22	69
Capricornus	The Goat	28	28	29	51
Aquarius	The Water-bearer	45	41	47	108
Pisces	The Fishes	38	36	39	113

THE ANCIENT CONSTELLATIONS CONTINUED.

		PTOLEMY.	TYCHO.	HEVELIUS.	FLAMSTEAD.
Cetus	The Whale	22	21	45	97
Orion	Orion	38	42	62	78
Eridanus	Eridanus	34	10	27	84
Lepus	The Hare	12	13	16	19
Canis Major	The Great Dog	29	13	21	31
Canis Minor	The Little Dog	2	2	13	14
Argo	The Ship	45	3	4	64
Hydra	The Hydra	27	19	31	60
Crater	The Cup	7	3	10	31
Corvus	The Crow	7	4		9
Centaurus	The Centaur	37			35
Lupus	The Wolf	19			24
Ara	The Altar	7			9
Corona Australis	The Southern Crown	13			12
Piscis Australis	The Southern Fish	18			24

THE NEW SOUTHERN CONSTELLATIONS.

Columba Naochi	Noah's Dove	10
Robur Carolinum	The Royal Oak	12
Grus	The Crane	13
Phœnix	The Phœnix	13
Indus	The Indian	12
Pavo	The Peacock	14
Apus, *Avis Indica*	The Bird of Paradise	11
Apis, *Musca*	The Bee or Fly	4
Chamæleon	The Chameleon	10
Triangulum Australis	The South Triangle	5
Piscis volans, *Passer*	The Flying Fish	8
Dorado, *Xiphias*	The Sword Fish	6
Toucan	The American Goose	9
Hydrus	The Water Snake	10

HEVELIUS's CONSTELLATIONS

Made out of the Unformed Stars.

		HEVELIUS.	FLAMSTEED.
Lynx	The Lynx	19	44
Leo Minor	The Little Lion		53
Asteron and Chara	The Greyhounds	23	25
Cerberus	Cerberus	4	
Vulpecula and Anser	The Fox and Goose	27	35
Scutum Sobieski	Sobieski's Shield	7	
Lacerta	The Lizard		16
Camelopardalus	The Camelopard	32	58
Monoceros	The Unicorn	19	31
Sextans	The Sextant	11	41

Besides the letters which are prefixed to the stars, many of them have names, as *Regulus*, *Syrius*, *Arcturus*, &c.

722. KEPLER, who was afterwards in this conjecture followed by Dr. HALLEY, has made a very ingenious observation upon the magnitudes and distances of the fixed stars. He observes, that there can be only 13 points* upon the surface of a sphere as far distant from each other as from the center; and supposing the nearest fixed stars to be as far from each other as from the sun, he concludes that there can be only 13 stars of the first magnitude. Hence, at twice that distance from the sun, there may be placed four times as many, or 52; at three times that distance, nine times as many, or 117; and so on. These numbers will give pretty nearly the number of stars of the first, second, third, &c. magnitudes. Dr. HALLEY further remarks, that if the number of stars be finite, and occupy only a part of space, the outward stars would be continually attracted towards those which are within, and in process of time they would coalesce and unite into one. But if the number be infinite, and they occupy an infinite space, all the parts would be nearly in equilibrio, and consequently each fixed star being drawn in opposite directions would keep its place, or move on till it had found an equilibrium. *Phil. Trans.* N°. 364.

* It is not here to be understood that there can be 13 points upon the surface of a sphere equidistant from each other and from the center of the sphere, but only that 12 equidistant points will be a little further from each other than from the center, so that if these points were reduced to the same distance as from the center, there would be left a space, greater than the other spaces, into which you might put another point, but not under the circumstances of the rest.

On the Catalogues of the Fixed Stars.

723. At the time of HIPPARCHUS of Rhodes, about 120 years before J. C. a new star appeared, upon which he set about numbering the fixed stars and reducing them to a *Catalogue*, that posterity might know whether any changes had taken place in the heavens. PTOLEMY however mentions that TYMOCHARIS and ARISTYLLUS left several observations made 180 years before. The catalogue of HIPPARCHUS contained 1022 stars, with their latitudes and longitudes, which PTOLEMY published, with the addition of four more. These Astronomers made their observations with an armillary sphere, placing the armilla, or hoop representing the ecliptic, to coincide with the ecliptic in the heavens by means of the sun in the day-time, and then they determined the place of the moon in respect to the sun by a moveable circle of latitude. The next night, by the help of the moon (whose place before found they corrected by allowing for its motion in the interval of time) they placed the hoop in such a situation as was agreeable to the present moment of time, and then compared, in like manner, the places of the stars with the moon. Thus they found the latitudes and longitudes of the stars; it could not however be done with such an instrument to any very great degree of accuracy. PTOLEMY adapted his catalogue to the year 137 after J. C.; but supposing, with HIPPARCHUS who made the discovery, the precession of the equinoxes to be 1° in 100 years, instead of about 72 years, he only added 2°. 40′ to the numbers in HIPPARCHUS for 265 years (the difference of the epochs) instead of 3°. 42′. 22″ according to Dr. MASKELYNE's Tables. To compare his Tables therefore with the present, we must first increase his numbers by 1°. 2′. 22″, and then allow for the precession from that time to this. The next Astronomer who observed the fixed stars anew, was ULUGH BEIGH, the Grandson of TAMERLANE the Great; he made a catalogue of 1022 stars, reduced to the year 1437. WILLIAM, the most illustrious Landgrave of Hesse, made a catalogue of 400 stars which he observed; he computed their latitudes and longitudes from their observed right ascensions and declinations. In the year 1610, TYCHO BRAHE's catalogue of 777 stars was published from his own observations, made with great care and diligence. It was afterwards, in 1627, copied into the *Rudolphine Tables*, and increased by 223 stars from other observations of TYCHO. Instead of a *zodiacal* armilla, TYCHO substituted the *equatorial* armilla, by which he observed the difference of right ascensions, and the declinations, out of the meridian, the meridian altitude being always made use of to confirm the others. From thence he computed the latitudes and longitudes. TYCHO compared *Venus* with the sun, and then the other stars with Venus, allowing for its parallax and refraction; and having thus ascertained the places of a few stars, he settled the rest from them; and although

his instrument was very large, and constructed with great accuracy, yet not having pendulum clocks to measure his time, his observations cannot be very accurate. The next catalogue was that of R. P. RICCIOLUS, which was taken from TYCHO's, except 101 stars which he himself had observed. HEVELIUS of Dantzick in 1690 published a catalogue of 1930 stars, of which 950 were known to the ancients; 603 he calls his own, because they had not been accurately observed by any one before himself; and 377 of Dr. HALLEY which were invisible to his hemisphere. Their places were fixed for the year 1660. The *British Catalogue*, which was published by Mr. FLAMSTEAD, contains 3000 stars, rectified for the year 1689. They are distinguished into seven degrees of magnitude (of which the seventh degree are telescopic) in their proper constellations. This catalogue is more correct than any of the others, the observations having been made with better instruments. He also published an *Atlas Cœlestis*, or maps of the stars, in which each star is laid down in its true place, and delineated of its own magnitude. Each star is marked with a letter, beginning with the first letter α of the Greek alphabet for the largest star of each constellation, and so on according to their magnitudes, following, in this respect, the charts of the same kind which were published by J. BAYER, a German, in 1603. In the year 1757, M. de la CAILLE published his *Fundamenta Astronomiæ*, in which there is a catalogue of 397 stars; and in 1763, he published a catalogue of 1942 southern stars, from the tropic of Capricorn to the south pole, with their right ascensions and declinations for 1750. He also published a catalogue of zodiacal stars in the *Ephemerides* from 1765 to 1774. Mr. MAYER also published a catalogue of 600 zodiacal stars. In the *Nautical Almanac* for 1773, there is published a catalogue of 380 stars observed by Dr. BRADLEY, with their longitudes and latitudes. In the year 1782, J. E. BODE, Astronomer at Berlin, published a set of *Celestial Charts*, containing a greater number of stars than in those of Mr. FLAMSTEAD, with many of the double stars and nebulæ He also published, in the same work, a catalogue of stars, that of FLAMSTEAD being the foundation, omitting some stars, whose positions were left incomplete, and altering the numbers of others; to which he has added stars from HEVELIUS, M. de la CAILLE, MAYER and others. In the year 1776, there was published at Berlin, a work entitled, *Recueil de Tables Astronomiques*, in which is contained a very large catalogue of stars from HEVELIUS, FLAMSTEAD, M. de la CAILLE, and Dr. BRADLEY, with their latitudes and longitudes for the beginning of 1800; with a catalogue of the southern stars of M. de la CAILLE;—of double stars;—of changeable stars, and of nebulous stars. This is a very useful Work for the Practical Astronomer. But the most complete catalogue is that published by the Rev. Mr. WOLLASTON, F. R. S. in 1789, entitled, *A Specimen of a General Astronomical Catalogue, arranged in Zones of North Polar Distance, and adapted to January* 1, 1790*; containing a Compara-*

tive View of the Mean Positions of Stars, Nebulæ, and Clusters of Stars, as they come out upon Calculation from the Tables of several principal Observers. By arranging the stars into zones parallel to the equator, an observer, with his telescope on an equatorial stand, will have the stars pass through in the order in which he finds them in the catalogue, by which he will more readily find out what he wants, being prepared for its appearance. The first Table contains a catalogue of the mean right ascensions of 36 principal stars for January 1, 1790, as settled by Dr. MASKELYNE, with their annual precessions, and proper motions. The second Table contains the general catalogue of all the stars whose places have been well ascertained, together with those nebulæ and clusters of stars which can easily be seen by a good common telescope, with their right ascensions and north polar distances, and their annual precessions; also their magnitudes, and the number, name or character of the object, and by whom it was observed. The third Table contains an index to the stars in the British Catalogue; referring to the zone of north polar distance in which each is to be found. The fourth Table contains an index of those stars in M. de la CAILLE's fundamental catalogue, which are not in FLAMSTEAD's. The fifth Table contains FLAMSTEAD's British Catalogue, and M. de la CAILLE's southern catalogue, with about eighty circumpolar stars from HEVELIUS which had been omitted by FLAMSTEAD, arranged in their order of right ascensions in time for January 1, 1790. The sixth Table contains a catalogue of the zodiacal stars for 9° of latitude, arranged in their order of longitude for January 1, 1790. The whole concludes with a plan for examining the heavens, proposing that different persons should undertake different zones and examine them very minutely; recommending a system of wires in a telescope which he has found very convenient for that purpose. The Practical Astronomer is under very great obligations to Mr. WOLLASTON for so useful and complete a Work.

On the Proper Motion *of the Fixed Stars.*

724. Dr. MASKELYNE, in the explanation and use of his Tables which he published with the first Volume of his *Observations*, observes, that many, if not all the fixed stars, have small motions among themselves, which are called their *Proper Motions;* the cause and laws of which are hid for the present in almost equal obscurity. From comparing his own observations at that time with those of Dr. BRADLEY, Mr. FLAMSTEAD, and M. ROËMER, he then found the annual proper motion of the following stars in right ascension to be, of *Sirius* $-0'',63$, of *Castor* $-0'',28$, of *Procyon* $-0'',8$, of *Pollux* $-0'',93$, of *Regulus* $-0'',41$, of *Arcturus* $-1'',4$, and of α *Aquilæ* $+0'',57$; and of *Sirius* in north polar distance $1'',20$, and of *Arcturus* $2'',01$ both southwards. But since that time he had

continued his observations, and from a catalogue of the mean right ascensions of 36 principal stars (which he communicated to Mr. WOLLASTON, and is found in his Work), it appears that 35 of them have a *proper motion* in right ascension.

725. In the year 1756, M. MAYER observed 80 stars, and compared them with the observations of ROËMER in 1706. M. MAYER is of opinion, that (from the goodness of the instruments with which the observations were made) where the disagreement is at least 10″ or 15″, it is a very probable indication of a proper motion of such a star. He further adds, that when the disagreement is so great as he has found it in some of the stars, amongst which is *Fomalhaut*, where the difference was 21″ in 50 years, he has no doubt of a proper motion. Dr. HERSCHEL, following MAYER's judgment of his own and ROËMER's observations, has compared the observations, and leaving out of his account all those stars which did not show a disagreement amounting to 10″, he found that 56 of them had a proper motion. From thence he attempts to deduce the motion of the solar system in the following manner.

726. If the sun be first at S, and then move from S to C in the line AB, a star at s would appear to move from a to b, hence if we suppose $BKAL$ to be the ecliptic, any star in the semicircle BKA, supposing that to be the order of the signs, will have its longitude, reckoned from the point to which the sun is moving, increased; but a star in the other semicircle will have its longitude, so reckoned, diminished. Those stars which do not lie in the ecliptic would have their latitudes altered; those would be increased, towards which the sun was moving, and those diminished, from which it was receding. The effect will be less in proportion as the distance of the star is greater, and as it is nearer to A and B in angular distance. These would be the appearances, if the stars themselves were at rest; but if any of them be in motion, these effects will be altered according to their motion compared with the motion of our sun. Some of them therefore from their own proper motions might destroy, or more than counteract the effects arising from the motion of the sun, and appear to have motions contrary to what is here described. Like effects will be produced, if our system move in any direction out of the ecliptic. Hence, in whatever direction our system should move, it would be very easy to find what effect of latitude and longitude would have taken place upon any star by means of a celestial globe, by conceiving the sun to move from the center upon any radius directed to the point to which the sun is moving. Dr. HERSCHEL describes the effect thus. Let an arc of 90° be applied to the surface of a globe, and always passing through that point to which the motion of the system is directed. Then whilst one end moves along the equator, the other will describe a curve passing through its pole and returning into itself; and the stars in the northern hemisphere, within this curve, will appear to move to the north; and the rest will go to the south. A similar curve may be described in the southern hemisphere, and like appearances will take place.

r. HERSCHEL first takes the seven stars before mentioned, whose
had been determined by Dr. MASKELYNE, and he finds, that if
assumed about the 77° of right ascension, and the sun to move
at it will account for all the motions in right ascension. And
supposing the sun to move in the plane of the equator, it should
nt near to γ *Herculis*, it will account for the observed change of
Sirius and *Arcturus*. In respect to the *quantity* of motion of
st depend upon their unknown relative distances; he only speaks
ections of the motions.
xt takes twelve stars from the catalogue of 56, whose proper mo-
n determined from a comparison of the observations of RÖEMER
and adds to them *Regulus* and *Castor*; these have all a proper
t. ascension and declination, except Regulus, which has none in
Of these 27 motions, the above supposed motion of the solar
tisfy 22. There are also some remarkable circumstances in the
hese motions. *Arcturus* and *Sirius* being the largest, and there-
the nearest, ought to have the greatest apparent motion, and so
ave. Also Arcturus is better situated to have a motion in right
d it has the greatest motion. Several other agreements of the
e found also to take place. But there is a very remarkable cir-
respect to Castor. Castor is a double star; now how extraordi-
ear the concurrence, that two such stars should both have a proper
tly alike, that they have never been found to vary a single second!
point out the common cause, the motion of the solar system.
ERSCHEL next takes 32 more of the same catalogue of 56 stars, and
ir motions agree very well with his supposed motion of the solar
the motions of the other 12 stars cannot be accounted for upon
s. In these therefore he supposes the effect of the solar motion
royed and counteracted by their own proper motions. The same
of 19 stars out of the 32, which only agrees with the solar motion
d are, as to sense, at rest the other. According to the rules of
g therefore, which direct us to refer all phænomena to as few and
ples as are sufficient to explain them, Dr. HERSCHEL thinks we
it the motion of the solar system. Perhaps, however, this argu-
be properly applied here, because, there is no new cause or prin-
iced by supposing each star to have a proper motion. Admitting
of universal gravitation, the fixed stars ought to move as well as
it the sun's motion, as here estimated, cannot be owing to the
body upon it which might give it a rotatory motion at the same
de la LANDE conjectures; because a body acting on the sun to

give it its rotation about its axis, would not, at the same time, give it that progressive motion. See Dr. HERSCHEL's Account in the *Phil. Trans.* 1783.

730. Let us now consider, how far this motion of the solar system agrees with the proper motion of the 35 stars determined by Dr. MASKELYNE. Now upon supposition that the sun moves, as conjectured by Dr. HERSCHEL, that motion will account for the motion of 20 of them, so far as regards their direction; but the motion of the other 15 is contrary to that which ought to arise from this supposition. As some of the stars must have a proper motion of their own, even upon the hypothesis of a solar motion, and which probably arises from their mutual attraction, it is very probable that they have all a proper motion from the same cause, but most of them so very small as not yet to have been discovered. And it might also happen, that such a motion might be the same as that which would arise from the motion of the solar system. Yet it must be confessed, that the circumstance of *Castor*, and the motions both in right ascension and declination of many of the stars being such as arise from this hypothesis, with the apparent motion being greatest of those stars which are probably nearest, form a strong argument in its favour.

On the Zodiacal Light.

731. The *Zodiacal Light* is a pyramid of light which sometimes appears in the morning before sun rise. It has the sun for its basis, and in appearance resembles the *Aurora Borealis.* Its sides are not straight, but a little curved, its figure resembling a lens seen edgeways. It is generally seen here about October and March, that being the time of our shortest twilight; for it cannot be seen in the twilight; and when the twilight lasts a considerable time, it is withdrawn before the twilight ends. It was observed by M. CASSINI, in 1683, a little before the vernal equinox, in the evening, extending along the ecliptic from the sun. He thinks however that it has appeared formerly and afterwards disappeared, from an observation of Mr. J. CHILDREY, in a book published in 1661, entitled, *Britannia Baconica.* He says, that "in the month of February, for several years, about six o'clock in the evening, after twilight, he saw a path of light tending from the twilight towards the *Pleiades*, as it were touching them. This is to be seen whenever the weather is clear, but best when the moon does not shine. I believe this phænomenon has been formerly, and will hereafter appear always at the abovementioned time of the year. But the cause and nature of it I cannot guess at, and therefore leave it to the enquiry of posterity." From this description, there can be no doubt but that this was the zodiacal light. He suspects also, that this is what the ancients called *Trabes*, which word they used for a meteor, or impression in the air like a beam. PLINY, lib.

the angle STA having been observed greater than 90°, ST must be less than SA, or the light must extend to a distance from the sun, greater than the earth's distance. Hence, when the earth is about the nodes of this light, or the points where the plane ABC intersects the ecliptic, it will be immersed in this zodiacal light, or, as it is also called, the solar atmosphere. M. de MAIRAN thinks the *Aurora Borealis* depends upon this.

734. M. FATIO conjectured, that this appearance arises from a collection of corpuscles encompassing the sun in the form of a lens, reflecting the light of the sun. M. CASSINI supposed that it might arise from an infinite number of planets revolving about the sun; so that this light might owe its existence to these bodies, as the milky way does to an innumerable number of fixed stars. It is now however generally supposed, that it is matter detached from the sun by its rotation about its axis. The velocity of the equatorial parts of the sun being the greatest, would throw the matter to the greatest distance, and, on account of the diminution of velocity towards its poles, the height to which the matter would there rise would be diminished; and as it would probably spread a little sideways, it would form an atmosphere about the sun something in the form of a lens, whose section perpendicular to its axis would coincide with the sun's equator. And this agrees very well with observation. There is however a difficulty in thus accounting for this phænomenon. It is very well known, that the centrifugal force of a point of the sun's equator is a great many times less than its gravity. It does not appear, therefore, how the sun, from its rotation, can detach any of its gross particles. If they be particles detached from the sun, they must be sent off by some other unknown force; and in that case they might be sent off equally in all directions, which would not agree with the observed figure. The cause is probably owing to the sun's rotation, although not immediately to the centrifugal force arising therefrom.

II. p. 26, says, *Emicant Trabes, quos docos vocant.* Des CARTES also speaks of a phænomenon of the same kind. M. FATIO de DUILLIER observed it immediately after the discovery by M. CASSINI, and suspected that it has always appeared. It was soon after observed by M. KIRCH and EIMMART in Germany. In the year 1707, on April 3, it was observed by Mr. DERHAM in Essex. It appeared in the western part of the heavens, about a quarter of an hour after sun set, in the form of a pyramid, perpendicular to the horizon. The base of this pyramid he judged to be the sun. Its vertex reached 15° or 20° above the horizon. It was throughout of a dusky red colour, and at first appeared pretty vivid and strong, but faintest at the top. It grew fainter by degrees, and vanished about an hour after sun set. This solar atmosphere has also been seen about the sun in a total solar eclipse, a luminous ring appearing about the moon at the time when the eclipse was total.

732. Let *HOR* be the horizon, *S* the sun 18° below at the end of twilight, FIG.
then will *AIO* represent the appearance and position of the zodiacal light seen 178.
at Paris on the last day of February, and *zge* will represent the same the next morning before the beginning of twilight, the sun being at *S′*, as determined by M. de MAIRAN in his treatise *De l'Aurore Boreale*. The distance *SA* was then about 90°, and *IO* about 20°. The axis *AZ*, *az* coincide with the sun's equator, and therefore makes an angle of about $7\frac{1}{2}$° with the ecliptic. Therefore as the angle which the ecliptic makes with the horizon changes at different times of the day, the angle which the axis of this light makes with the horizon will also be variable. Hence, if we determine the angle which the ecliptic makes with the horizon at any time, it will give us the position. If we set a celestial globe to the hour, it will show us its position, and through what stars it will pass, which will therefore direct us very accurately where to look for it. Hence it will be most visible, cæteris paribus, when the ecliptic makes the greatest angle with the horizon. On October 6, 1684, M. FATIO perceived the point *A* distinctly terminated, the angle of which was $26\frac{1}{2}$°. M. EIMMART observed the same on January 13, 1694. In 1683, when M. CASSINI first observed it, *SA* was 50° or 60°, and *IO* about 8° or 9°. In 1686 and 1687, *SA* extended from 90° to 103°, and *IO* was about 20°. On January 6, 1688, *SA* did not appear to be above 45°, but the horizon was then filled with fogs, and *Venus* shone very bright. The appearance therefore depends upon the state of the atmosphere, and situation of the planets, which may produce light enough partly to obscure it. *IO* has sometimes been extended to 30°. M. PINGRE′, in the torrid zone, has observed *SA* to be 120°. The thickness *IO* ought to appear different at different times of the year, because the earth will be in a different situation in respect to its edge.

733. Let *ABC* be a section of the zodiacal light perpendicular to its axis, *T* FIG.
the earth, and *TA* a line drawn to the highest point above the horizon; now 179.

tirely disappeared. It was observed by Tycho Brahe, who found that it had no sensible parallax; and he concluded that it was a fixed star. Some have supposed that this is the same which appeared in 945, and 1264, the situation of its place favouring this opinion.

705. On August 13, 1596, David Fabricius observed a new star in the *Neck of the Whale*, in 25°. 45′ of Aries, with 15°. 54′ south latitude. It disappeared after October in the same year. Phocyllides Holwarda discovered it again in 1637, not knowing that it had ever been seen before; and after having disappeared for nine months, he saw it come into view again. Bullialdus determined the periodic time between its greatest brightness to be 333 days. Its greatest brightness is that of a star of the second magnitude, and its least, that of a star of the sixth. Its greatest degree of brightness however is not always the same, nor are the same phases always at the same interval.

706. In the year 1600, William Jansenius discovered a changeable star in the *Neck of the Swan*. It was seen by Kepler, who wrote a treatise upon it, and determined its place, to be 16°. 18′ ♒, with 55°. 30′ or 32′ north latitude. Ricciolus saw it in 1616, 1621, 1624 and 1629. He is positive that it was invisible in the last years from 1640 to 1650. M. Cassini saw it again in 1655; it increased till 1660, and then grew less, and at the end of 1661 it disappeared. In November 1665, it appeared again, and disappeared in 1681. In 1715 it appeared of the sixth magnitude, as it does at present.

707. On June 20, 1670, another changeable star was discovered near the *Swan's Head*, by P. Anthelme. It disappeared in October, and was seen again on March 17, 1671. On September 11, it disappeared. It appeared again in March 1672, and disappeared in the same month, and has never since been seen. Its longitude was 1°. 52′. 26″. of ♒, and its latitude 47°. 25′. 22″ N. The days are here put down for the new style.

708. In 1686, Kirchius observed χ in the *Swan* to be a changeable star; and, from 20 years' observations, the period of the return of the same phases was found to be 405 days; the variations of its magnitude however were subject to some irregularity.

709. In the year 1604, at the beginning of October, Kepler discovered a new star near the heel of the right foot of *Serpentarius*, so very brilliant, that it exceeded every fixed star, and even *Jupiter* in magnitude. It was observed to be every moment changing into some of the colours of the rainbow, except when it was near the horizon, when it was generally white. It gradually diminished, and disappeared about October 1605, when it came too near the sun to be visible, and was never seen after. Its longitude was 17°. 40′ of ♐, with 1°. 56′ north latitude, and was found to have no parallax.

710. Montanari discovered two stars in the constellation of the *Ship*, marked β and γ by Bayer, to be wanting. He saw them in 1664, but lost

them in 1668. The star θ in the tail of the *Serpent*, reckoned by TYCHO of the third, was found, by him, of the fifth magnitude. The star ρ in *Serpentarius* did not appear, from the time it was observed by him, till 1695. The star ψ in the *Lion*, after disappearing, was seen by him in 1667. He observed also that β in *Medusa's Head* varied in its magnitude.

711. M. CASSINI discovered *one* new star of the fourth, and *two* of the fifth magnitude in *Cassiopea;* also *five* new stars in the same constellation, of which three have disappeared; *two* new ones in the beginning of the constellation *Eridanus*, of the fourth and fifth magnitude, and *four* new ones of the fifth or sixth magnitude, near the north pole. He further observed, that the star, placed by BAYER near ε of the *Little Bear*, is no longer visible; that the star A of *Andromeda*, which had disappeared, had come into view again in 1695; that in the same constellation, instead of one in the *Knee*, marked υ, there are two others come more northerly; and that ξ is diminished, that the star placed by TYCHO at the end of the *Chain of Andromeda*, as of the fourth magnitude, could then scarcely be seen, and that the star which, in TYCHO's catalogue, is the twentieth of *Pisces*, was no longer visible

712. M. MARALDI observed, that the star κ in the left leg of *Sagittarius*, marked by BAYER of the third magnitude, appeared of the sixth, in 1671; in 1676 it was found by Dr. HALLEY to be of the third; in 1692 it could hardly be perceived, but in 1693 and 1694 it was of the fourth magnitude. In 1704 he discovered a star in *Hydra* to be periodical; its position is in a right line with those in the tail marked π and γ. The time between its greatest lustre, of the fourth magnitude, was about two years; in the intermediate time it disappeared. In 1666, HEVELIUS says he could not find a star of the fourth magnitude in the eastern scale of *Libra*, observed by TYCHO and BAYER; but MARALDI, in 1709, says, that it had then been seen for 15 years, smaller than one of the fourth. See *Elem. d'Astron.* page 57.

713. J. GOODRICKE, Esq. has determined the periodic variation of *Algol*, or β *Persei* (observed by MONTANARI to be variable) to be about *2d. 21h.* Its greatest brightness is of the second magnitude, and least of the fourth. It changes from the second to the fourth in about three hours and a half, and back again in the same time, and retains its greatest brightness for the other part of the time. See the *Phil. Trans.* 1783. In the *Connoissance des Temps*, for 1792, M. de la LANDE has given the following Tables to find the time when the brightness is the least. I have reduced the epochs to the meridian of Greenwich.

TABLES OF THE VARIATION OF *ALGOL.*

EPOCHS.

YEARS.	D.	H.	M.
1796 *B*	2.	7.	38
1797	1.	11.	25
1798	0.	15.	12
1799	2.	15.	49
1800 *C*	1.	19.	36
1801	0.	23.	23
1802	0.	3.	10
1803	2.	3.	47
1804 *B*	0.	7.	34

In leap-year, we must add a day to the calculation, in January and February.

MEAN MOTION FOR MONTHS.

MONTHS.	D.	H.	M.
January	0.	0.	0
February	0.	12.	59
March	1.	5.	10
April	1.	18.	9
May	0.	10.	19
June	0.	23.	19
July	2.	12.	18
August	0.	4.	28
September	0.	17.	28
October	2.	6.	27
November	2.	19.	27
December	1.	11.	37

MEAN MOTION FOR YEARS.

YEARS.	D.	H.	M.
1	2.	0.	36
2	1.	4.	23
3	0.	8.	11
4 *B*	1.	8.	47
5	0.	12.	34
6	2.	13.	10
7	1.	16.	58
8 *B*	2.	17.	34

REVOLUTIONS.

	D.	H.	M.	S.
1	2.	20.	49.	2
2	5.	17.	38.	4
3	8.	14.	27.	6
4	11.	11.	16.	8
5	14.	8.	5.	10
6	17.	4.	54.	12
7	20.	1.	43.	14
8	22.	22.	32.	16
9	25.	19.	21.	18
10	28.	16.	10.	20

714. Mr. GOODRICKE also discovered, that β *Lyræ* was subject to a periodic variation. The following is the result of his observations. It completes all its phases in 12 days 19 hours, during which time, it undergoes the following changes.—1. It is of the third magnitude for about two days.—2. It diminishes in about $1\frac{1}{4}$ days.—3. It is between the fourth and fifth magnitude for less than a day.—4. It increases in about two days.—5. It is of the third magnitude for about three days.—6. It diminishes in about one day.—7. It is something larger than the fourth magnitude for a little less than a day.—8. It increases in about one day and three quarters to the first point, and so completes a whole period. See the *Phil. Trans.* 1785. He has also found, that δ *Cephei* is subject to a periodic variation of *5d.* 8*h.* $37'\frac{1}{2}$, during which time it undergoes the following changes. 1. It is at its greatest brightness about 1 day 13 hours.—2. Its diminution is performed in about 1 day 18 hours.—3. It is at its greatest obscuration about 1 day 12 hours.—4. It increases in about 13 hours. Its greatest and least brightness is that between the third and fourth, and between the fourth and fifth magnitudes.

715. E. PIGOTT, Esq. has discovered η *Antinoi* to be a variable star, with a period of 7 days 4 hours 38 minutes. The changes happen as follows. 1. It is at its greatest brightness 44 ± hours.—2. It decreases 62 ± hours.—3. It is at its least brightness 30 ± hours.—4. It increases 36 ± hours. When most bright it is of the third or fourth magnitude, and when least, of the fourth or fifth. See the *Phil. Trans.* 1785.

716. In the *Phil. Trans.* 1796, Dr. HERSCHEL has proposed a method of observing the changes that may happen to the fixed stars; with a catalogue of their comparative brightness, in order to ascertain the permanency of their lustre.

717. Dr. HERSCHEL, in a Paper in the *Phil. Trans.* 1783, upon the proper motion of the solar system, has given a large collection of stars which were formerly seen, but are now lost, also a catalogue of variable stars, and of new stars, and very justly observes, that it is not easy to prove that a star was never seen before; for though it should not be contained in any catalogue whatever, yet the argument for its former non-appearance, which is taken from its not having been observed before, is only so far to be regarded, as it can be made probable, or almost certain, that a star would have been observed, had it been visible.

718. There have been various conjectures to account for the appearances of the changeable stars. M. MAUPERTUIS supposes, that they may have so quick a motion about their axes, that the centrifugal force may reduce them to flat oblate spheroids, not much unlike a mill-stone; that its plane may be inclined to the plane of the orbits of its planets, by whose attraction the position of the body may be altered, so that when its plane passes through the earth, it may

be almost or entirely invisible, and then become again visible as its broad side is turned towards us. Others have conjectured, that considerable parts of their surfaces are covered with dark spots, so that when, by the rotation of the star, these spots are presented to us, the stars become almost or entirely invisible. Others have supposed, that these stars have very large opaque bodies revolving about and near to them, so as to obscure them when they come in conjunction with us. The irregularity of the phases of some of them, shows the cause to be variable, and therefore may perhaps be best accounted for, by supposing that a great part of the body of the star is covered with spots, which appear and disappear like those on the sun's surface. The total disappearance of a star may probably be the destruction of its system; and the appearance of a new star, the creation of a new system of planets.

719. The fixed stars are not all evenly spread through the heavens, but the greater part of them are collected into clusters, of which it requires a large magnifying power, with a great quantity of light, to be able to distinguish the stars separately. With a small magnifying power and quantity of light, they only appear small whitish spots, something like a small light cloud, and from thence they were called *Nebulæ*. There are some nebulæ, however, which do not receive their light from stars. In the year 1656, HUYGENS discovered a nebula in the middle of *Orion's Sword;* it contains only seven stars, and the other part is a bright spot upon a dark ground, and appears like an opening into brighter regions beyond. In 1612, SIMON MARIUS discovered a nebula in the *Girdle of Andromeda*. Dr. HALLEY, when he was observing the southern stars, discovered one in the *Centaur*, but this is never visible in England. In 1714, he found another in *Hercules*, nearly in a line with ζ and η of BAYER. This shows itself to the naked eye, when the sky is clear and the moon absent. M. CASSINI discovered one between the *Great Dog* and the *Ship*, which he describes as very full of stars, and very beautiful, when viewed with a good telescope. There are two whitish spots near the south pole, called, by sailors, the *Magellanic Clouds*, which, to the naked eye, resemble the milky way, but through telescopes they appear to be composed of stars. M. de la CAILLE, in his catalogue of fixed stars observed at the Cape of Good Hope, has remarked 42 nebulæ which he observed, and which he divided into three classes; fourteen, in which he could not discover the stars; fourteen, in which he could see a distinct mass of stars; and fourteen, in which the stars appeared of the sixth magnitude, or below, accompanied with white spots, and nebulæ of the first and third kind. In the *Connoissance des Temps*, for 1783, and 1784, there is a catalogue of 103 nebulæ, observed by MESSIER and MECHAIN, some of which they could resolve, and others they could not. But Dr. HERSCHEL has given us a catalogue of 2000 nebulæ and cluster of stars, which he himself has discovered. Some of them form a round compact system, others

are more irregular, of various forms, and some are long and narrow. The globular systems of stars appear thicker in the middle than they would do if the stars were all at equal distances from each other, they are therefore condensed towards the center. That the stars should be thus accidentally disposed, is too improbable a supposition to be admitted; he supposes therefore, that they are thus brought together by their mutual attractions, and that the gradual condensation towards the center, is a proof of a central power of such a kind. He further observes, that there are some additional circumstances in the appearance of extended clusters and nebulæ, that very much favour the idea of a power lodged in the brightest part. For although the form of them be not globular, it is plainly to be seen that there is a tendency towards sphericity, by the swell of the dimensions as they draw near the most luminous place, denoting, as it were, a course, or tide of stars, setting towards a center. As the stars in the same nebulæ must be very nearly all at the same relative distances from us, and they appear nearly of the same size, their real magnitudes must be nearly equal. Granting therefore that these nebulæ and clusters of stars are formed by their mutual attraction, Dr. HERSCHEL concludes that we may judge of their relative age by the disposition of their component parts, those being the oldest which are most compressed. He supposes the milky way to be a nebula, of which our sun is one of its component stars. See the *Phil. Trans.* 1786 and 1789.

720. Dr. HERSCHEL has discovered other phænomena in the heavens which he calls *Nebulous Stars,* that is, stars surrounded with a faint luminous atmosphere, of a considerable extent. Cloudy or nebulous stars, he observes, have been mentioned by several Astronomers; but this name ought not to be applied to the objects which they have pointed out as such; for, on examination, they proved to be either clusters of stars, or such appearances as may reasonably be supposed to be occasioned by a multitude of stars at a vast distance. He has given an account of seventeen of these stars, one of which he has thus described. "November 13, 1790. A most singular phænomenon! A star of the eighth magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ diameter. The star is perfectly in the center, and the atmosphere is so diluted, faint and equal throughout, that there can be no surmise of its consisting of stars, nor can there be a doubt of the evident connection between the atmosphere and the star. Another star not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance." Hence he draws the following consequences. Granting the connection between the star and the surrounding nebulosity, if it consist of stars very remote which gives the nebulous appearance, the central star, which is visible, must be immensely greater than the rest, or if the central star be no bigger than common, how extremely small and compressed must be those other lumi-

nous points which occasion the nebulosity? As, by the former supposition, the luminous central point must far exceed the standard of what we call a star, so, in the latter, the shining matter about the center will be much too small to come under the same denomination; we therefore either have a central body which is not a star, or a star which is involved in a shining fluid, of a nature totally unknown to us. This last opinion Dr. HERSCHEL adopts. The existence of this shining matter, he says, does not seem to be so essentially connected with the central points, that it might not exist without them. The great resemblance there is between the chevelure of these stars, and the diffused nebulosity there is about the constellation of *Orion*, which takes up a space of more than 60 square degrees, renders it highly probable that they are of the same nature. If this be admitted, the separate existence of the luminous matter is fully proved. Light reflected from the star could not be seen at this distance. And besides, the outward parts are nearly as bright as those near the star. In further confirmation of this, he observes, that a cluster of stars will not so completely account for the milkiness, or soft tint of the light of these nebulæ, as a self luminous fluid. This luminous matter seems more fit to produce a star by its condensation, than to depend on the star for its existence. There is a telescopic milky way extending in right ascension from $5h.\ 15'.\ 8''$ to $5h.\ 39'.\ 1''$, and in polar distance from $87^\circ.\ 46'$ to $98^\circ.\ 10'$. This, Dr. HERSCHEL thinks, is better accounted for, by a luminous matter, than from a collection of stars. He observes, that perhaps some may account for these nebulous stars, by supposing that the nebulosity may be formed by a collection of stars at an immense distance, and that the central star may be a near star accidentally so placed; the appearance however of the luminous part does not, in his opinion, at all favour the supposition that it is produced by a great number of stars; on the other hand, it must be granted that it is extremely difficult to admit the other supposition, when we know nothing but a solid body that is self-luminous, or, at least, that a fixed luminary must immediately depend upon such, as the flame of a candle upon the candle itself. See the *Phil. Trans.* 1791, for Dr. HERSCHEL's account.

On the Constellations.

721. As soon as Astronomy began to be studied, it must have been found necessary to divide the heavens into separate parts, and to give some representations to them, in order that Astronomers might describe and speak of the stars, so as to be understood. Accordingly we find that these circumstances took place very early. The ancients divided the heavens into *Constellations*, or collections of stars, and represented them by animals, and other figures accord-

ing to the ideas which the dispositions of the stars suggested. We find some of them mentioned by JOB, and although it has been disputed, whether our translation has sometimes given the true meaning to the Hebrew words, yet it is agreed, that they signify the constellations. Some of them are mentioned by HOMER and HESIOD, but ARATUS professedly treats of all the ancient ones, except three which were invented after his time. The number of the ancient constellations was 48, but the present number upon a globe is about 70, by rectifying which (as will be afterwards explained), and setting it to correspond with the stars in the heavens, you may, by comparing them, very easily get a knowledge of the different constellations and stars. Those stars which do not come into any of the constellations, are called *unformed stars.* The stars visible to the naked eye are divided into six classes, according to their magnitudes; the largest are called of the first magnitude, the next of the second, and so on. Those which cannot be seen without telescopes, are called *Telescopic Stars.* The stars are now generally marked upon maps and globes with BAYER's letters; the first letter in the Greek alphabet being put to the greatest star of each constellation, the second letter to the next greatest, and so on, and when any more letters are wanted, the Italic characters are generally used; this serves as a name to the star, by which it may be pointed out. Twelve of these constellations lie upon the ecliptic, including a space about 15° broad, called the *Zodiac*, within which all the planets move. The constellation *Aries*, or the *Ram*, about 2000 years ago, lay in the *first* sign of the ecliptic; but, on account of the precession of the equinox, it now lies in the *second.* The following are the names of the constellations, and the number of the stars observed in them by different Astronomers. *Antinous* was made out of the unformed stars near *Aquila;* and *Coma Berenices* out of the unformed stars near the *Lion's Tail.* They are both mentioned by PTOLEMY, but as unformed stars. The constellations as far as the Triangle, with Coma Berenices, are *northern;* those after Pisces, are *southern.*

THE ANCIENT CONSTELLATIONS.

		PTOLEMY.	TYCHO.	HEVELIUS.	FLAMSTEAD.
Ursa Minor	The Little Bear	8	7	12	24
Ursa Major	The Great Bear	35	29	73	87
Draco	The Dragon	31	32	40	80
Cæpheus	Cæpheus	13	4	51	35
Bootes	Bootes	23	18	52	54
Corona Borealis	The Northern Crown	8	8	8	21
Hercules	Hercules kneeling	29	28	45	113
Lyra	The Harp	10	11	17	21
Cygnus	The Swan	19	18	47	81
Cassiopea	The Lady in her Chair	13	26	37	55
Perseus	Perseus	29	29	46	59
Auriga	The Waggoner	14	9	40	66
Serpentarius	Serpentarius	29	15	40	74
Serpens	The Serpent	18	13	22	64
Sagitta	The Arrow	5	5	5	18
Aquila	The Eagle	15	12	23	71
Antinous	Antinous		3	19	
Delphinus	The Dolphin	10	10	14	18
Equulus	The Horse's Head	4	4	6	10
Pegasus	The Flying Horse	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	The Triangle	4	4	12	16
Aries	The Ram	18	21	27	66
Taurus	The Bull	44	43	51	141
Gemini	The Twins	25	25	38	85
Cancer	The Crab	23	15	29	83
Leo	The Lion	35	30	49	95
Coma Berenices	Berenice's Hair		14	21	43
Virgo	The Virgin	32	33	50	110
Libra	The Scales	17	10	20	51
Scorpius	The Scorpion	24	10	20	44
Sagittarius	The Archer	31	14	22	69
Capricornus	The Goat	28	28	29	51
Aquarius	The Water-bearer	45	41	47	108
Pisces	The Fishes	38	36	39	113

THE ANCIENT CONSTELLATIONS CONTINUED.

		PTOLEMY.	TYCHO.	HEVELIUS.	FLAMSTEAD.
Cetus	The Whale	22	21	45	97
Orion	Orion	38	42	62	78
Eridanus	Eridanus	34	10	27	84
Lepus	The Hare	12	13	16	19
Canis Major	The Great Dog	29	13	21	31
Canis Minor	The Little Dog	2	2	13	14
Argo	The Ship	45	3	4	64
Hydra	The Hydra	27	19	31	60
Crater	The Cup	7	3	10	31
Corvus	The Crow	7	4		9
Centaurus	The Centaur	37			35
Lupus	The Wolf	19			24
Ara	The Altar	7			9
Corona Australis	The Southern Crown	13			12
Piscis Australis	The Southern Fish	18			24

THE NEW SOUTHERN CONSTELLATIONS.

Columba Noachi	Noah's Dove	10
Robur Carolinum	The Royal Oak	12
Grus	The Crane	13
Phœnix	The Phœnix	13
Indus	The Indian	12
Pavo	The Peacock	14
Apus, *Avis Indica*	The Bird of Paradise	11
Apis, *Musca*	The Bee or Fly	4
Chamæleon	The Chameleon	10
Triangulum Australis	The South Triangle	5
Piscis volans, *Passer*	The Flying Fish	8
Dorado, *Xiphias*	The Sword Fish	6
Toucan	The American Goose	9
Hydrus	The Water Snake	10

HEVELIUS's CONSTELLATIONS

Made out of the Unformed Stars.

		HEVELIUS.	FLAMSTEAD.
Lynx	The Lynx	19	44
Leo Minor	The Little Lion		53
Asteron and Chara	The Greyhounds	23	25
Cerberus	Cerberus	4	
Vulpecula and Anser	The Fox and Goose	27	35
Scutum Sobieski	Sobieski's Shield	7	
Lacerta	The Lizard		16
Camelopardalus	The Camelopard	32	58
Monoceros	The Unicorn	19	31
Sextans	The Sextant	11	41

Besides the letters which are prefixed to the stars, many of them have names, as *Regulus*, *Syrius*, *Arcturus*, &c.

722. KEPLER, who was afterwards in this conjecture followed by Dr. HALLEY, has made a very ingenious observation upon the magnitudes and distances of the fixed stars. He observes, that there can be only 13 points* upon the surface of a sphere as far distant from each other as from the center; and supposing the nearest fixed stars to be as far from each other as from the sun, he concludes that there can be only 13 stars of the first magnitude. Hence, at twice that distance from the sun, there may be placed four times as many, or 52; at three times that distance, nine times as many, or 117; and so on. These numbers will give pretty nearly the number of stars of the first, second, third, &c. magnitudes. Dr. HALLEY further remarks, that if the number of stars be finite, and occupy only a part of space, the outward stars would be continually attracted towards those which are within, and in process of time they would coalesce and unite into one. But if the number be infinite, and they occupy an infinite space, all the parts would be nearly in equilibrio, and consequently each fixed star being drawn in opposite directions would keep its place, or move on till it had found an equilibrium. *Phil. Trans.* N°. 364.

* It is not here to be understood that there can be 13 points upon the surface of a sphere equidistant from each other and from the center of the sphere, but only that 12 equidistant points will be a little further from each other than from the center; so that if these points were reduced to the same distance as from the center, there would be left a space, greater than the other spaces, into which you might put another point, but not under the circumstances of the rest.

On the Catalogues of the Fixed Stars.

723. At the time of HIPPARCHUS of Rhodes, about 120 years before J. C. a new star appeared, upon which he set about numbering the fixed stars and reducing them to a *Catalogue*, that posterity might know whether any changes had taken place in the heavens PTOLEMY however mentions that TYMOCHARIS and ARISTYLLUS left several observations made 180 years before. The catalogue of HIPPARCHUS contained 1022 stars, with their latitudes and longitudes, which PTOLEMY published, with the addition of four more. These Astronomers made their observations with an armillary sphere, placing the armilla, or hoop representing the ecliptic, to coincide with the ecliptic in the heavens by means of the sun in the day-time, and then they determined the place of the moon in respect to the sun by a moveable circle of latitude. The next night, by the help of the moon (whose place before found they corrected by allowing for its motion in the interval of time) they placed the hoop in such a situation as was agreeable to the present moment of time, and then compared, in like manner, the places of the stars with the moon. Thus they found the latitudes and longitudes of the stars, it could not however be done with such an instrument to any very great degree of accuracy. PTOLEMY adapted his catalogue to the year 137 after J. C.; but supposing, with HIPPARCHUS who made the discovery, the precession of the equinoxes to be 1° in 100 years, instead of about 72 years, he only added 2°. 40′ to the numbers in HIPPARCHUS for 265 years (the difference of the epochs) instead of 3°. 42′. 22″ according to Dr. MASKELYNE's Tables. To compare his Tables therefore with the present, we must first increase his numbers by 1°. 2′. 22″, and then allow for the precession from that time to this. The next Astronomer who observed the fixed stars anew, was ULUGH BEIGH, the Grandson of TAMERLANE the Great, he made a catalogue of 1022 stars, reduced to the year 1437. WILLIAM, the most illustrious Landgrave of Hesse, made a catalogue of 400 stars which he observed, he computed their latitudes and longitudes from their observed right ascensions and declinations. In the year 1610, TYCHO BRAHE's catalogue of 777 stars was published from his own observations, made with great care and diligence. It was afterwards, in 1627, copied into the *Rudolphine Tables*, and increased by 223 stars from other observations of TYCHO. Instead of a *zodiacal* armilla, TYCHO substituted the *equatorial* armilla, by which he observed the difference of right ascensions, and the declinations, out of the meridian, the meridian altitude being always made use of to confirm the others From thence he computed the latitudes and longitudes. TYCHO compared *Venus* with the sun, and then the other stars with Venus, allowing for its parallax and refraction; and having thus ascertained the places of a few stars, he settled the rest from them; and although

his instrument was very large, and constructed with great accuracy, yet not having pendulum clocks to measure his time, his observations cannot be very accurate. The next catalogue was that of R. P. RICCIOLUS, which was taken from TYCHO's, except 101 stars which he himself had observed. HEVELIUS of Dantzick in 1690 published a catalogue of 1930 stars, of which 950 were known to the ancients; 603 he calls his own, because they had not been accurately observed by any one before himself; and 377 of Dr. HALLEY which were invisible to his hemisphere. Their places were fixed for the year 1660. The *British Catalogue*, which was published by Mr. FLAMSTEAD, contains 3000 stars, rectified for the year 1689. They are distinguished into seven degrees of magnitude (of which the seventh degree are telescopic) in their proper constellations. This catalogue is more correct than any of the others, the observations having been made with better instruments. He also published an *Atlas Cœlestis*, or maps of the stars, in which each star is laid down in its true place, and delineated of its own magnitude. Each star is marked with a letter, beginning with the first letter α of the Greek alphabet for the largest star of each constellation, and so on according to their magnitudes, following, in this respect, the charts of the same kind which were published by J. BAYER, a German, in 1603. In the year 1757, M. de la CAILLE published his *Fundamenta Astronomiæ*, in which there is a catalogue of 397 stars; and in 1763, he published a catalogue of 1942 southern stars, from the tropic of Capricorn to the south pole, with their right ascensions and declinations for 1750. He also published a catalogue of zodiacal stars in the *Ephemerides* from 1765 to 1774. Mr. MAYER also published a catalogue of 600 zodiacal stars. In the *Nautical Almanac* for 1773, there is published a catalogue of 380 stars observed by Dr. BRADLEY, with their longitudes and latitudes. In the year 1782, J. E. BODE, Astronomer at Berlin, published a set of *Celestial Charts*, containing a greater number of stars than in those of Mr. FLAMSTEAD, with many of the double stars and nebulæ. He also published, in the same work, a catalogue of stars, that of FLAMSTEAD being the foundation, omitting some stars, whose positions were left incomplete, and altering the numbers of others; to which he has added stars from HEVELIUS, M. de la CAILLE, MAYER and others. In the year 1776, there was published at Berlin, a work entitled, *Recueil de Tables Astronomiques*, in which is contained a very large catalogue of stars from HEVELIUS, FLAMSTEAD, M. de la CAILLE, and Dr. BRADLEY, with their latitudes and longitudes for the beginning of 1800; with a catalogue of the southern stars of M. de la CAILLE;—of double stars;—of changeable stars, and of nebulous stars. This is a very useful Work for the Practical Astronomer. But the most complete catalogue is that published by the Rev. Mr. WOLLASTON, F. R. S. in 1789, entitled, *A Specimen of a General Astronomical Catalogue, arranged in Zones of North Polar Distance, and adapted to January* 1, 1790; *containing a Compara-*

tive View of the Mean Positions of Stars, Nebulæ, and Clusters of Stars, as they come out upon Calculation from the Tables of several principal Observers. By arranging the stars into zones parallel to the equator, an observer, with his telescope on an equatorial stand, will have the stars pass through in the order in which he finds them in the catalogue, by which he will more readily find out what he wants, being prepared for its appearance. The first Table contains a catalogue of the mean right ascensions of 36 principal stars for January 1, 1790, as settled by Dr. MASKELYNE, with their annual precessions, and proper motions. The second Table contains the general catalogue of all the stars whose places have been well ascertained, together with those nebulæ and clusters of stars which can easily be seen by a good common telescope, with their right ascensions and north polar distances, and their annual precessions; also their magnitudes, and the number, name or character of the object, and by whom it was observed. The third Table contains an index to the stars in the British Catalogue; referring to the zone of north polar distance in which each is to be found. The fourth Table contains an index of those stars in M. de la CAILLE's fundamental catalogue, which are not in FLAMSTEAD's The fifth Table contains FLAMSTEAD's British Catalogue, and M. de la CAILLE's southern catalogue, with about eighty circumpolar stars from HEVELIUS which had been omitted by FLAMSTEAD, arranged in their order of right ascensions in time for January 1, 1790. The sixth Table contains a catalogue of the zodiacal stars for 9° of latitude, arranged in their order of longitude for January 1, 1790. The whole concludes with a plan for examining the heavens, proposing that different persons should undertake different zones and examine them very minutely, recommending a system of wires in a telescope which he has found very convenient for that purpose. The Practical Astronomer is under very great obligations to Mr. WOLLASTON for so useful and complete a Work.

On the Proper Motion *of the Fixed Stars.*

724. Dr. MASKELYNE, in the explanation and use of his Tables which he published with the first Volume of his *Observations*, observes, that many, if not all the fixed stars, have small motions among themselves, which are called their *Proper Motions*, the cause and laws of which are hid for the present in almost equal obscurity. From comparing his own observations at that time with those of Dr. BRADLEY, Mr. FLAMSTEAD, and M. ROËMER, he then found the annual proper motion of the following stars in right ascension to be, of *Sirius* $-0'',63$, of *Castor* $-0'',28$, of *Procyon* $-0'',8$, of *Pollux* $-0'',93$, of *Regulus* $-0'',41$, of *Arcturus* $-1'',4$, and of α *Aquilæ* $+0'',57$; and of *Sirius* in north polar distance $1'',20$, and of *Arcturus* $2'',01$ both southwards. But since that time he had

2

continued his observations, and from a catalogue of the mean right ascensions of 36 principal stars (which he communicated to Mr. WOLLASTON, and is found in his Work), it appears that 35 of them have a *proper motion* in right ascension.

725. In the year 1756, M. MAYER observed 80 stars, and compared them with the observations of ROËMER in 1706. M. MAYER is of opinion, that (from the goodness of the instruments with which the observations were made) where the disagreement is at least 10″ or 15″, it is a very probable indication of a proper motion of such a star. He further adds, that when the disagreement is so great as he has found it in some of the stars, amongst which is *Fomahand*, where the difference was 21″ in 50 years, he has no doubt of a proper motion. Dr. HERSCHEL, following MAYER's judgment of his own and ROËMER's observations, has compared the observations, and leaving out of his account all those stars which did not show a disagreement amounting to 10″, he found that 56 of them had a proper motion. From thence he attempts to deduce the motion of the solar system in the following manner.

726. If the sun be first at S, and then move from S to C in the line AB, a star at s would appear to move from a to b; hence if we suppose $BKAL$ to be the ecliptic, any star in the semicircle BKA, supposing that to be the order of the signs, will have its longitude, reckoned from the point to which the sun is moving, increased; but a star in the other semicircle will have its longitude, so reckoned, diminished. Those stars which do not lie in the ecliptic would have their latitudes altered; those would be increased, towards which the sun was moving, and those diminished, from which it was receding. The effect will be less in proportion as the distance of the star is greater, and as it is nearer to A and B in angular distance. These would be the appearances, if the stars themselves were at rest; but if any of them be in motion, these effects will be altered according to their motion compared with the motion of our sun. Some of them therefore from their own proper motions might destroy, or more than counteract the effects arising from the motion of the sun, and appear to have motions contrary to what is here described. Like effects will be produced, if our system move in any direction out of the ecliptic. Hence, in whatever direction our system should move, it would be very easy to find what effect of latitude and longitude would have taken place upon any star by means of a celestial globe, by conceiving the sun to move from the center upon any radius directed to the point to which the sun is moving. Dr. HERSCHEL describes the effect thus. Let an arc of 90° be applied to the surface of a globe, and always passing through that point to which the motion of the system is directed. Then whilst one end moves along the equator, the other will describe a curve passing through its pole and returning into itself; and the stars in the northern hemisphere, within this curve, will appear to move to the north; and the rest will go to the south. A similar curve may be described in the southern hemisphere, and like appearances will take place.

727. Now Dr. HERSCHEL first takes the seven stars before mentioned, whose proper motions had been determined by Dr. MASKELYNE, and he finds, that if the point *A* be assumed about the 77° of right ascension, and the sun to move from *S* to *C*, that it will account for all the motions in right ascension. And if, instead of supposing the sun to move in the plane of the equator, it should ascend to a point near to γ *Herculis*, it will account for the observed change of declination of *Sirius* and *Arcturus* In respect to the *quantity* of motion of each, that must depend upon their unknown relative distances; he only speaks here of the *directions* of the motions.

728. He next takes twelve stars from the catalogue of 56, whose proper motions have been determined from a comparison of the observations of ROEMER and MAYER, and adds to them *Regulus* and *Castor;* these have all a proper motion in right ascension and declination, except Regulus, which has none in declination. Of these 27 motions, the above supposed motion of the solar system will satisfy 22. There are also some remarkable circumstances in the *quantities* of these motions. *Arcturus* and *Sirius* being the largest, and therefore probably the nearest, ought to have the greatest apparent motion, and so we find they have. Also Arcturus is better situated to have a motion in right ascension, and it has the greatest motion. Several other agreements of the same kind are found also to take place But there is a very remarkable circumstance in respect to Castor. Castor is a double star, now how extraordinary must appear the concurrence, that two such stars should both have a proper motion so exactly alike, that they have never been found to vary a single second! This seems to point out the common cause, the motion of the solar system.

729. Dr. HERSCHEL next takes 32 more of the same catalogue of 56 stars, and shows that their motions agree very well with his supposed motion of the solar system. But the motions of the other 12 stars cannot be accounted for upon this hypothesis. In these therefore he supposes the effect of the solar motion has been destroyed and counteracted by their own proper motions. The same may be said of 19 stars out of the 32, which only agrees with the solar motion one way, and are, as to sense, at rest the other According to the rules of philosophizing therefore, which direct us to refer all phænomena to as few and simple principles as are sufficient to explain them, Dr. HERSCHEL thinks we ought to admit the motion of the solar system. Perhaps, however, this argument cannot be properly applied here, because, there is no new cause or principle introduced by supposing each star to have a proper motion. Admitting the doctrine of universal gravitation, the fixed stars ought to move as well as the sun. But the sun's motion, as here estimated, cannot be owing to the action of a body upon it which might give it a rotatory motion at the same time, as M. de la LANDE conjectures; because a body acting on the sun to

give it its rotation about its axis, would not, at the same time, give it that progressive motion. See Dr. HERSCHEL's Account in the *Phil. Trans.* 1783.

730. Let us now consider, how far this motion of the solar system agrees with the proper motion of the 35 stars determined by Dr. MASKELYNE. Now upon supposition that the sun moves, as conjectured by Dr. HERSCHEL, that motion will account for the motion of 20 of them, so far as regards their direction; but the motion of the other 15 is contrary to that which ought to arise from this supposition. As some of the stars must have a proper motion of their own, even upon the hypothesis of a solar motion, and which probably arises from their mutual attraction, it is very probable that they have all a proper motion from the same cause, but most of them so very small as not yet to have been discovered. And it might also happen, that such a motion might be the same as that which would arise from the motion of the solar system. Yet it must be confessed, that the circumstance of *Castor*, and the motions both in right ascension and declination of many of the stars being such as arise from this hypothesis, with the apparent motion being greatest of those stars which are probably nearest, form a strong argument in its favour.

On the Zodiacal Light.

731. The *Zodiacal Light* is a pyramid of light which sometimes appears in the morning before sun rise. It has the sun for its basis, and in appearance resembles the *Aurora Borealis.* Its sides are not straight, but a little curved, its figure resembling a lens seen edgeways. It is generally seen here about October and March, that being the time of our shortest twilight; for it cannot be seen in the twilight; and when the twilight lasts a considerable time, it is withdrawn before the twilight ends. It was observed by M. CASSINI, in 1683, a little before the vernal equinox, in the evening, extending along the ecliptic from the sun. He thinks however that it has appeared formerly and afterwards disappeared, from an observation of Mr. J. CHILDREY, in a book published in 1661, entitled, *Britannia Baconica.* He says, that "in the month of February, for several years, about six o'clock in the evening, after twilight, he saw a path of light tending from the twilight towards the *Pleiades*, as it were touching them. This is to be seen whenever the weather is clear, but best when the moon does not shine. I believe this phænomenon has been formerly, and will hereafter appear always at the abovementioned time of the year. But the cause and nature of it I cannot guess at, and therefore leave it to the enquiry of posterity." From this description, there can be no doubt but that this was the zodiacal light. He suspects also, that this is what the ancients called *Trabes*, which word they used for a meteor, or impression in the air like a beam. PLINY, lib.

II. p. 26, says, *Emicant Trabes, quos docos vocant.* Des CARTES also speaks of a phænomenon of the same kind. M. FATIO de DUILLIER observed it immediately after the discovery by M. CASSINI, and suspected that it has always appeared. It was soon after observed by M. KIRCH and EIMMART in Germany. In the year 1707, on April 3, it was observed by Mr. DERHAM in Essex. It appeared in the western part of the heavens, about a quarter of an hour after sun set, in the form of a pyramid, perpendicular to the horizon. The base of this pyramid he judged to be the sun. Its vertex reached 15° or 20° above the horizon. It was throughout of a dusky red colour, and at first appeared pretty vivid and strong, but faintest at the top. It grew fainter by degrees, and vanished about an hour after sun set. This solar atmosphere has also been seen about the sun in a total solar eclipse, a luminous ring appearing about the moon at the time when the eclipse was total.

732. Let *HOR* be the horizon, *S* the sun 18° below at the end of twilight, then will *AIO* represent the appearance and position of the zodiacal light seen at Paris on the last day of February, and *zge* will represent the same the next morning before the beginning of twilight, the sun being at *S'*, as determined by M. de MAIRAN in his treatise *De l'Aurore Boreale.* The distance *SA* was then about 90°, and *IO* about 20°. The axis *AZ, az* coincide with the sun's equator, and therefore makes an angle of about $7\frac{1}{3}$° with the ecliptic. Therefore as the angle which the ecliptic makes with the horizon changes at different times of the day, the angle which the axis of this light makes with the horizon will also be variable. Hence, if we determine the angle which the ecliptic makes with the horizon at any time, it will give us the position. If we set a celestial globe to the hour, it will show us its position, and through what stars it will pass, which will therefore direct us very accurately where to look for it. Hence it will be most visible, cæteris paribus, when the ecliptic makes the greatest angle with the horizon. On October 6, 1684, M. FATIO perceived the point *A* distinctly terminated, the angle of which was $26\frac{1}{2}$°. M. EIMMART observed the same on January 13, 1694. In 1683, when M. CASSINI first observed it, *SA* was 50° or 60°, and *IO* about 8° or 9°. In 1686 and 1687, *SA* extended from 90° to 103°, and *IO* was about 20°. On January 6, 1688, *SA* did not appear to be above 45°, but the horizon was then filled with fogs, and *Venus* shone very bright. The appearance therefore depends upon the state of the atmosphere, and situation of the planets, which may produce light enough partly to obscure it. *IO* has sometimes been extended to 30°. M. PINGRE', in the torrid zone, has observed *SA* to be 120°. The thickness *IO* ought to appear different at different times of the year, because the earth will be in a different situation in respect to its edge. FIG. 178.

733. Let *ABC* be a section of the zodiacal light perpendicular to its axis, *T* the earth, and *TA* a line drawn to the highest point above the horizon; now FIG. 179.

the angle STA having been observed greater than 90°, ST must be less than SA, or the light must extend to a distance from the sun, greater than the earth's distance. Hence, when the earth is about the nodes of this light, or the points where the plane ABC intersects the ecliptic, it will be immersed in this zodiacal light, or, as it is also called, the solar atmosphere. M. de Mairan thinks the *Aurora Borealis* depends upon this.

734. M. Fatio conjectured, that this appearance arises from a collection of corpuscles encompassing the sun in the form of a lens, reflecting the light of the sun. M. Cassini supposed that it might arise from an infinite number of planets revolving about the sun; so that this light might owe its existence to these bodies, as the milky way does to an innumerable number of fixed stars. It is now however generally supposed, that it is matter detached from the sun by its rotation about its axis. The velocity of the equatorial parts of the sun being the greatest, would throw the matter to the greatest distance, and, on account of the diminution of velocity towards its poles, the height to which the matter would there rise would be diminished; and as it would probably spread a little sideways, it would form an atmosphere about the sun something in the form of a lens, whose section perpendicular to its axis would coincide with the sun's equator. And this agrees very well with observation. There is however a difficulty in thus accounting for this phænomenon. It is very well known, that the centrifugal force of a point of the sun's equator is a great many times less than its gravity. It does not appear, therefore, how the sun, from its rotation, can detach any of its gross particles. If they be particles detached from the sun, they must be sent off by some other unknown force; and in that case they might be sent off equally in all directions, which would not agree with the observed figure. The cause is probably owing to the sun's rotation, although not immediately to the centrifugal force arising therefrom.

CHAP. XXVIII.

ON THE LONGITUDE OF PLACES UPON THE SURFACE OF THE EARTH

Art. 735. THE situation of any place upon the earth's surface is determined from its latitude and longitude. The latitude may be found from the meridian altitude of the sun, or a known fixed star, from two altitudes of the sun, and the time between; and by a variety of other methods. These operations are so easy in practice, and opportunities are so continually offering themselves, that the latitude of a place may generally be determined, even under the most unfavourable circumstances, to a degree of accuracy sufficient for all nautical purposes. But the longitude cannot be so readily found. PHILIP III. King of Spain, was the first person who offered a reward for its discovery, and the States of Holland soon after followed his example, they being at that time rivals to Spain, as a maritime power. During the minority of LEWIS XV. of France, the regent power promised a great reward to any person who should discover the longitude at sea. In the time of CHARLES II. about 1675, the Sieur de St. PIERRE, a Frenchman, proposed a method of finding the longitude by the moon. Upon this, a commission was granted to Lord Viscount BROUNKER, president of the Royal Society, Mr. FLAMSTEAD, and several others, to receive his proposals, and give their opinion respecting it. Mr. FLAMSTEAD gave his opinion, that if we had Tables of the places of the fixed stars, and of the moon's motions, we might find the longitude, but not by the method proposed by the Sieur de St. PIERRE. Upon this, Mr FLAMSTEAD was appointed Astronomer Royal, and an Observatory was built at Greenwich for him; and the instructions to him and his successors were, "that they should apply themselves with the utmost care and diligence to rectify the Tables of the motions of the heavens, and the places of the fixed stars, in order to find out the so much desired longitude at sea, for the perfecting of the art of navigation."

736. In the year 1714, the British Parliament offered a reward for the discovery of the longitude, the sum of £.10000, if the method determined the longitude to 1 degree of a great circle, or 60 geographical miles; of £.15000, if it determined it to 40 miles; and of £.20000, if it determined it to 30 miles, with this proviso, that if any such method extend no further than to 80 miles adjoining to the coast, the proposer shall have no more than half such rewards*. The Act also appoints the First Lord of the Admiralty, the Speaker

* See WHISTON's Account of the proceedings to obtain this Act, in the preface to his *Longitude discovered by Jupiter's Planets*

of the House of Commons, the First Commissioner of Trade, the Admirals of the Red, White, and Blue Squadrons, the Master of the Trinity House, the President of the Royal Society, the Royal Astronomer at Greenwich, the two Savilian Professors at Oxford, and the Lucasian and Plumian Professors at Cambridge, with several other persons, as Commissioners for the Longitude at Sea. The Lowndian Professor at Cambridge was afterwards added. After this Act of Parliament, several other Acts passed in the reigns of GEORGE II. and III. for the encouragement of finding the longitude. At last, in the year 1774, an Act passed, repealing all other Acts, and offering separate rewards to any person who shall discover the longitude, either by the lunar method, or by a watch keeping true time, within certain limits, or by any other method. The Act proposes as a reward for a time-keeper, the sum of £.5000, if it determine the longitude to one degree, or 60 geographical miles; the sum of £.7500, if it determine the same to 40 miles; and the sum of £.10000, if it determine the same to 30 miles, after proper trials specified in the Act. If the method be by improved solar and lunar Tables, constructed upon Sir I. NEWTON's theory of gravitation, the author shall be intitled to £.5000, if such Tables shall show the distance of the moon from the sun and stars within fifteen seconds of a degree, answering to about seven minutes of longitude, after making an allowance of half a degree for the errors of observation. And for any other method, the same rewards are offered as those for the time-keeper, provided it gives the longitude true within the same limits, and be practicable at sea. The commissioners have also a power of giving smaller rewards, as they shall judge proper, to any one who shall make any discovery for finding the longitude at sea, though not within the above limits. Provided however, that if such person or persons shall afterwards make any further discovery as to come within the above-mentioned limits, such sum or sums as they may have received, shall be considered as part of such greater reward, and deducted therefrom accordingly.

737. After the decease of Mr. FLAMSTEAD, Dr. HALLEY, who was appointed to succeed him, made a series of observations on the moon's transit over the meridian, for a complete revolution of the moon's apogee, which observations being compared with the places computed from the Tables then extant, he was enabled to correct the Tables of the moon's motion. And as Mr. HADLEY had then invented an instrument by which altitudes could be taken at sea, and also the moon's distance from the sun or a fixed star, Dr. HALLEY strongly recommended the method of finding the longitude from such observations*, having

* The idea of finding the longitude by the moon, was first thought of by JOHN WERNER of Nuremberg, in 1514; it was afterwards recommended by PETER APIAN, in 1524; and by ORONCE FINE, and GEMMA FRISIUS; the latter of which proposed to find the place of the moon at any time, by observing its distance from a fixed star, and then to calculate the time when the moon ought to be at that distance, by which you will have the difference of the meridians of the place of observation and

 the

found from experience the impracticability of all other methods, particularly at sea.

738. To discover the longitude of any place from Greenwich, we must be able to ascertain the time at that place, and compare it with the time at the same instant at Greenwich. The methods which have been proposed to effect this are—By the moon's distance from the sun or a fixed star—By the moon's transit over the meridian compared with that of a fixed star—By the occultation of a fixed star by the moon—By a solar eclipse—By a time-keeper—And by an eclipse of the moon, or of *Jupiter's* satellites.

By the Moon's Distance from the Sun or a Fixed Star.

739. Dr MASKELYNE, our late worthy Astronomer Royal, in his two voyages, one to St. Helena, and the other to Barbadoes, proved the utility of this method of finding the longitude at sea, and which he very fully explained in a Treatise, entitled, *The British Mariner's Guide*. But the great labour and nicety of the calculations seemed to be a material objection to it; particularly the calculation of the moon's latitude and longitude, which are necessary to compute its distance from the sun or a fixed star. To facilitate this, and many other parts of the computation, Dr. MASKELYNE proposed the publication of the *Nautical Almanac*, in which, amongst a great many other things, the moon's true distance from the sun or proper fixed star is put down for every three hours; so that its distance at any other time may be found by only one proportion. Another requisite was, an easy practical rule for finding the true distance of the moon from the sun or a star from their apparent distance and altitudes. Dr. MASKELYNE gave a practical method of doing this, in the above-mentioned Work, and afterwards he improved it. The first *Nautical Almanac* was published in 1767, in which are given two other methods of finding the moon's true distance from the sun or star from their observed distance, one by Mr. LYONS, and the other by Mr. DUNTHORNE. In the Requisite Tables these two methods are improved. Another method is also given by Mr. WITCHELL in that Work. Various other methods have been also given. For the same purpose, a set of Tables were published by order of the Board of Longitude, containing the corrections for refraction and parallax to every degree of the moon's distance from the sun or a fixed star, and for every degree of altitude of each, under the care of Dr. SHEPHERD, the late Plumian Professor of Astro-

the place for which the calculation was made KEPLER also mentioned this as an excellent method of finding the longitude, and after him LONGOMONTANUS But without correct Tables of the moon's motions, and proper instruments to measure its distance from a fixed star, this method could not be put in practice

nomy and Experimental Philosophy, at Cambridge. They were computed by Mr. LYONS, Mr. PARKINSON, and Mr. WILLIAMS. The objection to the direct method of solving this problem was, partly from the length of the operation, and partly from the tediousness of proportioning to find the logarithms to seconds. But since the publication of Mr. TAYLOR's Logarithms, this latter objection is taken away.

740. The steps by which we find the longitude by this method, are these:

From the observed altitudes of the moon and the sun or a fixed star, and their observed distance, compute the moon's true distance from the sun or star.

From the *Nautical Almanac* find the time at Greenwich when the moon was at that distance.

From the altitude of the sun or star, find the time at the place of observation.

The difference of the times thus found, gives the difference of the longitudes.

741. To find the true distance of the moon from the sun or star by observation, let Z be the zenith, S the apparent place of the sun or a star, s the true place, M the apparent place of the moon, m its true place; then in the triangle ZSM, we know SM the apparent distance, SZ, ZM the complements of the apparent altitudes, to find the angle Z; and then in the triangle sZm, we know the angle Z, and sZ, mZ the complements of the true altitudes, to find sm the true distance. But the problem may be otherwise solved thus:

742. By spherical Trigonometry, ver. sin. $\angle Z = r^2 \times \frac{\cos.\ \overline{ZS - ZM} - \cos.\ SM}{\sin.\ ZS \times \sin.\ ZM}$ $= r^2 \times \frac{\cos.\ \overline{Zs - Zm} - \cos.\ sm}{\sin.\ Zs \times \sin.\ Zm.}$; but if $\frac{1}{2} SM + \frac{1}{2}.\ \overline{ZS - ZM} = A$, $\frac{1}{2} SM - \frac{1}{2}.\ \overline{ZS - ZM} = B$, then by plane Trigonometry, $\cos.\ \overline{ZS - ZM} - \cos.\ SM = \frac{2 \sin.\ A \times \sin.\ B}{r}$; hence, $\cos.\ sm = \cos.\ \overline{Zs - Zm} - \frac{2 \sin.\ A \times \sin.\ B}{r} \times \frac{\sin.\ Zm}{\sin.\ ZM} \times \frac{\sin.\ Zs}{\sin.\ ZS}$. Now the *ninth* of the Requisite Tables gives the arithmetic complement of the difference between the logarithmic sines of ZM and Zm, increased by 120; for at all altitudes above 25°, this number is uniformly the difference between the logarithmic sines of Zs and ZS for all celestial objects not affected by parallax. At altitudes less than 25° this uniformity ceases, and the difference between the sines is less than 120 by the numbers in Table XI. for a star. But for the sun, which is sensibly affected by parallax, the differ-

ence between the sines is less than 120 by the numbers in Table X. In these cases therefore the logarithm in Table IX. must be diminished by the numbers contained in Tables X, or XI. Hence we have the following Rule.

To log. 2 add the log. sines of A and B, also the log. from the *ninth* of the Requisite Tables, corrected if necessary by Tables X, or XI. and reject 20 from the index, and find the natural number corresponding, the difference between which and the natural cosine of the difference of the true zenith distances, gives the natural cosine of the true distance required.

Ex. Suppose, on June 29, 1793, the sun's apparent zenith distance ZS was observed to be 70°. 56′. 24″, the moon's apparent zenith distance ZM to be 48°. 53′. 58″, their apparent distance SM to be 103°. 29′. 27″, and the moon's horizontal parallax to be 58′. 35″; to find their true distance sm.

By Requisite Table VIII. the correction Mm for the moon's parallax and refraction is 43′. 3″, and by Table I. and III. the correction Ss for the sun's parallax and refraction is 2′. 36″, hence, Zm = 48°. 10′. 55″, and Zs = 70°. 59′.

$ZS-ZM$	= 22°. 2′. 26″				
$\frac{1}{2}(ZS-ZM)$	= 11. 1. 13				
$\frac{1}{2}SM$	= 51. 44. 43				
		Log. 2.	0,301030		
A	= 62 45. 56	Log sin.	9,948971		
B	= 40. 43. 30	Log. sin.	9,814533		
		Log. from Req. Tab. IX, X.	9,995307		
			10,059841	Nat. Num.	1147741
$Zs-Zm$	= 22. 48. 5	Nat. cos.	- -	- -	921854
sm	= 103. 3. 18	Nat. cos	- -	- -	225887

The radius to the Table of natural sines and cosines to six figures is 1000000, and the index to the log. for the radius in the Tables of log. sines, cosines, &c. is 10; in this case therefore, an index 10 points out 7 places of whole numbers, and consequently an index 9 points out 6 places, &c. When the natural cosine of $Zs-Zm$ is less than the natural number standing above it, the difference gives the natural cosine of an arc above 90°, as in this case; otherwise the arc is below 90°. In this method there is no distinction of cases, and it only requires three logarithms and one natural cosine to be taken corresponding to a given angle, one natural number corresponding to a logarithm, and an

arc corresponding to a natural cosine; Mr. DUNTHORNE's method was by natural cosines, and required only the same number of quantities to be taken; but Dr. MASKELYNE* has deduced from it the following method of computing by logarithms only.

743. By the last Article, $\cos. ms = \cos. \overline{Zs - Zm} - \overline{\cos. \overline{ZS - ZM} - \cos. SM} \times \frac{\sin. Zm}{\sin. ZM} \times \frac{\sin. Zs}{\sin. ZS}$; put $H = Zs - Zm$, $h = ZS - ZM$, $Q = \frac{\sin. Zm}{\sin. ZM} \times \frac{\sin. Zs}{\sin. ZS}$, $A = \overline{\cos. h - \cos. SM} \times Q$; then $\cos. ms = \cos. H - A$. Now $\overline{\cos. \tfrac{1}{2} ms}^2 = \tfrac{1}{2} + \tfrac{1}{2} \cos. ms = \tfrac{1}{2} + \tfrac{1}{2} \cos. H - \tfrac{1}{2} A$; let $\tfrac{1}{2} A$ be the square of the sine of an arc $\tfrac{1}{2} B$, then $\overline{\sin. \tfrac{1}{2} B}^2 = \tfrac{1}{2} . \overline{\cos. h - \cos. SM} \times Q = \sin. \overline{\tfrac{1}{2} SM + \tfrac{1}{2} h} \times \sin. \overline{\tfrac{1}{2} SM - \tfrac{1}{2} h} \times Q$, hence, the arc B is known. But $\tfrac{1}{2} \cos. B = \tfrac{1}{2} - \overline{\sin. \tfrac{1}{2} B}^2 = \tfrac{1}{2} - \tfrac{1}{2} A$; hence, $\overline{\cos. \tfrac{1}{2} ms}^2 = \tfrac{1}{2} \cos. B + \tfrac{1}{2} \cos. H = \cos. \overline{\tfrac{1}{2} B + \tfrac{1}{2} H} \times \cos. \overline{\tfrac{1}{2} B - \tfrac{1}{2} H}$. Hence we have the following Rule.

Add together, log. sine of $\overline{\tfrac{1}{2}\text{ obs. dist. } + \tfrac{1}{2}\text{ diff. of app. alt}^s}$. log. sine of $\overline{\tfrac{1}{2}\text{ obs. dist.} - \tfrac{1}{2}\text{ diff. of app. alt}^s}$. and arith. comp. of Q taken from Requisite Tables IX. and X. or XI. as the case may require, and subtract 10 from the index, divide the sum by 2, and you have the sine of $\tfrac{1}{2} B$.

Add log. cos. $\overline{\tfrac{1}{2} B + \tfrac{1}{2}\text{ diff. of true alt}^s}$. to log. cos. $\overline{\tfrac{1}{2} B - \tfrac{1}{2}\text{ diff. of true alt}^s}$. take half this sum, and you get the log. cosine of half the true distance.

To apply this to the last Example, we have,

* The last method given by Dr. MASKELYNE for clearing the moon's distance from the sun or a fixed star, is in the Supplement to the Requisite Tables, where the reader will find some considerable improvements in the solution of this problem.

$\frac{1}{2}$ Obs. dist.	51°. 44′. 43″$\frac{1}{2}$		
$\frac{1}{2}.ZS-ZM$	11. 1. 13		
Sum -	62. 45. 56 $\frac{1}{2}$	- -	log sine 9,9489713
Diff. -	40. 43. 30 $\frac{1}{2}$	- - -	log. sine 9,8145346
		Log. from Tab. IX. & X.	9,9953070
			2)19,7588129
$\frac{1}{2}B=$ -	49°. 14′. 52″ $\frac{1}{2}$	- -	log. sin. 9,8794064
$\frac{1}{2}.Zs-Zm$	11. 24. 2 $\frac{1}{2}$		
Sum -	60. 38. 55	- - -	cos. 9,6903418
Difference	37 50. 50	- - -	cos. 9,8974344
			2)19,5877762
$\frac{1}{2}.sm=$.	51. 31. 39	- - - -	cos. 9,7938881

Hence, the true distance is 103°. 3′. 18″.

As we have now logarithmic Tables to every second of the quadrant, this is a considerable improvement upon Mr. DUNTHORNE's rule. There is also no distinction of cases in this, which there is in Mr. DUNTHORNE's method. As we deduce, by this rule, half the true distance, it is manifest, that any error in the seconds will be doubled in the true distance; upon that account we were obliged to take in the half seconds, for if we had not, the half distance would have come out 51°. 31′. 38″, and consequently the true distance would have been found 103°. 3′. 16″. This is a circumstance very necessary to be attended to in all the rules that first give half the true distance.

This last Rule may be applied without the Requisite Tables, by considering, that the logarithms taken from Tables IX, X, or XI. in that Work, give the arithm. complem. of $\frac{\sin. Zm}{\sin. ZM} \times \frac{\sin\ Zs}{\sin. ZS}$, which quantity may be taken from the logarithmic Tables, by adding to the log. sines of Zm, Zs, the arith. comp. of the log. sines of ZM and ZS, and subtracting 10 from the index. If we apply this to the last Example, we have,

$Zm = 48°. 10'. 55''$	- - -	sin. 9,8723113	
$ZM = 48. 53. 58$	- - - -	0,1228839	arith. comp. of sine.
$Zs = 70. 59. 0$	- - -	sin. 9,9756265	
$ZS = 70. 56. 24$	- - - -	0,0244868	arith. comp. of sine.
		9,9953085	

This differs a little from the number taken from the Requisite Tables, which gives only six figures. It would indeed lengthen the work a little to take this quantity from the logarithmic Tables, but it would add to the accuracy. Dr. MASKELYNE, in his Preface to the Tables of Logarithms by Mr. TAYLOR, has given a Rule in which those Tables only are requisite; and it is certainly best to use as few auxiliary Tables as possible, as, by that means, you subject the operation to fewer probable errors.

744. Dr. MASKELYNE's Rule for clearing the moon's apparent distance from a star or the sun from the effect of parallax and refraction.

I. To the log. sine of the moon's horizontal parallax add the log. cosine of the moon's apparent altitude, using five decimal places of the logarithms; the sum, abating 10 from the index, is the log. sine of the moon's parallax in altitude, from which subtract the moon's refraction taken with the moon's apparent altitude out of Table I. (Requisite Tables) and you will have the correction of the moon's altitude. Add this to the moon's apparent altitude, and you will have the moon's true altitude. Also, with the star's apparent altitude take the star's refraction out of Table I. which subtract from the star's apparent altitude, and you will have the star's true altitude. But if the moon's distance was observed from the sun, with the sun's apparent altitude take the refraction out of Table I. and its parallax out of Table III. and take the difference, and subtract it from the sun's apparent altitude, and you will have the sun's true altitude. Take the difference of the true altitudes of the moon and star, or moon and sun, and the difference of their apparent altitudes.

II. Take half the sum and half the difference of the apparent distance and difference of the apparent altitudes.

III. To the log. sines of the above half sum and half difference add the log. cosines of the true altitudes, and the arithmetical complements of the log. cosines of the apparent altitudes; and take half the sum.

IV. From this half sum take the log. sine of half the difference of the true altitudes, and look for the remainder among the tangents, and take out the corresponding log. cosine, without taking out the arc, which is unnecessary.

V. Subtract the said log. cosine from the log. sine of half the difference of the true altitudes increased by 10 in the index, the remainder will be the log. sine of half the true distance.

DEMONSTRATION. Put $ZS-ZM=X$, $Zs-Zm=x$, $D=SM$, $d=sm$; now by Art. 742 sin $ZS\times$ sin. ZM : sin. $Zs\times$ sin. Zm :: cos. $X-$ cos. D : cos. $x-$ cos. d, or ver. sin. $d-$ ver. sin. x; but by plane Trigonometry, cos. $X-$ cos. FIG. 180.

$D=2\times\sin.\frac{D+X}{2}\times\sin.\frac{D-X}{2}$, and ver. sin. $d-$ ver. sin. $x=2.\overline{\sin.\frac{1}{2}d}^2-2\times$

$\overline{\sin.\frac{1}{2}x}^2$; hence, sin. $ZS\times$ sin. ZM : sin. $Zs\times$ sin. Zm :: $\sin.\frac{D+X}{2}\times\sin.\frac{D-X}{2}$

: $\overline{\sin.\frac{1}{2}d}^2-\overline{\sin.\frac{1}{2}x}^2$, consequently $\dfrac{\sin.Zs\times\sin.Zm\times\sin.\frac{D+X}{2}\times\sin.\frac{D-X}{2}}{\sin.ZS\times\sin.ZM\times\overline{\sin.\frac{1}{2}x}^2}$

$=\dfrac{\overline{\sin.\frac{1}{2}d}^2}{\overline{\sin.\frac{1}{2}x}^2}-1$, which put $=\overline{\tan.a}^2$, hence, tan. $a=$

$$\frac{\sqrt{\sin.Zs\times\sin.Zm\times\sin.\frac{D+X}{2}\times\sin.\frac{D-X}{2}}}{\sqrt{\sin.ZS\times\sin.ZM}\times\sin.\frac{1}{2}x},$$ but

$\overline{\sin.\frac{1}{2}d}^2=\overline{\sin.\frac{1}{2}x}^2\times\overline{1+\tan.a}^2=\overline{\sin.\frac{1}{2}x}^2\times\overline{\sec.a}^2$; consequently sin. $\frac{1}{2}d=$

sin. $\frac{1}{2}x\times$ sec. $a=\dfrac{\sin.\frac{1}{2}x}{\cos.a}$.

EXAMPLE.

Let the apparent altitude of ☽'*s center be* 5°. 17′, *that of* ☉ 84°. 7′, *and their apparent distance* 90°. 21′. 13″, *and* ☽'*s horizontal parallax* 61′. 48″; *required the true distance of* ☉ *and* ☽.

☽'s horizontal parallax - -	1°. 1′. 48″	Log. sine - -	8,25469
☽'s apparent altitude - -	5. 17. 0	Log. cosine -	9,99815
☽'s parallax in altitude - -	1. 1. 32	Log. sine - -	8,25284
☽'s refraction from Tab. I. -	− 9. 28		
Correct. of ☽'s altitude - -	+ 52. 4		
☽'s apparent altitude -	5. 17. 0		
☽'s true altitude - -	6. 9. 4		
☉'s apparent altitude -	84. 7. 0		
Diff. of refraction and parallax	− 0. 5		
☉'s true altitude - -	84. 6. 55		
☽'s true altitude - -	6. 9. 4		
Diff. of true alt^s. of ☉ and ☽	77. 57. 51		
☉'s apparent altitude -	84. 7. 0		
☽'s apparent altitude -	5. 17. 0		
Diff. of app. alt^s. of ☉ and ☽	78. 50. 0		
Apparent distance - -	90. 21. 13		
Sum - - - -	169. 11. 13		
Difference - - -	11. 31. 13		
Half sum - - -	84. 35. 36	Log. sine - - -	9,9980635
Half difference - - -	5. 45. 36	Log. sine - - -	9,0015681
☽'s apparent altitude -	5. 17. 0	Co-ar. log. cosine -	0,0018490
☽'s true altitude - -	6. 9. 4	Log. cosine -	9,9974924
☉'s apparent altitude -	84. 7. 0	Co-ar. log. cosine -	0,9892626
☉'s true altitude - -	84. 6. 55	Log. cosine -	9,0108395
			2)38,9990751
			19,4995375
½ Diff. of true alt^s. of ☽ and ☉	38. 58. 55	Log. sine - - -	9,7987027
		Log. tang. of arc. -	9,7008348
		Corresp. log. cosine	9,9511707
Half true distance - -	44. 44. 36½	Log. sine - - -	9,8475320
	2		
True distance - -	89. 29. 13		

745. The true distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich. For this purpose, the sun or such fixed stars are chosen, as lie in or very near the moon's way, so that looking upon the moon's motion to be uniform for a small time, the moon may be considered as approaching to, or receding from the sun or star uniformly. To determine therefore the time at Greenwich corresponding to any given true distance of the moon from the sun or star, the true distance is computed in the *Nautical Almanac* for every three hours for the meridian of Greenwich. Hence, considering that distance as varying uniformly, the time corresponding to any other distance may be thus computed. Look into the *Nautical Almanac* and take out two distances, one next greater and the other next less than the true distance deduced from observation, and the difference D of these distances gives the access of the moon to, or recess from the sun or star in three hours; then take the difference d between the moon's distance at the beginning of that interval and the true distance deduced from observation; and then say, $D : d :: 3$ hours : the time the moon is acceding to, or receding from the sun or star by the quantity d; which added to the time at the beginning of the interval, gives the apparent time at Greenwich, corresponding to the given true distance of the moon from the sun or star. To find the fourth term in the above proportion, there is, in the Requisite Tables, a Table of proportional logarithms, where the log. of 3 hours is made $=0$, and therefore the log. of the fourth term is found by subtraction only. The same Table will serve, if one of the terms be three degrees instead of three hours.

Ex. On June 29, 1793, in latitude 52°. 12′. 35″ the sun's altitude in the morning was by observation 19°. 3′. 36″, the moon's altitude was observed to be 41°. 6′ 2″, the sun's declination at that time was 23°. 14′. 4″, and the moon's horizontal parallax 58°. 35′, to find the apparent time at Greenwich.

True dist. of ☾ from ☉ by Art. 742 - -	103°. 3′ 18″		
——— by *Naut. Alm.* on June 29, at 3*h.* -	103. 4. 58		
——————— at 6*h.* -	101. 26. 42		
	0. 1. 40	pr. log.	2,0334
	1. 38. 16	pr. log.	0,2629
Time of approaching 0°. 1′. 40″ - - -	0. 3. 3	pr. log.	1,7705
Beginning of the interval - - - - -	3h. 0. 0		
Apparent time at Greenwich, June 29, - - -	3. 3. 3		

746. Now to find the apparent time at the place of observation, we have the sun's declination 23°. 14′. 4″, its altitude 19°. 3′. 36″, its refraction 2′. 44″, and parallax 8″; hence its true altitude was 19°. 1′, and therefore its true zenith distance was 70°. 59′; also, the complement of declination was 66°. 45′. 56″; hence, by Art. 92.

66°. 45′. 56″	arith. comp. sin.	0,0367325
37. 47. 25	arith. comp. sin.	0,2127004
70. 56. 24		
175. 29. 45		
87. 44. 52	sin.	9,9996644
70. 56. 24		
16. 48. 28	sin.	9,4601408
		2)19,7092381

9,8546190 the cosine of 44°. 18′. 52″, which doubled gives 88°. 37′. 44″ the hour angle from apparent noon, which in time gives 5*h*. 54′. 31″ the time before apparent noon, or 18*h*. 5′. 29″ on June 28. Hence,

Apparent time at place of observ.	June 28,	18*h*. 5′. 29″
———— at Greenwich	June 29,	2. 3. 3
Difference of meridians in time		8. 57. 34

Which converted into degrees gives 134°. 23′. 30″ the longitude of the place of observation west of Greenwich.

If a star be observed, find the time by Art. 106. The sun's declination is first taken from the *Nautical Almanac*, and then corrected by Req. Tab. VI. If a star be observed, take its declination from Requisite Table VII. The longitude being nearly known by account, will be sufficiently exact to enter Table VI. with.

747. In order to apply this method of finding the longitude, three observers are convenient, two to take the altitudes of the moon and sun or a star, and one to take their distance; the latter must be taken with great care, as the deter-

mination of the true distance depends principally upon that, a small error in the altitudes not sensibly affecting it. If a single observer should want to apply this method, he may do it with a very considerable degree of accuracy in the following manner. Let him first take the altitude of the moon and then of the sun or star, his assistant noting the times, then let him take several distances of the moon from the sun or star at one or two minutes distance of time from each other, and note the times, and lastly, let him again take the altitude of the moon and then of the sun or star, noting the times. Then taking the mean of all the distances, and the mean of the times when they were taken, he will have the moon's distance from the sun or star at that mean time. Take the difference of the moon's altitudes at the two observations, and the difference of the times, and then say, as that difference of times, is to the difference between the time of the first observation of its altitude and the mean of the times at which the distances were taken, so is the variation of the moon's altitude between the first and second observations, to the variation of its altitude from the time of the first observation to the above mean time, which added to or subtracted from its first observed altitude, according as the moon ascends or descends, gives its altitude at that mean time. In the same manner he may get the sun's or star's altitude at the same time. Thus he may get the two altitudes and the corresponding distance.

748. In general, the altitudes of the stars at sea are too uncertain for finding the time; they may do in a fine summer's night, or in twilight; and if the sun be used, it may be so near the meridian, or the horizon may be so hazy and ill-defined, that the altitude cannot be determined with sufficient accuracy to deduce the time from it, although it may be sufficiently exact to calculate their true distance. In this case, the observer must be careful to find the error of his watch by some altitude taken near to the time of observation, by which he may correct the time shown by the watch at the time of observation. But as, in this case, the watch shows the time at the meridian under which the altitude of the sun or star was taken in order to correct it, the longitude thus found is that under which the watch was regulated, and not that where the distance of the moon from the sun or star was observed. If the watch cannot be depended upon to keep time tolerably well for a small interval, the error of the watch must be found at two observations, from which you get its rate of going; by this means you may determine the time very accurately. If this be done at sea, the altitude at the second observation must be reduced to the altitude at that time at the place of the first observation; the method of doing which is as follows. Let T be the place on the earth to which the sun was vertical at the first observation, Z the place of a spectator at the first observation, ZV or ZV' the distance run between the observations, then TV or TV' would have been the zenith distance at the first observation, if it had been made at the place FIG. 181.

where the spectator was when he made the second observation; draw VW, $V'W'$ perpendicular to ZT; and then, as the angle T is small, TV is very nearly equal to TW, and TV' to TW', and therefore they may be considered as respectively equal; hence ZW, ZW' may be considered as the difference of the zenith distances, *increasing* the distance in the former case, and *decreasing* it in the latter. To find which, observe the angle VZT or $V'ZT$ between the ship's course and the sun's bearing; and then in the right angled plane* triangles VZW, $V'ZW'$, we know all the angles, and the side ZV or ZV', the observed distance run by the ship, to find ZW or ZW', which must be reduced into degrees at the rate of $69\frac{1}{2}$ miles for a degree. Or the same thing may be done by the *Traverse Table*, which is a Table ready calculated to take out these quantities at once.

749. The observer should be furnished with a good HADLEY'S *Quadrant* to observe the altitudes and distance. Great care must be taken to examine the error of adjustment as near to the time of observation as possible, as it is very liable to alter. Altitudes should not be taken nearer the horizon than $5°$ or $6°$, on account of the uncertainty of refraction at lower altitudes. The principal observer is he who takes the distance; and as soon as he has completed his observation he must give notice to the other two observers, who ought to complete their observations as soon as possible, at least within a minute. Note the time also by the watch when the sun's or star's altitudes were taken, by which, and the estimated longitude at the place of observation, you will have nearly the time at Greenwich, which is necessary in order to get the sun's declination at the time of observation, in order to compute the time. A full account of the adjustments and uses of HADLEY'S *Quadrant*, may be seen in my *Practical Astronomy*.

750. The accuracy of this method of finding the longitude was established by Dr. MASKELYNE from his own experience in two voyages, one to St. Helena, and the other to Barbadoes, by the following irrefragable proofs. 1. On the near agreement of the longitude inferred by his observations, made within a few days or hours of making land, with the known longitude of such land. 2. From the near agreement of the longitude of the ship from observations made on a great many different days near to one another, when connected together by the help of the common reckoning. 3. From the near agreement of the longitude of the ship, deduced from observations of stars on different sides of the moon, taken on the same night. For here, all the most probable kinds of error, whether arising from a faulty division of the limb of the instrument, a refraction of the speculums or dark glasses, a wrong allowance for the error of ad-

* The triangles may be considered as plane, on account of the small distance run by the ship.

ustment, or from a bad habit of estimating the contact of the star with the moon's limb, operating different ways, their effect, if any, must be sensible in the result. But in all the double longitudes thus determined, the difference was so small as to warrant him to say, that by good instruments and careful observers, these errors may be so far reduced as to be of very little consequence, and all the observations which have been made since, agree in confirming it; and show that the longitude thus deduced may be determined to a very great degree of accuracy, and fully sufficient for all nautical purposes.

751. At sea, the moon and sun or star's altitude must be corrected for the dip of the horizon, by subtracting the dip; for the observer being on the deck of the ship must see below his own horizon, and the altitudes are taken above his visible horizon. The following Table gives the dip corresponding to the observer's height.

Height.	Dip	Height.	Dip.	Height.	Dip.
1	0'. 57"	13	3'. 26"	26	4'. 52"
2	1. 21	14	3. 34	28	5. 3
3	1. 39	15	3. 42	30	5. 14
4	1. 55	16	3. 49	35	5. 39
5	2. 8	17	3. 56	40	6. 2
6	2. 20	18	4. 3	45	6. 24
7	2. 31	19	4. 10	50	6. 44
8	2. 42	20	4. 16	60	7. 23
9	2. 52	21	4. 22	70	7. 59
10	3. 1	22	4. 28	80	8. 32
11	3. 10	23	4. 34	90	9 3
12	3. 18	24	4. 40	100	9. 33

752. The moon's true distance from the sun or a fixed star as put down in the *Nautical Almanac*, is thus calculated. Let Z be the pole of the ecliptic, s the true place of the *star*, m the true place of the *moon*, then Zs, Zm are the complements of latitudes, and the angle Z the difference of their longitudes, draw st perpendicular to Zm; and by spher. Trig. log. tan. Zt = log. cos. Z + log. tan. $Zs - 10$, and log. cos. sm = log. cos. Zs + log. cos. tm + arith. comp. log. cos. $Zt - 10$. If s be the *Sun*, $Zs = 90°$; hence, rad. × cos. sm = cos. Z × sin. Zm, or log. cos. Z + log. sin. $Zm - 10$, = log. cos. sm. If Z be the pole of the equator, then Zs, Zm will be the complements of declination, and the angle Z the difference of right ascensions. FIG. 180.

Ex. I. To find the moon's distance from the sun, on August 24, 1796, at apparent noon at Greenwich.

The sun's longitude at the given time is $5^s.\ 1°.\ 55'.\ 58''$, the moon's longitude is $1^s.\ 13°.\ 50'.\ 53''$, the difference of which is $108°.\ 5'.\ 5''$ the angle sZm; also, the moon's latitude is $4°.\ 13'.\ 51''$; hence, $Zm = 85°.\ 46'.\ 9''$.

Log. cos. $Z = 108°.$	$5'.\ 5''$	- -	9,4919538
Log. sin. $Zm = 85.$	46. 9	- -	9,9988149
Log. cos. $sm = 108.$	2. 1	- -	9,4907687

Ex. II. Let the right ascension of a star be $2^s.\ 7°.\ 2'.\ 25''$, and north declination $5°.\ 28'.\ 40''$, also the right ascension of the moon $0^s.\ 11°.\ 54'.\ 4''$, and south declination $3°.\ 22'.\ 32''$; to find the moon's distance from the star.

In this case, Z represents the pole of the equator, and the difference of right ascensions is $55°.\ 8'.\ 21'' =$ the angle Z, also $Zs = 84°.\ 31'.\ 20'$, and $Zm = 93°.\ 22'.\ 32''$; hence,

Log. cos. - - -	$55°.\ 8'.\ 21$	- - -	9,7570809
Log. tan. - - -	84. 31. 20	- - -	11,0181922
Log. tan. Zt - -	= 80. 28. 33	- - -	10,7752731
Zm - -	= 93. 22. 32		
Log. cos. tm - -	= 12. 53. 59	- - -	9,9888987
Log. cos. Zs - -	= 84. 31. 20	- - -	8,9798200
Arith. comp. log. cos. Zt	= 80. 28. 33	- - -	0,7812975
Log. cos. sm - -	= 55. 46. 50	- - -	9,7500162

Thus the moon's true distance from the sun or a fixed star may be calculated for every three hours, as given in the *Nautical Almanac*.

To find the Difference of the Longitudes of two Places, by the observed Transits of the Moon and a fixed Star over the Meridian at each Place.

753. This method of finding the longitude was proposed by Dr. MASKELYNE in the *Nautical Almanac* for 1769. It is extremely easy in practice, and capable of great accuracy. The Rule is thus investigated. Let P be the pole of the earth RQ, PG a meridian of Greenwich passing through the moon at M, PD the meridian of any other place, and when it comes into the situation Pd let it pass through the moon at m. At each transit, observe the differences MPS, mPS, between the right ascensions of the moon and a fixed star S, the difference of which is the angle MPm, or the increase of the moon's right ascension in the interval of the transits. From the *Nautical Almanac*, find the increase (A) of the moon's right ascension in 12 hours apparent time, and reduce it into mean time thus; let $a =$ the variation of the equation of time in 12 hours, then 12 hours apparent time is $12h. \pm a$ of mean time, where the sign $+$ is used if the equation be *increasing* and *additive*, or *decreasing* and *subtractive*; and the sign $-$, when *increasing* and *subtractive*, or *decreasing* and *additive*. Now $A : Mpm :: 12h. \pm a : x$ the angle (expressed in mean time) described by a meridian of the earth in the time the moon describes Mpm, hence, $x \times \frac{366}{365} =$ the angle DPm of longitude described by a meridian in that time, because in 12 hours mean time the earth revolves through $180° \times \frac{366}{365}$ of longitude, very nearly. Consequently the difference DPG of the meridians $= x \times \frac{366}{365} - Mpm$. If the places do not differ much in longitude, $x \times \frac{366}{365} = x + \frac{x}{6 \times 60}$ sufficiently near; in this case also, the apparent may be used for the mean time. FIG. 182.

Ex. On June 13, 1791, the following observations of the passage of the moon and *α Serpentis* were made at Greenwich and Dublin Observatories.

AT DUBLIN.

Right ascension of ☽'s first limb - -	$15^h.\ 6'.\ 12'',49$
——— α Serpentis - - -	15. 33. 36, 91
	27. 24, 42
Daily rate of clock, $-16'',88$ - - - -	+ 0, 32
	27. 24, 74

AT GREENWICH.

A. R. ☽'s first limb	$15^h.\ 5'.\ 3'',52$	at $9^h.\ 36'$ apparent time.
A. R. of α Serpentis	$15.\ 33.\ 34,70$	
	$28.\ 31,18$	
	$27.\ 24,74$	
Difference - - -	$1.\ 6,44 = 16'.\ 36'',6$	in space.

As the places do not differ much in longitude, it is unnecessary to reduce apparent to mean time.

This difference $16'.\ 36'',6$ is the increase of the moon's right ascension in the interval of its passages over the meridians at Greenwich and Dublin Observatories. By the *Nautical Almanac*, we find the following differences of the right ascensions of the same limb of the moon, and the star, about the same time;

			Difference.
June 12, midnight -	$213^\circ.\ 15'$		
		- - -	$7^\circ.\ 23'$
13, noon - -	$220.\ 38$		
		- - -	$7.\ 33$
13, midnight -	$228.\ 11$		
		- - -	$7.\ 42$
14, noon - -	$235.\ 53$		
		- - -	$7.\ 50$
14, midnight -	$243.\ 43$		

If the places differ much in longitude, the motion in right ascension should be calculated to seconds.

The second differences are always sufficiently uniform, that we may take $7^\circ.\ 37',5$, the middle of the first differences, for the rate of increase for 12 hours at the middle time. Hence, $7^\circ.\ 37',5 : 16'.\ 36'',6 :: 12h. : x = 1568'',418$, and $x + \frac{x}{6 \times 60} = 26'.\ 12'',77$, consequently the difference of the longitudes is $26'.\ 12'',77 - 1'.\ 6'',44 = 25'.\ 6'',33$. Dr. Brinkley was so good as to favour me with this; and he further observes, that when the two places differ much in longitude, an allowance ought to be made for the change of the moon's semidiameter in the interval of the passages arising from its change of distance, and also for the change of semidiameter in right ascension from its change of declination. He very strongly recommends this method, as being extremely easy in practice, and capable of great accuracy, far beyond that from the eclipses of *Jupiter's* satellites.

To find the Difference of the Longitudes of two Places, from the Occultation of a fixed Star by the Moon.

754. The principal part of the calculation is made by the following Rule, given by Dr MASKELYNE, for finding the true longitude and latitude of the point of occultation in the moon's limb.

I. Find the angle between the parallels to the ecliptic and equator passing through the star, by saying, cos. star's latitude : cos. of its right ascension :: sine of the obliquity of the ecliptic : sine of the angle between the parallels. This may also be found by Table XXVII. and XXVIII. at the end of Volume the second.

II. From 9 signs to 3 signs of the right ascension of the star, in a place of *north* latitude, the parallel to the ecliptic *ascends above* the parallel to the equator, but from 3 signs to 9 signs, it *descends below* the same. The contrary for a place of south latitude.

III. If the parallel to the ecliptic *ascend above* the parallel to the equator, subtract the angle just found from 90°, but if it *descend below*, add it to 90°, and you will have the angle between the meridian passing through the star and the parallel to the ecliptic.

IV. Subtract the right ascension of the star from that of the meridian of the place, or the right ascension of the meridian from that of the star, borrowing 360° if necessary, so that the remainder may be under 180°, and you will have the horary angle of the star, which will be east or west, according as the right ascension of the meridian was subtracted from that of the star, or the right ascension of the star subtracted from that of the meridian.

V. With this angle, and the star's declination and latitude of the place (corrected for the spheroidical figure of the earth,) compute the star's altitude, and the angle of position at the star.

VI. If the star be *east* of the meridian, *add* the angle of position to the angle between the meridian and parallel to the ecliptic, but if the star be to the *west* of the meridian, *subtract* the former from the latter, borrowing 360° if necessary, and you have the angle between the vertical circle and the parallel to the ecliptic.

VII. To the sine of the moon's equatorial parallax (corrected for the spheroidical figure of the earth) add the cosine of the star's altitude and the sine of the angle between the vertical circle and the parallel to the ecliptic, and the sum, rejecting 20 from the index, is the sine of the principal part of the parallax in *latitude*. This must be *added* to the star's latitude, if of the *same*

denomination with the latitude of the place, but *subtracted*, if of a *contrary* denomination, unless the angle between the vertical circle and the parallel to the ecliptic is greater than 180°, when it must be applied in a contrary manner, to obtain the true latitude nearly of the point of the moon's limb at which the occultation happens. This is to be corrected by a small quantity found hereafter.

VIII. To the sine of the moon's equatorial parallax (corrected as before) add the cosine of the star's altitude, the cosine of the angle between the vertical circle and the parallel to the ecliptic and the arithmetical complement of the cosine of the latitude of the true point of occultation, found nearly in the last Article, and the sum, rejecting 20 from the index, is the sine of the parallax in *longitude*.

IX. To the constant logarithm 4,7124 add twice the sine of the parallax in longitude and the sine of twice the true latitude of the point of occultation found nearly, and the sum, rejecting 30 from the index, is the logarithm of a number of seconds, which subtracted from the true latitude of the point of occultation of the moon's limb found nearly by Art. VII. gives the true latitude of that point correctly.

X. If the angle between the vertical circle and the parallel to the ecliptic be *more* than 270° or *less* than 90°, *add* the parallax of longitude to the longitude of the star; but if that angle be *more* than 90° and *less* than 270°, *subtract* the parallax in longitude from the longitude of the star, and you will have the true longitude of the point of the moon's limb where the star immerges or emerges.

DEMONSTRATION. Let ♈C be the ecliptic, ♈K the equator, S the star, Z the zenith, P the pole of the equator, p the pole of the ecliptic, and draw the great circles PSA, PZB, pSv, ZSH, and Sm, Sn perpendicular to pS and PS respectively. Then by Trig. Art. 212.

$$\begin{array}{l}
\text{Cos. } ♈zA : \text{sin. } ♈ :: \text{cos. } ♈A : \text{rad.} \\
\underline{\text{Cos. } vS : \text{rad.} :: \text{cos. } ♈zA : \text{sin. } zSv} \\
\therefore \text{Cos. } vS : \text{sin. } ♈ :: \text{cos. } ♈A : \text{sin. } zSv, \text{ or } nSm,
\end{array}$$

which proves the first proportion of the Rule; and by taking the star in all possible situations, the second and third articles are found to be true.

As $PSn = 90°$, $90° \pm mSn = PSm$, according to the cases in the Rule.

Also ♈$B \sim$ ♈$A = AB$ the measure of the angle ZPS.

With ZPS, PS and PZ, compute ZS, and ZSP which is here called the angle of position, but it is not the angle generally understood under this appellation, as defined in Art. 53.

Next, $PSm \pm ZSP = ZSm$, which will be found to agree with the Rule in the different cases, as there stated.

Let p be the north or south pole of the ecliptic, according as the place is in north or south latitude (no matter whether p be elevated above the horizon or not), S the star touching the moon's limb, Sx the parallax in altitude of that part of the moon's limb, then Spx is the parallax in *longitude*, and $pS - px$ the parallax in *latitude*. Draw xr a portion of a parallel to the ecliptic, and xs a portion of a great circle perpendicular to Sp; then the true latitude s (a point of the moon's limb) $= zr = zs - rs = zS \pm Ss - rs = zS \pm Sx \times \cos. xSs - \frac{xs^2}{2 \tan. xp}$* $=$ (because $ZSm = 90° \pm xSs$) $zS \pm Sx \times \sin. ZSm - \frac{xs^2}{2 \tan. xp}$, if therefore from this we subtract zS, the apparent latitude of the point S of the moon's limb, we have the parallax in latitude $= \pm Sx \times \sin. ZSm - \frac{xs^2}{2 \tan. xp}$, but if $h =$ the horizontal parallax of the moon, then (154) $Sx = h \times \sin.$ star's app. zenith dist., therefore the first and principal part $= \pm h \times \sin.$ star's app. zenith dist. $\times \sin. ZSm$, where the sign is $+$ or $-$, according as the latitude of the place is north or south, except ZSm is more than 180°, when it becomes of a contrary denomination, that is, $-$ in north latitude, and $+$ in south latitude. Having gotten the true latitude nearly, before we find the second part of the parallax in latitude, we will find the parallax in longitude.

$$\begin{array}{l}
\text{Rad.} : \sin. ZS :: \sin. h : \sin. Sx \ (154) \\
\text{Rad.} : \sin. xSr\ (\cos. ZSm) : \sin. Sx : \sin. xr \\
\text{Cos. } rv : \text{rad.} :: \sin. xr : \sin. xpS \\
\hline
\therefore \text{Rad.} \times \cos. rv : \sin. ZS \times \cos. ZSm :: \sin. h : \sin. xpS,
\end{array}$$

hence, $\sin. xpS = \frac{\sin. ZS \times \cos. ZSm \times \sin. h}{\text{rad.} \times \cos. rv}$ the sine of the parallax in *longitude*, agreeing with the Rule. This parallax will be $+$, that is, easterly, or $-$, that is, westerly, according as ZSm is acute or obtuse, reckoning those angles acute which are from 270° to 90°, and those obtuse which are from 90° to 270°.

Now to find the second part of the parallax in latitude, we may further reduce the expression thus: $xs = \frac{\sin. xpS \times \sin. xp}{\text{rad.}}$, and $\tan. xp = \frac{\text{rad.} \times \sin. xp}{\cos. xp}$;

* See the Note to Ex. Art. 164

hence, this second part $= -\frac{\overline{\sin.\ xpS}^2}{2 \times \text{rad.}^3} \times \sin.\ xp \times \cos.\ xp = -\frac{\overline{\sin.\ xpS}^2}{2 \times \text{rad.}^3} \times \sin.\ vr \times \cos.\ vr = -\frac{\overline{\sin.\ xpS}^2}{4 \times \text{rad.}^3} \times \sin.\ 2vr$; and to reduce this to seconds of a degree, we have 206264″,8 the seconds in an arc equal to radius; hence, rad. $= 1 : -\frac{\overline{\sin.\ xpS}^2}{4 \times \text{rad.}^3} \times \sin.\ 2vr :: 206265'' : -51566'' \times \frac{\overline{\sin.\ xpS}^2 \times \sin.\ 2vr}{\text{rad.}^3}$; but the log. of 51566 is 4,7124, and xpS is the parallax in longitude; this therefore proves the truth of the Rule for the second part of the parallax in latitude. This is always of a contrary denomination to the moon's latitude.

755. This calculation being made, we very readily find the difference of the longitudes of two places by the following Rule.

I. From the meridian observations of the moon, compute its true latitude and longitude, and compare it with the latitude and longitude computed from the Tables (which may be taken from the *Nautical Almanac*), and we get the error of the Tables in latitude and longitude.

II. Compute, by the above Rule, the true latitude and longitude of the point of the moon's limb where the occultation takes place.

III. Take the difference CP of the latitudes of the point of occultation and the moon's center, and knowing Cs the moon's semidiameter, we have log. sP $=$ log. $\sqrt{Cs^2 - CP^2} = \frac{1}{2} \times \log.\ \overline{Cs + CP} - \log.\ \overline{Cs - CP}$.

IV. Find the value of sP in longitude, by dividing it (108) by the cosine of the star's latitude, and we get the true difference of longitudes of the moon and star, which difference applied to the true longitude of the star, gives the true longitude of the moon's center at the time of the occultation.

V. Find the same for any other place, and take the difference, and then say, as the moon's horary motion : that difference :: one hour : the time between the immersions at the two places.

VI. Apply that time to the time of occultation at one place, and the difference between that result and the time of occultation at the other place gives the difference of longitudes.

EXAMPLE.

On March 27, 1792, the immersion of *Aldebaran* at the moon's dark limb at Greenwich was at 8*h*. 37′. 36″,8 apparent time, and at the Observatory of Trinity College, Dublin, it was at 8*h*. 4′. 51″,5 apparent time; to find the difference of their longitudes.

As the immersions only were observed, it is necessary to have the exact latitude of the moon at the immersions, for finding which we have the following observations.

By observations at Greenwich on the day of occultation when the moon passed the meridian, the right ascension of its first limb was found to be 63°. 35′. 55″,8, and the zenith distance of its lower limb, corrected for refraction and the error of the line of collimation, was 35°. 41′. 0″,7, and the apparent time of its passage was 3*h*. 46′. 10″. Hence, its latitude by observation was 4°. 38′. 40″ S. and its longitude was 64°. 55′. 57″,2. By the *Nautical Almanac*, its latitude computed was 4°. 38′ 40″, and its longitude was 64°. 56′. 47″,6, hence, on that day there was no error in the Tables of the moon's latitude, but an error of + 50″,4 in its longitude.

By the *Nautical Almanac*, the moon's latitude at the immersion at Greenwich was 4° 44′. 52″,7 S., and at the Observatory at Dublin it was 4°. 44′ 43″,7. The apparent longitude of Aldebaran was 66°. 52′. 59″,2, and its apparent latitude 5°. 29′. 5″,6; its right ascension was 66°, and its declination was 16′. 4″,4; the obliquity of the ecliptic was 23°. 27′. 48″, and the moon's horary motion in longitude was 30′. 9″,2 by the *Nautical Almanac*.

Log. cos.	5°. 29′. 5″,6	- - arith. comp.	0,00200
cos.	66. 0. 0	- - - - -	9,60931
sin.	23. 27. 48	- - - - -	9,60006
sin.	9. 21. 48	- ∠ bet. ‖ ecl. and ‖ equat.	9,21137
	90. 0 0		
	80. 38. 12	the angle between the meridian passing through	

the star and a parallel to the ecliptic.

Calculation for the Observatory at Greenwich.

Right ascen. of the mid-heaven at immersion	9h. 6′. 34″,5
——— star - - - -	4. 24. 0
Horary angle 70°. 38′,6 - - - -	4. 42. 34, 5

With this, and the declination 16°. 4′,4, and latitude reduced 51°. 14′,1, we find the angle of position=40°. 29′,7, and the star's altitude=24°. 32′,3; hence,

2

80°. 38′,2 − 40°. 29′,7 = 40°. 8′,5 the angle made by a vertical circle and a parallel to the ecliptic.

The moon's equatorial parallax	= 54′. 45″,9	(*Naut. Alm.*)
Reduction (169) - - - -	− 8, 7	
Horizontal parallax reduced -	54. 37, 2	

Sin.	54′. 37″,2	hor. par. red. - - -	8,20106
Cos.	24°. 32′,3	star's altitude - - -	9,95889
Sin.	40. 8, 5	∠ bet. ver. circle and ∥ ecl. -	9,80934
Sin.	32′. 1″,9	parallax in *latitude* nearly -	7,96929
	5°. 29. 5, 6	S. lat.	
	4. 57. 3, 7	true lat. of the point of occultation nearly.	

Sin.	54′. 37″,2	horizontal parallax - -	8,20106
Cos.	24°. 32′,3	star's altitude - - -	9,95889
Cos.	40. 8, 5	∠ bet. ver. circle and ∥ ecl. -	9,88335
Cos.	4. 57	tr. lat. of point of occ. nearly — ar. com.	0,00162
Sin.	38′. 7″,5	parallax in *longitude* - -	8,04492

Constant logarithm - - - - -	4,7124
2 × sin. par. in long. - - - - -	16,0898
Sin. twice true latitude - - - -	9,2353
Logarithm of 1″,1 - - - - -	0,0375
4°. 57′. 3, 7	
4. 57. 2, 6 true latitude of the point of occultation.	

Apparent longitude of *Aldebaran* -	$2^s.\ 6^\circ.\ 52'.\ 59'',2$
Parallax - - - - - -	$38.\ \ 7,5$
Longitude of the point of occultation .	$2.\ 7.\ 31.\ \ 6,7$

Lat. of point s of moon – lat. of center $C = 4^\circ.\ 57'.\ 2'',6 - 4^\circ.\ 44'.\ 52'',7 = 12'.\ 9'',9 = CP$; also $Cs = 14'.\ 55'',4$;

Hence $Cs = 14'.\ 55'',4$
$CP = 12.\ \ 9,9$

Sum $= 27.\ \ 5,3 = 1625'',3$	- -	3,2109335
Diff. $= 2.\ 45,5 = 165,3$	- -	2,2187980
		2)5,4297315
		2,7148657
Log. cos. lat. - - -	arith. comp.	0,0016228
Log. $520'',6 = 8'.\ 40'',6 = sP$ - -		2,7164885

$2^s.\ 7^\circ.\ 31.\ \ 6,7$ longitude of s

$2.\ \ 7.\ 22.\ 26,1$ long. of ☾'s center at immer. at Greenwich.

Calculation for the Observatory of Trinity College, Dublin.

Right ascen. of the mid-heaven at immersion	$8^h.\ 33'.\ 48'',5$
———————— star - - -	$4.\ 24.\ \ 0$
Horary angle $62^\circ.\ 27'$ - - - -	$4.\ \ 9.\ 48,5$

With this, and the declination $16^\circ.\ 4'.\ 26''$, and latitude reduced $53^\circ.\ 9'$, we find the angle of position $= 37^\circ.\ 32'$, and the star's altitude $= 29^\circ.\ 12',9$, hence, $80^\circ.\ 38',2 - 37^\circ.\ 32' = 43^\circ,\ 6',2$ the angle made by a vertical circle and a parallel to the ecliptic.

The moon's equatorial parallax - - -	54'. 45",9
Reduction - - - - - - -	−9, 1
Horizontal parallax - - - - -	54. 36, 8

Sin.	54'. 36",8	hor. par. red. - - -	8,20100
Cos.	29°. 12',9	star's altitude - - -	9,94091
Sin.	43. 6, 2	∠ bet. ver. circle and ‖ eclip.	9,83462
Sin.	32'. 34",2	parallax in *latitude* nearly -	7,97653
	5°. 29. 5, 6	S. lat.	
	4. 56. 31, 4	true lat. of the point of occultation nearly.	

Sin.	54'. 36",8	hor. par. red. - - -	8,20100
Cos.	29°. 12',9	star's altitude - - -	9,94091
Cos.	43. 6, 2	∠ bet. vert. circle and ‖ ecl.	9,86340
Cos.	4. 57	tr. lat. of point of occ. nearly - ar. com.	0,00162
Sin.	34'. 55",9	parallax in *longitude* - -	8,00693

Constant logarithm - - - - -	4,7124
2 × sin. par. in long. - - - - -	16,0154
Sin. twice true latitude - - - -	9,2346
Logarithm of 0",9 - - - - - -	−1,9624
4°. 56'. 31, 4	
4. 56. 30, 5 true latitude of the point of occultation.	

Apparent longitude of *Aldebaran* -	2ˢ. 6°. 52'. 59",2
Parallax - - - - - - -	34. 55, 9
Longitude of the point of occultation -	2. 7. 27. 55, 1

Lat. of the point s of moon − lat. of center C = 4°. 56'. 30",5 − 4°. 44'. 43",7 = 11'. 46",8 = CP; also Cs = 14'. 55",4;

Hence, $Cs = 14'. 55'',4$
$CP = 11. 46, 8$

Sum 26. 42, 2 = 1602'',2	- - - -	3,2047167
Diff. 3 8, 6 = 188, 6	- - - -	2,2755417
		2)5,4802584
		2,7401292
Log. cos. lat. - - -	arith. comp.	0,0016228
Log. 551'',8 = 9'. 11'',8 = sP	- - -	2,7417520

$2^s. 7°. 27. 55, 1$	longitude of s
2. 7. 18. 43,3	long. of ☽'s cent. at immer. at Dublin Obser.
2. 7. 22. 26,1	——— Greenwich.
Diff. 0. 0. 3. 42,8	

Hence, 30'. 9'',2 (☾'s hor. mot. in long.) . 3'. 42'',8 . 1 hour · 7'. 23'',3 the time between the immersions at Greenwich and the Observatory of Trinity College, Dublin.

Immersion at Obser. of Trin. Coll. Dublin -	8^h. 4'. 51'',5
	7. 23, 3
Time at the Obser. of Trin. Coll. Dublin, when the occult. happened at Greenwich	8. 12. 14, 8
Time of occultation at Greenwich - -	8. 37. 36, 8
Longitude of Obser. Trin. Coll. Dublin -	25. 22, 0 W.

For this computation I am indebted to Dr. BRINKLEY, who observes, that the accuracy in the result will not be affected by an error in the longitude of the star, and that a small error in its latitude will not sensibly affect the result, when the places do not differ much in longitude and latitude.

To find the Difference of Longitudes of two Places from a Solar Eclipse.

I. Find (164) the moon's parallax in latitude and longitude for the given time and place of observation.

II. Compute the moon's true latitude, and to it apply the error* of the Tables, and you get the true latitude correctly; to which apply the parallax in latitude, and you get the apparent latitude ME, M being the center of the moon, S of the sun, SE the ecliptic, and ME perpendicular to it.

III. Hence, for the beginning or end of the eclipse, knowing SM the sum of the semidiameters, or at any other time knowing the distance SM of their centers from observation, we get $SE = \sqrt{SM^2 - ME^2} = \sqrt{\overline{SM+ME} \times \overline{SM-ME}}$ the apparent difference of longitudes, to which apply the parallax in longitude and you get the true difference of longitudes of the centers.

IV. Then say, as the horary motion of the moon from the sun : that difference :: 1 hour : the time between the observation and the time of the true conjunction, which applied to the time of observation gives the time of the true conjunction.

V. Find the same for any other place, and the difference of the times gives the difference of the longitudes.

EXAMPLE.

On September 4, 1793, the beginning of a solar eclipse at Greenwich was observed to be at $21h.\ 39'.\ 21''$ apparent time; at the Observatory of Trinity College, Dublin, the beginning was at $8h.\ 4'.\ 50'',2$ sidereal time, or $21h.\ 6'.\ 47''$ apparent time, the middle at $9h.\ 36'.\ 12''$ sidereal time, or $22h.\ 37'.\ 54'',6$ apparent, and the breadth of the lucid part at the middle, measured with a divided object glass micrometer, was $6'.\ 47''$; to find the difference of the longitudes.

Calculation for the Observatory at Dublin.

The latitude is $53°.\ 23',3$, and the reduction (173) $14',3$, hence the latitude reduced is $53°.\ 9'$. The obliquity of the ecliptic was $23°.\ 27',7$, the moon's

* The error of latitude of the Tables is found by comparing the latitude deduced from observation with the computed latitude; in this Example, it is found from the observation of the middle of the eclipse. It cannot here be found as it was at the occultation of a fixed star by the moon, from a meridian observation of the moon, as such an observation cannot be made at, or near to the time of the eclipse, the moon being invisible.

horary motion in longitude was 29′. 37″,9, in latitude 2′. 42″,8, and its horary motion from the sun 27′. 12″,1.

First, to find the error of the Tables in latitude from the observation of the middle. At the middle, the moon's latitude by the Tables was 37′. 43″, and longitude 162°. 51′, and the right ascension of medium cœli was 9*h*. 36′. 12″ =144°. 3′, hence the following calculation* to find the parallax in latitude and longitude.

Cos. - -	144° 3′ right ascension med. cœli -	9,90823
Cos. - -	53. 9 latitude reduced - - - - -	9,77795
Cos. arc I. =	119. 2,8 - - -	9,68618
Cot. 53°. 9′ lat. red.	- - - -	9,87474
Sin. 144. 3 right ascen. med. cœli	- -	9,76870
Cot. arc II. =	66°. 15′ - - - -	9,64344
Obliq. ecl. =	23. 27,7	
Sin. arc III. =	42. 47,3 - - - -	9,83205
Sin. arc I. =	119. 2,8 - - - -	9,94161
Cos. *alt.* non. =	53°. 34′,2 - - - -	9,77366
Cos arc III. =	42°. 47′,3 - - - -	9,86561
Tan. arc I. =	119. 2,8 - - - -	10,25541
Tan. *long.* non. -	= 127°. 7′ - -	10,12102
Moon's longitude -	= 162. 51	
Moon's dist. from nonag.	35. 44	

* Dr. Brinkley made his Calculations by this Rule. Let L=the latitude reduced, O=obl. ecl. A=AR of medium cœli, then cos. A+cos. L=cos. arc I. which is greater than a quadrant in the second and third quadrants of med. cœli. Cot. L+sin. A=arc II. which is always less than a quadrant. Arc II ± O=arc III. where — takes place when Aries is West of the meridian, and +, when East. Cos. of *alt. nonag.*=sin. arc I + sin. arc III. Tang. *long. nonag.*=cos. arc III + tan. arc I. When arc III. is *less* than a quadrant, the *long. nonag.* is of the same affection as A, when *greater*, of the same affection as arc I.

The moon's equatorial parallax -	54′. 12	
Reduction - - - - -	−8,9	
Moon's horizontal parallax - -	54. 3,1=3243″,1	
Sun's horizontal parallax - - - - -		8, 6
Hor. par. ☽ from ☉ - - - - -		3234, 5

Log. 3234,5 - - - - -		3,50981
Sin. 53°. 34′,2 alt. nonag. - - -		9,90557
Cos. 37′. 43″ ☾'s lat. by Tab. - -	arith. comp.	0,00003
		3,41541
Sin. 35°. 44′ ☾'s dist. from nonag. - -		9,76642
Log. 1520″=25′. 20″ par. in long. nearly - - -		3,18183
35°. 44		
Sin. - 36. 9 - - - - - -		9,77078
		3,41541
Log. 1535″,3=25′. 35″,3 paral. in *Longitude* -		3,18619

Log. 3234,5 - - - - - -		3,50981
Cos. 53°. 34′,2 alt. nonag. - - -		9,77366
Cos. app. lat. ☾ - -	arith. comp.	0,00000
Log. 1920″,8=32′. 0″,8 first part par. in lat. -		3,28347
37. 43 ☾'s lat. by Tab.		
5. 42,2 apparent latitude very nearly.		

Log. 3234,5	- - - - - -	3,510
Sin. 53°. 34',2 alt. nonag.	- - - -	9,906
Sin. 5' 42",2 app. lat. of ☾	- - - -	7,220
$\text{Cos. } 35°.\ 44' + \frac{25'.\ 35''}{2}$	- - - - - - - -	9,908
Log. - 3",5 second part par. lat.	- -	0,544
32'. 0, 8 first part		
31. 57, 3 parallax in *Latitude*.		

Moon's horizontal semidiameter - - -	14'. 46"
Inflex. light - - - - - - -	−3
	14. 43
Augmen. for ☽'s alt. 35° - - - - -	+9
Moon's semidiameter - - - - -	14. 52
Lucid part at the middle - - - -	6'. 47"
	—6*
Correction of lucid part - - - -	6. 41
The moon's semidiameter - - -	14 52
Sum - - - - - - -	21. 33
The sun's semidiameter - - -	15. 53
	5. 40 the dis-

* Dr. BRINKLEY observes, that imperfect or bad achromatic telescopes are found to give the sun's diameter greater than it really is MAYER's telescope, which was not achromatic, gave, according to the late observations of Dr. MASKELYNE, the sun's diameter too great as set down in the *Nautical Almanac* by 6" An achromatic with a divided object glass micrometer may be considered as an indifferent telescope, perhaps little better than a telescope which is not achromatic. I have therefore diminished the measure 6" I could give several reasons why I fix it at 6" The correction may be disputed, but it is of little or no consequence in the result, except when the true conjunction is determined separately from the beginning and end, and not then of much, except the eclipse be small Dr MASKELYNE has found the sun's diameter as put down in the *Nautical Almanac* 6" too much; that correction therefore to the diameter in the *Nautical Almanac* is here applied.

tance of the center of the sun from that of the moon; and as the apparent path of the moon makes an angle of only about 1°. 5'. 40" with the ecliptic, it may be considered as the moon's apparent latitude at the middle of the eclipse.

Apparent latitude - - -	5'. 40"
Parallax in latitude - - -	31. 57, 3
True latitude by observation - -	37. 37, 3
——— by the Tables - -	37. 43
Error of the Tables in latitude - - -	— 5, 7

To find the true Time of the Conjunction at Dublin Observatory.

Latitude of Dublin Observatory reduced = 53°. 9'.

Cos. 121°. 12',6 right ascen. med. cœli - -	9,71448
Cos. 53. 9 lat. red. - - - -	9,77795
Cos. arc I. = 108°. 6',5 - - - -	9,49243

Cot. 53°. 9' lat. red. - - - -	9,87474
Sin. 121. 12,6 right ascen. med. cœli - -	9,93211
Cot. arc II. = 57°. 20',5 - - - -	9,80685
Obliq. ecl. = 23. 27,7	
Sin. arc. III. = 33. 52, 8 - - - -	9,74620
Sin. arc I. = 108. 6, 5 - - - -	9,97793
Cos. *alt.* nonag. = 58°. 0',5 - - - -	9,72413

Cos. arc III. = 33°. 52′,8 - - - -	9,91920
Tan. arc I. = 108. 6, 5 - - - -	10,48545
Tan. *long.* nonag. = 111°. 29′,9 - - -	10,40465
Moon's longitude = 162. 6 (*Naut. Alm.*)	
☾'s dist. à nonag. = 50. 36, 1	

Moon's equatorial parallax - -	54′. 11″
Reduction - - - - -	−8,9
Moon's horizontal parallax - -	54. 2,1 = 3242″,1
Sun's horizontal parallax - - - - =	8, 6
Horizontal parallax of ☽ from ⊙ - - -	3233, 5

Moon's latitude by Tables - - -	33′. 36″ N.
Error of Tables - - - - - -	−5, 7
Moon's true latitude - - - -	33 30, 3

Log. 3233,5 - - - - - -	3,50967
Sin. 58°. 0′,5 alt. nonag. - - - -	9,92846
Cos. 33′. 30″,3 ☾'s true lat. - arith. comp.	0,00002
	3,43815
Sin. 50°. 36′,1 ☾'s dist. from nonag. - -	9,88804
Log. 2119″,3 = 35′. 19″,3 par. in long. nearly -	3,32619
50°. 36′,1	
Sin. - 51. 11,4 - - -	9,89166
	3,43815
Log. 2136″,8 = 35′.36″,8 par. in *longitude* -	3,32981

Log. 3233,5 - - - -	3,50967
Cos. 58°. 0',5 alt. nonag. - - -	9,72410
Log. 1713",1 = 28'. 33",1 first part par. in lat. -	3,23377
33. 30, 3 moon's true latitude.	
4. 57, 2 moon's apparent latitude nearly.	

Log. 3233,5 - - - - -	3,510
Sin. 58°. 0',5 alt. nonag. - - -	9,928
Sin. 4'. 57",2 ☾'s app. latitude nearly - -	7,159
Cos. $\overline{50^\circ.\ 36',1 + \frac{35'.\ 36'',8}{2}}$ - - -	9,799
Log. 2",5 second part of par. in latitude - -	0,396
28'. 33, 1 first part of par. in latitude.	
28. 30, 6 parallax in *latitude*.	

Moon's true latitude - - -	33'. 30",3
Parallax in latitude - - -	28. 30, 6
$ME = 299'',7 =$ - - - - -	4. 59, 7

The moon's semidiameter - - -	14'. 46"
Inflexion of light - - - - -	− 3
	14. 43
Augmen. for ☽'s alt. 32° - - -	+ 8,4
Moon's semidiameter - - - -	14. 51,4

The sun's semidiameter reduced - - -	15'. 53"
Moon's semidiameter - - - - -	14. 51,4
$SM = 1844'',4 =$ - - - - -	30. 44,4

	1844,4	
	299,7	
Sum	2144,1 - - - -	log. 3,33125
Diff.	1544,7 - - - -	log. 3,18884
		2)6,52009
Log. $SE=1819'',9=$	$30'.\ 19'',9$ - -	3,26004
Parallax in long. $=$	$35.\ 36,8$	
	$1^\circ.\ 5.\ 56,7$ true diff. long. ☉ and ☾.	

Hence, $27'.\ 12'',1 : 1^\circ.\ 5'.\ 56'',7 :: 1h. : 2h.\ 25'.\ 28''$ the interval from the beginning to the time of the true conjunction; consequently $21h.\ 6'.\ 47''$ (beg.) $+2h.\ 25'.\ 28''=23h.\ 32'.\ 15''$ for the time of the conjunction at the Observatory of Trinity College, Dublin.

To compute the same for Greenwich.

Beginning at - - - -	$21^h.\ 39'.\ 21''$ apparent time.
Sun's right ascension - - -	$10.\ 58.\ 3$
Right ascension med. cœli - -	$8.\ 37.\ 24=129^\circ.\ 21'.$

Latitude of Greenwich reduced $=51^\circ.\ 14',1$

Cos. $129^\circ.\ 21'$ right ascen. of med. cœli -	9,80213
Cos. $51.\ 14,1$ latitude reduced - -	9,79668
Cos. arc I. $=113^\circ.\ 23',5$ - - -	9,59881

Cot. 51°. 14′,1 latitude reduced	- -	9,90475
Sin. 129. 21 right ascen. of med. cœli	-	9,88834
Cot. arc II. = 58°. 9′,7	- - -	9,79309
Obliq. ecl. = 23. 27,7		
Sin. arc III. = 34. 42	- - -	9,75532
Sin. arc I. = 113. 23,5	- - -	9,96275
Cos. *alt.* nonag. = 58°. 30′	- - -	9,71807

Cos. arc III. = 34°. 42′	- - - -	9,91495
Tan. arc I. = 113. 23,5	- - - -	10,36394
Tan. *longitude* nonag. = 117°. 45′	- -	10,27889
Moon's longitude = 162. 10 (*Naut. Alm.*)		
Moon's dist. from non. 44. 25		

Moon's equatorial parallax - -	54′. 11″	
Reduction - - - - -	−7,1	
Moon's horizontal parallax - -	54. 3,9	=3243″,9
Sun's horizontal parallax - - - -		8, 6
Horizontal parallax ☽ from ☉ - - -		3235, 3

Moon's latitude by the Tables - - -	33′. 55″,9
Error of the Tables - - - - -	−5, 7
Moon's true latitude - - - -	33. 50, 2

Log. 3235, 3	- - - - - -	3,50991
Sin. 58°. 30′ alt nonag.	- - - -	9,93077
Cos. 33′. 50″ moon's true lat.	- - arith. comp.	0,00002
		3,44070
Sin. 44°. 25′ moon's dist. from nonag	- -	9,84502
Log. 1930″,7 = 32′. 10″,7 par. in long. nearly	-	3,28572
44°. 25		
Sin. - 44. 57. 10,7	- - - -	9,84912
		3,44070
Log. 1949″,1 = 32′. 29″,1 par. in *Longitude*	- -	3,28982

Log. 3235, 3	- - - - - -	3,50991
Cos. 58°. 30′ alt. nonag.	- - - -	9,71807
Cos. moon's apparent latitude	- arith. comp.	0,00000
Log. 1690″,4 = 28′. 10″,4 first part par. in lat.	-	3,22798
33. 50, moon's true latitude.		
5. 40 apparent latitude nearly.		

Log. 3235, 3	- - - - - -	3,510
Sin. 58°. 30′ alt. nonag.	- - - -	9,931
Sin. 5′. 40″ apparent latitude of moon	- -	7,217
Cos. $\overline{44°.\ 25' + \frac{32'.\ 29''}{2}}$	- - - - -	9,852
Log. 3″,2 second part of par. in latitude	- -	0,510
28′. 10, 4 first part of par. in latitude.		
28. 7, 2 parallax in *Latitude*.		

Moon's true latitude - - - - - -	33'. 50",2
Parallax in latitude - - - - - -	28. 7, 2
ME = 343" = - - - - - -	5. 43

The moon's semidiameter - - - -	14'. 46"
Inflexion of light - - - - - -	−3
	14. 43
Augmen. for ☽'s alt. 37° - - - - -	+9,5
Moon's semidiameter - - - - -	14. 52,5

The sun's semidiameter reduced - - -	15'. 53"
The moon's semidiameter - - - -	14. 52,5
SM = 1845",5 - - - - - -	30. 45

1845,5
343

Sum 2188,5 - - - - -	log. 3,34015
Diff. 1502,5 - - - - -	log. 3,17681
	2)6,51696
Log. *SE* = 1813"3 = 30'. 13",3 - - -	3,25848
Parallax in longitude = 32. 29, 1	
1°. 2. 42, 4 true diff. of long. ☉ and ☾.	

Hence, 27'. 12",1 : 1°. 2'. 42',4 :: 1 hour : 2*h*. 18'. 18",9 the interval from the beginning to the time of the true conjunction; consequently 21*h*. 39'. 21" (beg.) + 2*h*. 18'. 18",9 = 23*h*. 57'. 39",9 the time of the true conjunction at Greenwich.

Time of conjunction at Greenwich Observ.	$23^h. 57'. 39'',9$
——————— at Dublin Observatory	23. 32. 15
Difference of the meridians - - -	25 24, 9

To find the error of the Tables in longitude, we have

☉'s long. at time of conj. by MAYER's Tab.	$5^s. 13^\circ. 17'. 6''$
☽ 's long. ———————————	5. 13. 17. 48
Error of the lunar Tables in longitude, supposing the solar Tables to be accurate	+ 42

Dr. BRINKLEY observes, that in an occultation, or eclipse of the sun, when the calculation is made for the difference of longitudes to be deduced from the beginnings or endings at two places, it will be sufficient to use the equatorial parallax to the nearest second, and not to regard the inflexion and irradiation of light; but when the difference of longitudes is to be deduced from the beginning at one place and the ending at the other, these circumstances ought to be strictly attended to.

To find the Longitude by a Time-keeper.

756. Let the *Time-keeper* be well regulated, and set to the meridian of Greenwich; then if it neither gain nor lose, it will always show the time at Greenwich. Hence, to find the longitude of any other place, find the mean time, either by the sun's altitude or that of a fixed star by Art. 92, or 106. and observe, at the instant of taking the altitude, the time by the watch; and the difference of these times, converted into degrees, at the rate of 15° for an hour, gives the longitude from Greenwich. If, for example, the time by the watch when the altitude was taken, was 6*h*. 19′, and the mean time deduced from that altitude was 9*h*. 23′, the difference 3*h*. 4′ converted into degrees gives 46° the longitude of the place *east* from Greenwich, because the time at the place of observation is *forwarder* than that at Greenwich. Thus the longitude could be very readily determined, if you could depend upon the watch. But as a watch will always gain or lose, before it is sent out its gaining or losing every day for some time, a month for instance, is observed; this is called the *rate of going* of the watch, and from thence the *mean* rate of going is thus found.

757. Suppose, for instance, I examine the rate of a watch for 30 days, on

some of those days I find it has gained, and on some it has lost; add together all the quantities which it has gained, and suppose they amount to 17″; add together all the quantities which it has lost, and let the sum be 13″; then the difference 4″ is the *mean* rate of gaining for 30 days, which divided by 30 gives 0″,133 for a *mean daily rate* of gaining. Or you may get the mean daily rate thus. Take the *difference* between what the watch was too fast, or too slow on the first and last days of observation, if it be too fast or too slow on each day; but take the *sum*, if it be too fast on one day and too slow on the other, and divide by the number of days between the observations*. And to find the time at the place of trial at any future period by this watch, you must put down, at the end of the above trial how much the watch is too fast or too slow; then subtract from the time shown by the watch, 0″,133 × number of days from the end of the trial, being the quantity which it has gained according to the above mean rate of gaining, and you are then supposed to get the true time affected with the error at the end of the trial. This would be all the error if the watch had continued to gain according to the above rate; and although, from the different temperatures of the air to which the watch may be exposed, and from the imperfection of the workmanship, this cannot be expected, yet by taking it into consideration, the probable error of the time will be diminished. In watches which are under trial at the Royal Observatory at Greenwich, as candidates for the rewards offered by Parliament for the discovery of the longitude, this allowance of a mean rate to be applied in order to get the time, is not granted by the Act of Parliament, but it requires that the watch itself should go within the limits assigned; the Commissioners, however, are so indulgent, as to grant the application of a mean rate, which is undoubtedly favourable to the watches.

758. As the rate of going of a watch is subject to vary from so many circumstances, the observer, whenever he goes ashore and has sufficient time, should compare his watch for several days with the mean time deduced from the altitude of the sun or a star, by which he will be able to determine its rate of going. And whenever he comes to a place whose longitude is known, he may correct his watch and set it to Greenwich time. For instance, if he go to a place known to be 30° east longitude from Greenwich, his watch should be two hours slower than the time at that place. Find therefore the time at that place by the altitude of the sun or a fixed star, and correct it by the equation of time, and compare the time so found with the time by the watch when the altitude was taken, and if the watch be two hours slower than the time deduced from observation, it is right; if not, correct it by the difference, and it again gives Greenwich time.

* For further information on this subject, see Mr. WALES's *Method of finding the Longitude at Sea.*

759 In long voyages, unless you have sometimes the means of adjusting the watch to Greenwich time, its error will probably be very considerable, and consequently the longitude deduced from it will be subject to a proportional error. In short voyages a watch is undoubtedly very useful, and also in long ones, where you have the means of correcting it from time to time It serves to carry on the longitude from one known place to another, supposing the interval of time not to be very long, or to keep the longitude from that which is deduced from a lunar observation, till you can get another observation. Thus the watch may be rendered of great service in Navigation.

To find the Longitude by an Eclipse of the Moon, and of Jupiter's Satellites.

760. By an eclipse of the moon. This eclipse begins when the umbra of the earth first touches the moon, and it ends when it leaves the moon. Having the times calculated when the eclipse begins and ends at Greenwich, observe the times when it begins and ends at any other place, the difference of these times converted into degrees, gives the difference of longitudes. For as the phases of the moon in an eclipse happen at the same instant to every observer, the difference of the times at different places when any phase is observed will give the difference of the longitudes This would be a very ready and accurate method, if the time of the first and last contact could be accurately observed, but the darkness of the penumbra continues to increase till it comes to the umbra, so that until the umbra actually gets upon the moon, it is not discovered. The umbra itself is also very badly defined The beginning and end of a lunar eclipse cannot, in general, be determined nearer than 1′ of time, and very often not nearer than 2′ or 3′ Upon these accounts, the longitude, from the observed beginning and end of an eclipse, is subject to a considerable degree of uncertainty. Astronomers therefore determine the difference of the longitudes of two places by corresponding observations of other phases, that is, when the umbra bisects any of the spots upon the moon's surface. And this can be determined with a greater degree of accuracy than the beginning and end, because, when the umbra is gotten upon the moon's surface, the observer has leisure to consider and fix upon the proper line of termination, in which he will be assisted by running his eye along the circumference of the umbra. Thus the coincidence of the umbra with the spots may be observed to a tolerable accuracy The observer therefore should have a good map of the moon at hand, that he may not mistake The telescope to observe a lunar eclipse should have but a small magnifying power with a great deal of light. The shadow comes upon the moon on the east side, and goes off on the west, but if the telescope invert, the appearance will be contrary.

761. The eclipses of *Jupiter's* satellites afford the readiest method of determining the longitude of places at land. It was also hoped that some method might be invented to observe them at sea, and Mr. IRWIN made a chair to swing for that purpose, for the observer to sit in; but Dr. MASKELYNE, in a voyage to Barbadoes, under the direction of the Commissioners of longitude, found it totally impracticable to derive any advantage from it; and he observes that "considering the great power requisite in a telescope for making these observations well, and the violence as well as irregularities of the motion of a ship, I am afraid the complete management of a telescope on ship-board will always remain among the desiderata. However, I would not be understood to mean to discourage any attempt, founded upon good principles, to get over this difficulty." The telescopes proper for making these observations are common refracting ones from 15 to 20 feet, reflecting ones of 18 inches or 2 feet, or the 46 inch achromatic with three object glasses which were first made by Mr. DOLLOND. On account of the uncertainty of the theory of the satellites, the observer should be settled at his telescope a few minutes before the expected time of an immersion. And if the longitude of the place be also uncertain, he must look out proportionably sooner. Thus, if the longitude be uncertain to 2°, answering to eight minutes of time, he must begin to look out eight minutes sooner than is mentioned above. However, when he has observed one eclipse and found the error of the Tables, he may allow the same correction to the calculations of the Ephemeris for several months, which will advertise him very nearly of the time of expecting the eclipses of the same satellite, and dispense with his attending so long. Before the opposition of Jupiter to the sun, the immersions and emersions happen on the west side of Jupiter, and after opposition, on the east side; but if the telescope invert, the appearance will be the contrary. Before opposition, the immersions only of the first satellite are visible; and after opposition, the emersions only. The same is generally the case with respect to the second satellite; but both immersion and emersion are frequently observed in the two outer satellites. See Art. 456.

762. When the observer is waiting for an emersion, as soon as he suspects that he sees it, he should look at his watch and note the second, or begin to count the beats of the clock, till he is sure that it is the satellite, and then look at the clock and subtract the number of seconds which he has counted from the time then observed, and he will have the time of emersion. If Jupiter be 8° above the horizon and the sun as much below, an eclipse will be visible; this may be determined near enough by a common globe.

763. The immersion or emersion of a satellite being observed according to apparent time, the longitude of the place from Greenwich is found by taking the difference between that time and the time set down in the *Nautical Almanac*, which is calculated for apparent time.

Ex. Suppose the emersion of a satellite to have been observed at the Cape of Good Hope, May 9, 1767, at 10*h*. 46'. 45" apparent time; now the time in the *Nautical Almanac* is 9*h*. 33' 12", the difference of which times is 1*h*. 13'. 33" the longitude of the Cape *east* of Greenwich in time, or 18°. 23' 15".

764 But to find the longitude of a place from an observation of an eclipse of a satellite, it is better to compare it with an observation made under some well known meridian, than with the calculations of the Ephemeris, because of the imperfections of the theory, but where a corresponding observation cannot be obtained, find what correction the calculations of the Ephemeris require by the nearest observations to the given time that can be obtained; and this correction applied to the calculation of the given eclipse in the Ephemeris, renders it almost equivalent to an actual observation. The observer must be careful to regulate his clock or watch by apparent time, or at least to know the difference, this may be done either by equal altitudes of the sun or proper stars; or the latitude being known, from one altitude at a distance from the meridian, by the methods already explained.

765 In order the better to determine the difference of longitudes of two places from corresponding observations, the observers should be furnished with the same kind of telescopes. For at an immersion, as the satellite enters the shadow it grows fainter and fainter, till at last the quantity of light is so small that it becomes invisible even before it is wholly immersed in the shadow, the instant therefore that it becomes invisible will depend upon the quantity of light which the telescope receives, and its magnifying power. The instant therefore of the disappearance of a satellite will be later the better the telescope is, and the sooner it will appear at its emersion Now the immersion is the instant the satellite is wholly gotten into the shadow, and the emersion is the instant before it begins to emerge from the shadow, if therefore two telescopes show the disappearance or appearance of the satellite at the same distance of time from the immersion or emersion, the difference of the times will be the same as the difference of the true times of their immersions or emersions, and therefore will show the difference of longitudes accurately. But if the observed time at one place be compared with the computed time at another, then we must allow for the difference between the apparent and true time of immersion or emersion in order to get the true time where the observation was made to compare with the true time from computation at the other place. This difference may be found by observing an eclipse at any place whose longitude is known, and comparing it with the time by computation. Observers, therefore, should settle the difference accurately by the mean of a great number of observations thus compared with the computations, by which means the longitude will be ascertained to a much greater accuracy and certainty. After all this

precaution, however, the different states of the air at different times, and also the different states of the eye, will introduce a small degree of uncertainty; the latter case may perhaps, in a great measure, be obviated, if the observer will be careful to remove himself from all warmth and light for a little time before he makes the observation, that the eye may be reduced to a proper state; which precaution the observer should also attend to when he settles the difference between the apparent and true times of immersion and emersion. Perhaps also the difference arising from the different states of the air might, by proper observations, be ascertained to a considerable degree of accuracy; and as this method of determining the longitude is, of all others, the most ready, no means ought to be left untried to reduce it to the greatest certainty. For further directions, see Art. 460.

CHAP. XXIX.

ON THE USE OF THE GLOBES

Art 766. THERE are two Globes, one called *Terrestrial*, upon which the places on the earth are delineated, and the other called *Celestial*, upon which the principal fixed stars are put down in their proper places, and the figures of the constellations drawn. The terrestrial globe is a perfect map of the earth, representing accurately the relative situations of all the places upon its surface. The celestial globe serves to explain all the phænomena arising from the diurnal motion of the earth about its axis, and also the variation of seasons arising from its motion about the sun, only supposing the sun to move in the ecliptic instead of the earth, which will not alter any of the appearances. Fig. 186. represents the construction of each globe. *HR* is a flat circular frame of wood supported by semicircular pieces coming from the foot, the plane of which passes through the center of the globe. *PQpE* is a brass circle called the *Brazen Meridian*, it is supported at its lowest point upon a roller on which it turns in its own plane, and passes through the horizon *HR* in two groves cut for that purpose. The globe itself is supported within this circle by an axis *Pp* on which it turns; this axis passes through the brazen meridian at *P* and carries an index round with it over a circular plate *hc*, which is divided into hours, &c. *EQ* represents the equator, and *CL* the ecliptic; to each of which circles on the celestial globe secondaries are drawn to every 10 or 15 degrees, but on the terrestrial, they are drawn only to the equator. From *C* and *L* are drawn the two tropical circles; and on the terrestrial globe are drawn the parallels of latitude. There is also part of another circle *Za*, called a *Quadrant of Altitude*, which is occasionally fixed to the brazen meridian, it is a thin plate of brass, having a nut and a screw at one end to fasten it to the meridian in its zenith *Z*, and then the lower end is put between the globe and horizon, and can be turned round to any point, it is divided into degrees, &c. by which the altitude of any object above the horizon may be measured, and at the same time it refers the object to the horizon, by which its azimuth may be determined. From one point *E* of the brazen meridian corresponding to the equator, the degrees begin and are continued up to 90° at each pole *P*, *p*, but for the other semicircle of the meridian, the degrees begin at the poles and are continued to 90° at the equator. On the horizon, the degrees begin at the east and west points, and are continued both ways to 90°, or to

FIG. 186.

the north and south points. The points of the compass are also generally put upon the horizon; and on two other circles drawn thereon are put the signs of the zodiac, and the months and days corresponding to the sun's place, which serves as a calendar to show the place of the sun on any day; this however cannot be accurate, as the sun is not always in the same point of the ecliptic on the same day. The ecliptic and equator begin their degrees at one of their intersections, called *Aries*, which are continued the same way all round up to 360°, and the former is divided into and marked with the twelve signs; the equator is also divided from the same point into 24 hours, which is therefore sometimes made use of instead of the hour circle. Upon the foot of the globe there is often put a compass, by which the brazen meridian may be set north and south. In the *Phil. Trans.* 1789, Mr. SMEATON has given a description of an improved quadrant of altitude. Instead of a strip of thin flexible brass, he makes it of a more solid construction. It is fixed to a brass socket, and made to turn upon an upright steel spindle, fixed in the zenith, by which you measure altitudes and azimuths with as much accuracy as you do any other arcs. He approves of the common hour circle, and says, that one of four inches diameter may be divided into 720 distinguishable divisions, answering to two minutes of time; and if instead of a *Pointer*, an *Index Stroke* is used in the same plane with that of the divisions, half minutes may be easily distinguished. He therefore thinks the hour circle should rather be improved than omitted, as it is upon some globes.

ON THE USE OF THE TERRESTRIAL GLOBE.

To find the Latitude of a Place.

767. Bring the place under that semicircle of the brazen meridian where the divisions begin at the equator, and observe what degree it is under, and it is the latitude required.

To rectify the Globe for the Latitude of a Place.

768. Elevate the pole above the horizon till its altitude is equal to the latitude of the place, and it then stands right for the solution of all problems for that latitude.

To find the Longitude of a Place from any given Meridian.

769. Bring the place to the brazen meridian, and observe the point of the equator which lies under it, and the distance of that point from the point where the given meridian cuts the equator, is the longitude required.

Given the Latitude and Longitude of a Place, to find that Place.

770. Bring the given degree of longitude to the meridian, and then under the degree of latitude upon the meridian you have the place required.

When it is Noon at any Place A, *to find the Hour at any other Place* B.

771. Bring *A* to the meridian, and set the index to XII.; then turn the globe till *B* comes under the meridian, and the index will show the hour at *B*. If it be not noon at *A*, set the index to the hour, and proceed as before, and you get the corresponding hour at *B*.

To find the Distance of A *from* B.

772. Bring *A* to the meridian, and screw the quadrant of altitude over it, and carry it over *B*, and you get the number of degrees between *A* and *B*, which multiply by 69,2, the miles in one degree, and you have the distance.

To find the Bearing of B *from* A.

773. Rectify the globe for the latitude of *A*, and bring it to the meridian, and fix the quadrant of altitude to it, then direct the quadrant to *B*, and the point where it cuts the horizon shows the bearing required.

To find the Place A *to which the Sun is vertical at any Hour of the Day, at a given Place* B.

774. Find the sun's place in the ecliptic, and bring it to the meridian, and mark the declination; then bring *B* to the meridian, set the index to the given

hour, and turn the globe till the index comes to XII. at noon, and the place under the sun's declination upon the meridian is that required.

To find on any Day and Hour, the Places where the Sun is rising, setting, or on the Meridian; also, those places which are enlightened, and where the Twilight is beginning and ending.

775. Find (774) the place to which the sun is vertical at the given time, and bring the same to the meridian, and rectify the globe for a latitude equal to the sun's declination; then to all those places in the *western* semicircle of the horizon, the sun is *rising;* to those in the *eastern,* it is *setting;* and to those under the *meridian,* it is *noon.* Also, all the places above the horizon are enlightened; and the altitude of the sun above the horizon at any one place at that time, is equal to the distance of that place from the horizon, which may be measured by the quadrant of altitude. Lastly, in all those places 18° below the western horizon the twilight is just beginning in the morning, and in those on the eastern, it is just ending in the evening.

To find all the Places to which a Lunar Eclipse is visible at any instant.

776. Find the place to which the sun is vertical at the given time, and bring that place to the zenith, and the eclipse will be visible to all the hemisphere *under* the horizon, because the moon is opposite to the sun.

777. We cannot, by a globe only, determine the same for a *solar* eclipse, because that eclipse does not happen to the whole hemisphere of the earth next the sun, nor does it happen at the same time to those places where it is visible.

778. The inhabitants of the earth are distinguished by the different directions of their shadows. They who live in the torrid zone are called *Amphiscii,* because their shadows at noon are cast sometimes to the north and sometimes to the south. But when the sun is vertical to them at noon, they then cast no shadows, and are called *Ascii.* The inhabitants of the temperate zones are called *Heteroscii,* because they never cast their meridian shadows but one way. They who inhabit the frigid zones are called *Periscii,* because the sun is sometimes above their horizon for a day, or for a longer time even to six months, so that their shadows turn all round them.

779. The inhabitants of the earth have also been distinguished into three sorts, in respect to their relative situations in latitude or longitude. They who live under opposite points of the same parallel to the equator are called, in respect to each other, *Periœci.* These have the same seasons at the same time,

only they differ 12 hours in time, it being midnight to one when it is noon to the other. They who live in the same meridian, but on opposite sides of the equator and equidistant from it, are called *Antœci*. These have day and night at the same time, the hours being the same, but they have different seasons, it being summer with one when it is winter with the other. They who live in opposite parallels to the equator, and in opposite meridians, or who live on opposite points of the globe, are called *Antipodes*. With these, it is day to one when it is night to the other, and summer to one when it is winter to the other.

ON THE USE OF THE CELESTIAL GLOBE.

To find the Sun's Right Ascension and Declination.

780 Bring the sun's place in the ecliptic to the meridian, and it points out upon the meridian, the declination; and the degree of the equator which is cut by the meridian is the right ascension.

Given the Right Ascension and Declination of an heavenly Body, to find its Place.

781. Bring the degree of right ascension on the equator to the meridian, and the point corresponding to the declination, is the place required.

Given the Latitude of the Place, the Day and Hour, to find the Altitude and Amplitude of the heavenly Bodies.

782. Rectify the globe (768) to the latitude of the place, and bring the sun's place in the ecliptic to the meridian, and set the index to XII.; then turn the globe till the index points to the given hour, and in that position the globe represents the proper situation of all the heavenly bodies upon it, in respect to the meridian and horizon. Then fix the quadrant of altitude to the zenith, and direct its graduated edge to the place of the body, and it shows the altitude of the body, and the degree where it cuts the horizon shows its amplitude. If the body be the moon or a planet, after having found its place, put a very small patch upon it to denote its place.

Given as before, to set the Globe so that the Stars upon it may correspond to their Situations in the Heavens.

783. The globe being fixed as in the last Article, let the meridian be set in the meridian of the place, with the north pole to the north; then will all the stars upon the globe correspond to their places in the heavens, so that an eye at the center of the globe would refer every star upon its surface to the place of the star in the heavens. By comparing therefore the stars in the heavens with their places on the globe, you will very easily get acquainted with all the stars.

To find the Time when any of the heavenly Bodies rise, set, or come to the Meridian; also their Azimuth at rising or setting.

784. Every thing remaining as in Art. 782, turn the globe till the given body comes to the eastern part of the horizon, and the index shows the time of its rising; bring it to the meridian, and the index shows the time of its coming to the meridian; lastly, bring the body to the western horizon, and the index shows the time of its setting. When the body is in the horizon, the arc upon the horizon between it and the north or south will give its azimuth. If you thus find the time of the sun's rising and setting, you get the length of the day. If you turn the globe about its axis, all those stars which do not descend below the horizon in a revolution, never set in that place; and those which do not come above it, never rise.

To explain, in general, the Alteration of the Length of the Days, and Difference of the Seasons.

785. Put several patches upon the ecliptic from Aries both ways to the two tropics; and then the globe being rectified to the latitude of the place, turn it about, and you will see, for north latitudes, that as the patches approach the tropic of Cancer, the corresponding diurnal arcs will increase; and as the patches approach the tropic of Capricorn, the corresponding diurnal arcs will decrease; also, the former arcs are greater than a semicircle, and the latter less; and the patch in the equator will describe a semicircle above the horizon. Therefore when the sun is in the equator, days and nights are equal; as he advances towards the tropic of Cancer, the days increase and the nights decrease, till he comes to that tropic, where the days are longest and the nights shortest; then as he approaches the equator, the lengths of the days diminish and those of the nights increase, and when he comes to the equator, there will be again equal

days and nights. Then as he advances towards the tropic of Capricorn, the days diminish and the nights increase, until he comes to that tropic, where the days are shortest and the nights are longest, and then as he approaches the equator, the days increase and the nights decrease, and when he comes to the equator, the length of the days and nights are equal. Whatever be the latitude of the place, when the sun is in the equator the days and nights are equal in length. To an inhabitant at the pole, the sun will appear to be half a year above the horizon, and half a year below. To an inhabitant at the equator, the days and nights will appear to be always equal, also, all the heavenly bodies will be found to be as long above the horizon as below. At the arctic circle, the longest day will be found 24 hours, and the longest night of the same length; this appears, by rectifying the globe to that latitude, and putting the patch, first at the tropic of Cancer, and then of Capricorn. Lastly, it will be found that all places enjoy equally the sun in respect to time, and are equally deprived of it, the length of the days at one time of the year being exactly equal to that of the nights at the opposite season. This will appear, by putting a patch upon the ecliptic at equal distances on each side of the equator.

To find the Latitude and Longitude of a given Star, also, the Distance of two Stars.

786. Bring the solstitial colure to the meridian, and fix the quadrant of altitude over the pole of the ecliptic; then turn the quadrant over the given star, and the arc contained between the star and the ecliptic will be the *latitude*, and the degree on the ecliptic cut by it will be the *longitude*. The distance of two stars may be found, by laying the quadrant of altitude over both, and counting the degrees between.

To explain the Phænomena of the Harvest Moon.

787. Rectify the globe for any northern latitude, for instance, that of London, and as the moon's orbit makes but a small angle with the ecliptic, let us suppose the ecliptic to represent the moon's orbit. Now in September, when the sun is in the beginning of Libra, the moon, at its full, is in the beginning of Aries; and as the mean motion of the moon in its orbit is about 13° in a day, put a patch on the first point of Aries, and another at the distance of 13° from it, bring the former patch to the horizon, and then turn the globe till the other comes to it, and the motion of the index will show about 17 minutes, which is the difference of the times of rising on two successive nights at that time.

This small difference arises from the small angle which that point of the ecliptic, or moon's orbit, makes with the horizon at its rising. If you continue the patches at every 13° till you come to Libra, you will find the difference of the times of rising will increase up to that point, and there the difference will be found to be about 1*h.* 17′; for this point of the ecliptic makes the greatest angle with the horizon at its rising. Hence, whenever the moon comes to the first point of Aries, there will be the least difference of the times of its rising; and this happens at the time of the full moon, when the full moon happens about September 21. That point of the ecliptic which rises at the least angle with the horizon will appear to set at the greatest, and therefore when there is the least difference in the times of rising, there will be the greatest difference in the times of setting; and the contrary.

To find the Time of the Year when a Star rises or sets Cosmically *and* Achronically.

788. Rectify the globe to the latitude of the place, and bring the star to the horizon on the east side, and see what degree of the ecliptic cuts the horizon, and upon the horizon seek what day of the month that degree answers to, and that is the day when the star rises *cosmically;* bring the star to the western horizon, and the degree of the ecliptic cut by the horizon, will give the day when it sets *cosmically.* Bring the star to the eastern horizon, and the degree of the ecliptic which cuts the western horizon will give the day when the star rises *achronically;* and if you bring the star to the western horizon, the degree of the ecliptic cut by the eastern horizon shows the day when the star sets *achronically.*

To find the Time of the Year when a Star rise or sets Heliacally.

789. Having rectified the globe to the latitude of the place, bring the star to the eastern horizon, and apply the quadrant of altitude to the western side, so that it may cut the ecliptic 12° above the horizon, then will the opposite point of the ecliptic be 12° below the horizon, and the day corresponding to that point is the day when the star rises *heliacaly;* bring the star to the western horizon, and apply the quadrant of altitude to the eastern to cut the ecliptic 12° above the horizon, and the opposite point will give the day when the star sets *heliacally.* This is for a star of the first magnitude, which may be seen when the sun is about 12° below the horizon; but for one of the second, third, fourth, fifth, or sixth magnitude, the sun must be 13°, 14°, 15°, 16° or 17° below the horizon.